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# Exploring Inflation Options for Warm Dark Matter Coupled to the Higgs Boson

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## Abstract

We extend the Standard Model with a scalar warm dark matter field $S$ with an interaction $-\frac{1}{2}\lambda_{hS}\left(\phi^\dagger\phi\right)S^2$ with the Higgs boson $\phi$. This warm dark matter scenario is in agreement with cosmological observations if $S$ and $\phi$ come into thermal and diffusive equilibrium before the temperature drops below the Higgs boson mass $m_H$. We study inflation driven by the fields $\phi$ or $S$, and also study preheating and reheating, in order to constrain the parameters of this extension of the Standard Model. It is remarkable that, with the current data, these models pass a closure test with no free parameters.

## Keywords

Warm Dark Matter, Inflation, Preheating, Reheating

## 1. Introduction

The present study is motivated by two recent observations. Firstly, the precise measurements of the Higgs boson mass $m_H$ at the Large Hadron Collider (LHC), and of the top quark mass $m_t$ at the Tevatron and LHC, within their $3\sigma$ uncertainties, allow the Standard Model to be valid all the way up to the Planck energy scale, without a compelling need for new physics beyond the Standard Model at high energies [1] [2] [3]. And secondly, recent measurements of the dark matter particle comoving root-mean-square velocity $v_{hrms}(1) = a v_{hrms}(a)$ happen to agree with the prediction of the "no freeze-in and no freeze-out" scenario of spin zero dark matter $S$ that decouples early on from the Standard Model sector, e.g. scalar dark matter coupled to the Higgs boson [4]. $a(t)$ is the expansion parameter of the universe normalized to $a(t_0)=1$ at the present time $t_0$. Rotation curves of dwarf galaxies obtain $v_{hrms}(1) = 406 \pm 69\ \text{m/s}$ [5]. Galaxy rest-frame ultraviolet (UV) luminosity distributions, and stellar mass distributions, at redshifts $z = 4, 6, 8$ and 10, obtain $v_{hrms}(1) = 410^{+140}_{-120}$ m/s [6]. The predictions of the "no freeze-in and no freeze-out" scenario, for dark matter $S$ coupled to the Higgs boson, with the measured dark matter density, are $v_{hrms}(1) = 490 \pm 5$ m/s, 842 ± 13 m/s, and 1471 ± 19 m/s, for spin 0, $\frac{1}{2}$, and 1 dark matter particles, respectively [4]. Note that spin $\frac{1}{2}$ and spin 1 particles $S$ are disfavored by the measurements. $S$ has a predicted mass $M_S = 150 \pm 2$ eV [4]. If $S$ is real, then $S$ is the dark matter particle.[1] If $S$ is complex, it may decay to two massive gauge bosons $V$ which are the dark matter in this case [10]. Additional observables studied, that obtain consistent results, are the redshift of first galaxies [11], the ultra-violet luminosity of first galaxies [6], and the related "reionization optical depth" [6]. For a summary of measurements see [6].

For these reasons, in the present study we explore inflation options of the Standard Model extended with a scalar warm dark matter field $S$ with a contact coupling $-\frac{1}{2}\lambda_{hS}(\phi^\dagger\phi)S^2$ to the Higgs boson $\phi$.

In Section 2 we take a journey towards the past, based on current observations and on the assumption of the validity of the Standard Model up to high energies. In Sections 3 to 7 we take a journey towards the future, starting from an assumed early stage of inflation of the universe driven by either $S$ or $\phi$, and the assumption that quantum fluctuations during inflation are the seeds of galaxies and of the observed large-scale power spectrum of density perturbations. The meeting, or not, of the two journeys provides a consistency test of the preceding assumptions. It is quite amazing, and non-trivial, that current data and the listed assumptions do pass the consistency test *with no adjustable parameters*! A summary of results is presented in the concluding section.

## 2. The Expansion History of the Universe

A summary of the expansion history of the universe is presented in **Figure 1**. The horizontal axis $x \equiv M_P r/a$ is the comoving distance in Planck units, and the vertical axis $y = a$ is the expansion parameter $a$, both with natural logarithmic scales. $M_P \equiv \sqrt{1/G_N} \approx 1.2 \times 10^{19}$ GeV is the Planck mass (we adopt the definition of [1]). The two vertical lines correspond to two particles fixed in space: the vertical axis is the observer, *i.e.* us, and the red line is a point fixed in space at the reference comoving distance $1/k = \text{Mpc}/0.05 = \exp(134.9)/M_P$. The comoving distance to the horizon relative to us, indicated by the blue line, is $M_P/(aH)$, where the Hubble parameter $H$ is the relative expansion rate. The

[1] "Thermal relic" warm dark matter with mass $\gg 150$ eV freezes-out and decouples from the Standard Model sector while non-relativistic (to not exceed the measured dark matter density). Dark matter $S$ is colder than this "thermal relic" by a factor $< T_S/T_\gamma = 0.345$. Hence, to compare $M_S$ with limits that can be found in the literature on the "thermal relic" dark matter mass it is necessary to multiply the latter by a factor $<0.345^{1.11} = 0.31$. In addition to this power spectrum free-streaming cut-off $k_{fs}$ correction, there is the "velocity dispersion correction", and the non-linear regeneration of small scale structure [6] [7] [8] [9].

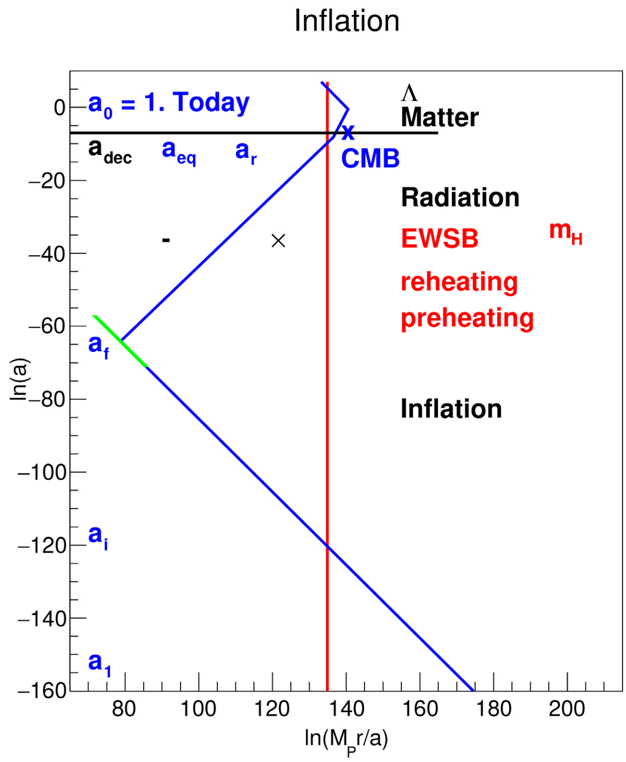


**Figure 1.** Past and future history of the universe. Vertical axis: natural logarithm of the expansion parameter $a$, *i.e.* "e-folds". Horizontal axis: natural logarithm of comoving distances $M_P r/a$ in Planck units. The red line corresponds to a fixed point in space at a reference comoving distance $1/k = \text{Mpc}/0.05 = \exp(134.9)/M_P$. The blue line is the comoving distance to the horizon $M_P/(aH)$ in Planck units. The thin green box (with a width too small to be resolved on this scale) is the *prediction* $\left(M_P/(a_f H_f), a_f\right)$ for the end of inflation with the observed values of $P_\zeta(k)$ and $n_s$, at 68% confidence (from Table 2), *with no free parameter*! The Cosmic Microwave Background radiation (CMB) that we see today propagates to us from point "X". Symbols "−" and "×" illustrate segments of "light" rays (or should we say "dark" rays?) inside and outside of the horizon (see text).

expansion parameter $a(t)$, in a spatially flat and homogeneous universe, evolves according to the Friedmann equation [1]:

$$H^2 \equiv \left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 = \frac{8\pi}{3M_P^2}\rho \equiv H_0^2 \frac{\rho}{\rho_{\text{crit}}}. \tag{1}$$

where $\rho_{\text{crit}} \equiv 3H_0^2/(8\pi G_N)$ is the critical density. Today, $H(t_0) \equiv H_0$. The density has contributions from radiation, and matter, and we have included the cosmological constant $\Lambda$ (or vaccuum energy) contribution into $\rho$: $\rho_{\text{crit}}\Omega_\Lambda = \rho_{\text{crit}}\Lambda/(3H_0^2)$:

$$\rho \equiv \rho_{\text{crit}}\left(\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda\right). \tag{2}$$

It is handy to have the solutions to (1) when one term in $\rho$ dominates, see Table 1.

**Table 1.** $t$ or $a$ dependence of some cosmological quantities when only one type of density dominates. $1/H$ is the proper distance to the horizon. $1/(aH)$ is the comoving distance to the horizon. The proper distance to a "galaxy", *i.e.* a fixed point in space, is $\propto a$. The comoving distance to a "galaxy" is a constant. See **Figure 1**.

| | $\rho(a)$ | $a(t)$ | $H(t)$ | $1/(aH)$ | $w \equiv p/\rho$ |
|---|---|---|---|---|---|
| $\Lambda$ | constant | $e^{Ht}$ | constant | $a^{-1}$ | $-1$ |
| Radiation | $a^{-4}$ | $t^{1/2}$ | $1/(2t)$ | $a$ | 1/3 |
| Matter | $a^{-3}$ | $t^{2/3}$ | $2/(3t)$ | $a^{1/2}$ | 0 |

The blue straight lines in **Figure 1** can be understood by noting that the comoving distance to the horizon $1/(aH)$ is proportional to $a^{-1}$, $a$, $a^{1/2}$, and $a^{-1}$, for $\rho$ dominated, respectively, by vacuum energy during inflation until $a = a_f$, radiation until $a_{\text{eq}}$, matter until $a_\Lambda$, and the cosmological constant (extended to the future to be seen on the scale of the figure). (There are intervals of $a$ in the range $a_f < a < a_r$ with $\rho \propto a^{-3}$ if the inflaton field has a dominating quadratic potential, but this will not be the case in the present scenario.)

Note that a point in space at the reference comoving distance $1/k$ exits the horizon at $a_i$, and re-enters the horizon at $a_r$, *i.e.*

$$k = a_i H_i = a_r H_r. \tag{3}$$

Radiation decouples from matter at $a_{\text{dec}}$ as indicated by the black horizontal line in **Figure 1**. The Cosmic Microwave Background Radiation that we observe today propagates freely since $t_{\text{dec}}$ from the "CMB" point marked with an X in **Figure 1**.

The slope of "light" rays is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\pm(rH)^{-1} \pm 1}, \tag{4}$$

so $\mathrm{d}y/\mathrm{d}x \approx 0$ if $r$ is inside the horizon, while $\mathrm{d}y/\mathrm{d}x \approx \pm 1$ if $r$ is outside the horizon. Segments of light rays are shown with symbols "–" and "×" in **Figure 1**.

The Einstein equation of General Relativity obtains (1), and also the energy conservation equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + 3H(\rho + p) = 0, \tag{5}$$

where $p$ is the pressure:

$$p = \rho_{\text{crit}}\left(\frac{\Omega_r}{3a^4} + 0 - \Omega_\Lambda\right). \tag{6}$$

Let us now obtain the temperature in the range $m_t < T(a) < T_{\text{reh}}$, where $T_{reh}$ is the reheating temperature when the Standard Model particles and $S$ obtain thermal equilibrium. From conservation of entropy

$$g_{\text{reh}} a^3 T^3 = g_\gamma a_0^3 T_0^3 + g_\nu a_0^3 T_{\nu 0}^3 + g_S a_0^3 T_{S0}^3, \tag{7}$$

where $T_0 = 2.7255\ \text{K}$ is the present photon temperature [1], and $g \equiv N_b + \frac{7}{8} N_f$

for each particle species, with $N_b$ or $N_f$ the number of spin states for bosons or fermions respectively. We have neglected gauge singlet neutrinos with both Yukawa and Majorana mass terms (needed to give the observed mass and mixing to neutrinos, and for baryogenesis via leptogenesis [12]). $g_\gamma = 2$ for the photon, $g_\nu = 7/8 \cdot 3 \cdot 2$ for three left-handed neutrinos plus right-handed anti-neutrinos, and $g_S = 1$ for the scalar warm dark matter $S$, $T_{\nu 0} = (4/11)^{1/3} T_0$, and $T_{S0} = \left(43/(11 \cdot 95.25)\right)^{1/3} T_0$. $g_{\text{reh}}$ is the effective number of degrees of freedom at the end of the reheating phase. For the present model with warm dark matter, $g_{\text{reh}} = 106.75 + 1$. So,

$$\frac{a}{a_0} = 0.332 \frac{T_0}{T}. \tag{8}$$

The corresponding density is

$$\rho = \frac{\pi^2}{30} g_{\text{reh}} T^4. \tag{9}$$

## 3. Inflation with a Quadratic Potential

In this Section we take $S$ to be the field that drives inflation. Our metric for flat space is

$$\mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2 \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right). \tag{10}$$

Greek indices go from 0 to 3, while latin indices go from 1 to 3. The Lagrangian, wave equation, energy-momentum tensor, energy density, and pressure of a real scalar field $S$, are [1]

$$\mathcal{L} = \frac{1}{2} \partial_\mu S \cdot \partial^\mu S - V(S), \tag{11}$$

$$0 = \frac{\partial \mathcal{L}}{\partial S} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu S)} = -\frac{\mathrm{d}V(S)}{\mathrm{d}S} - \partial_\mu \partial^\mu S, \tag{12}$$

$$T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu S)} \partial_\nu S - g^\mu{}_\nu \mathcal{L} = \partial^\mu S \cdot \partial_\nu S - g^\mu{}_\nu \mathcal{L} \tag{13}$$

$$T^0{}_0 = \rho = \frac{1}{2} \dot{S}^2 + V(S), \tag{14}$$

$$-T^i{}_j = p \delta^i_j = \left[\frac{1}{2} \dot{S}^2 - V(S)\right] \delta^i_j. \tag{15}$$

We have neglected spatial derivatives in these last two equations. $\rho$ and $p$ are the density and pressure of the homogeneous fluid in its rest frame. From (5), (10) and (14) we obtain

$$\ddot{S} + (3H + \Gamma) \dot{S} = -\frac{\mathrm{d}V}{\mathrm{d}S}. \tag{16}$$

Dots are derivatives with respect to $t$. In this equation we have included the total decay rate $\Gamma$ of $S$ to discuss reheating in Sections 6 and 7. It is convenient to define the "slow-roll" parameters:

$$\varepsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{\mathrm{d}V/\mathrm{d}S}{V}\right)^2 \quad \text{and} \quad \eta \equiv \frac{M_P^2}{8\pi}\frac{\mathrm{d}^2V/\mathrm{d}S^2}{V}. \tag{17}$$

Initial conditions for $S(t_1) \equiv S_1$ and $a(t_1) \equiv a_1$ are such that $V(S_1) \gg \dot{S}^2/2$ at $t_1$. While $\ddot{S}$ can be neglected in (16) we are in the "slow-roll" regime. Necessary conditions for the slow-roll approximation to be valid are $\varepsilon \ll 1$ and $\eta \ll 1$. In this approximation $\varepsilon \approx -\dot{H}/H^2$. Note that inflation ends, *i.e.* $\ddot{a} = 0$, when $-\dot{H}/H^2 = 1$.

In the slow-roll approximation, $H$, $\rho$, and $p$ are approximately constant, and $a(t) \propto \mathrm{e}^{Ht}$ grows approximately exponentially with time. In the slow-roll approximation, the number of "e-folds" of growth of $a(t)$ until $\varepsilon = 1$, is

$$N \equiv \ln\left(\frac{a_f}{a_i}\right) = \int_{t_i}^{t_f} H\mathrm{d}t = -\frac{2\sqrt{\pi}}{M_P}\int_{S_i}^{S_f}\frac{\mathrm{d}S}{\sqrt{\varepsilon}}. \tag{18}$$

The dimensionless power spectrum[2] of density fluctuations at horizon re-entry is [1] [13] [14]

$$P_\zeta(k) = \frac{H_i^2}{\pi\varepsilon_i M_P^2} \approx \frac{1}{24\pi^2}\frac{V_i}{\varepsilon_i}\left(\frac{8\pi}{M_P^2}\right)^2, \tag{19}$$

and the tensor gravitational wave power spectrum at horizon re-entry is

$$P_t(k) = \left(\frac{H_i}{2\pi}\right)^2 \frac{64\pi}{M_P^2} \approx \frac{2}{3\pi^2}V_i\left(\frac{8\pi}{M_P^2}\right)^2. \tag{20}$$

Note that $H_i$ and $\varepsilon_i$ correspond to $a_i$. The power spectrum $P_\zeta(k)$ at the reference wavenumber $k_{\mathrm{ref}} = 0.05\,\mathrm{Mpc}^{-1}$ is measured to be $\exp(3.044 \pm 0.014)\times 10^{-10} \approx 2.1\times 10^{-9}$ [1]. The tensor-to-scalar ratio for single-field slow-roll inflation is [1]

$$r \equiv \frac{P_t}{P_\zeta} = 16\varepsilon_i \tag{21}$$

A "pedestrian" derivation of (19) is presented in **Appendix A**.

Let us now consider the quadratic potential

$$V(S) = \frac{1}{2}\bar{m}_S^2 S^2. \tag{22}$$

The slow-roll parameters are

$$\varepsilon = \eta = \frac{1}{4\pi}\left(\frac{M_P}{S}\right)^2. \tag{23}$$

The solution of the evolution equations in the slow-roll approximation are

$$S = S_i - \frac{\bar{m}_S M_P}{\sqrt{12\pi}}(t - t_i), \tag{24}$$

[2]The dimensionless power specturm $P(k)$ is defined as follows [1]. $\delta(\boldsymbol{x}) \equiv (\rho(\boldsymbol{x}) - \langle\rho\rangle)/\langle\rho\rangle = \sum \delta_{\boldsymbol{k}} \exp(-i\boldsymbol{k}\cdot\boldsymbol{x})$ (with periodic boundary conditions in a box of volume $L^3$). The density variance is then $\langle\delta^2\rangle = \sum|\delta_{\boldsymbol{k}}|^2 = \int \frac{4\pi k^2 \mathrm{d}k}{(2\pi/L)^3}|\delta_{\boldsymbol{k}}|^2 \equiv \int \mathrm{d}\ln(k)P(k)$, so the dimensionless power spectrum is $P(k) \equiv \frac{k^3}{2\pi^2}L^3|\delta_{\boldsymbol{k}}|^2$. In [13], $P(k)$ is written as $\Delta_R^2(k)$.

$$a(t) = a_i \exp\left(\sqrt{\frac{4\pi}{3}} \frac{\bar{m}_S}{M_P} \int_{t_i}^{t} S\mathrm{d}t\right). \tag{25}$$

End of inflation, *i.e.* $\varepsilon = 1$, corresponds to $S_f = M_P / \sqrt{4\pi}$. The number of e-folds of inflation is:

$$N \equiv \ln\left(\frac{a_f}{a_i}\right) = 2\pi \frac{S_i^2 - S_f^2}{M_P^2}. \tag{26}$$

The power spectrum of density fluctuations, and the spectral tilt, are

$$P_\zeta = \frac{16\pi}{3}\left(\frac{\bar{m}_S}{M_P}\right)^2 \left(\frac{S_i}{M_P}\right)^4, \tag{27}$$

$$n_s - 1 \equiv \frac{\mathrm{d}\ln P_\zeta}{\mathrm{d}\ln k} \approx -6\varepsilon_i + 2\eta_i \approx -\frac{2}{N}. \tag{28}$$

From the measured $n_s \approx 0.965$, we obtain $N \approx 57.1$ and $S_i \approx 3.0 M_P$, and from the measured $P_\zeta \approx 2.1 \times 10^{-9}$, we obtain $\bar{m}_S \approx 1.2 \times 10^{-6} M_P$. In conclusion, a quadratic potential is appropriate for cold dark matter, not for the warm dark matter studied in this article. Also, preheating with a quadratic potential requires $M_S \gg m_H$, *i.e.* cold dark matter. We therefore turn to a study of inflation with a quartic potential.

## 4. Inflation with a Quartic Potential

To the Standard Model we add a warm dark matter real scalar field $S$, with $Z_2$ symmetry $S \leftrightarrow -S$, with a contact coupling $\lambda_{hS}$ to the Higgs boson field $\phi$. (An alternative is complex $S$ with a Lagrangian with $U(1)$ symmetry that is, or is not, broken by the $S$ and $\phi$ ground state [10]. Complex $S$ only requires minimal changes of notation, e.g. $S^2 \to S^* S$.) The action is

$$\mathcal{S}_J = \int \mathrm{d}^4 x \sqrt{-g}\left\{\mathcal{L}_{\mathrm{grav}} + \mathcal{L}_S + \mathcal{L}_{S\phi} + \mathcal{L}_{SM}\right\}, \tag{29}$$

$$\mathcal{L}_{\mathrm{grav}} = -\left(\frac{M_P^2}{2(8\pi)} + \xi_h \phi^\dagger \phi + \xi_S S^2\right) R, \tag{30}$$

$$\mathcal{L}_S = +\frac{1}{2}\partial_\mu S \cdot \partial^\mu S - \frac{1}{2}\bar{m}_S^2 S^2 - \frac{\lambda_S}{4} S^4, \tag{31}$$

$$\mathcal{L}_{S\phi} = -\frac{1}{2}\lambda_{hS}\left(\phi^\dagger \phi\right) S^2. \tag{32}$$

Note that the Higgs field $\phi$ and dark matter field $S$ have non-minimal couplings $\xi_h$ and $\xi_S$ to the curvature scalar $R$. These terms are needed for inflation (see **Table 2** and **Table 3**) and also for renormalization [15]. The Higgs part of the Standard Model Lagrangian $\mathcal{L}_{SM}$, in the unitary gauge with

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_h + h \end{pmatrix}, \tag{33}$$

is

$$\mathcal{L}_h = +\frac{1}{2}\partial_\mu h \cdot \partial^\mu h - \frac{\lambda_h}{4} h^4 - \lambda_h v_h h^3 - \lambda_h v_h^2 h^2. \tag{34}$$

**Table 2.** Parameters of the metric version of general relativity, obtained from $n_s = 0.965 \pm 0.004$, $P_\zeta(k_{\text{ref}}) \equiv \Delta_R^2 = \exp(3.044 \pm 0.014) \times 10^{-10}$ [1], and $k = 0.05\ \text{Mpc}^{-1}$, and Equations (42) to (49). The Higgs boson $h$ is assumed to drive inflation. For $S$ driving inflation replace $h \to S$. Uncertainties have 68% confidence, and are correlated.

| | Metric |
|---|---|
| $N \equiv \ln(a_f/a_i)$ | 57.1 ± 6.6 |
| $r$ | $(3.7 \pm 0.9) \times 10^{-3}$ |
| $\sqrt{\xi_h} h_i$ | $(1.74 \pm 0.10) M_P$ |
| $\varepsilon_i$ | $(2.30 \pm 0.54) \times 10^{-4}$ |
| $\eta_i$ | $(-1.75 \pm 0.21) \times 10^{-2}$ |
| $\lambda_h^{1/4} h_i$ | $(8.05 \pm 0.03) \times 10^{-3} M_P$ |
| $\lambda_h/\xi_h^2$ | $(4.6 \pm 1.1) \times 10^{-10}$ |
| $\rho_i$ | $(1.81 \pm 0.43) \times 10^{-13} M_P^4$ |
| $\rho_f$ | $(2.42 \pm 0.57) \times 10^{-14} M_P^4$ |
| $H_i$ | $(1.23 \pm 0.15) \times 10^{-6} M_P$ |
| $H_f$ | $(4.50 \pm 0.54) \times 10^{-7} M_P$ |
| $\ln(a_i)$ | −121.28 ± 0.12 |
| $\ln(a_f)$ | −64.1 ± 6.8 |
| $\ln(M_P/(a_i H_i))$ | 134.89 ref. |
| $\ln(M_P/(a_f H_f))$ | 78.8 ± 6.6 |

**Table 3.** Parameters of the Palatini version of general relativity, obtained from $n_s = 0.965 \pm 0.004$, $P_\zeta(k_{\text{ref}}) = \exp(3.044 \pm 0.014) \times 10^{-10}$ [1], and $k = 0.05\ \text{Mpc}^{-1}$, and Equations (42) to (49). Other parameters remain unconstrainted. The Higgs boson $h$ is assumed to drive inflation. For $S$ driving inflation replace $h \to S$. Uncertainties have 68% confidence, and are correlated.

| | Palatini |
|---|---|
| $N \equiv \ln(a_f/a_i)$ | 57.1 ± 6.6 |
| $h_i$ | $(4.26 \pm 0.25) M_P$ |
| $\eta_i$ | $(-1.75 \pm 0.21) \times 10^{-2}$ |
| $(\lambda_h/\xi_h)^{1/4} h_i$ | $(1.26 \pm 0.01) \times 10^{-2} M_P$ |
| $\lambda_h/\xi_h$ | $(7.6 \pm 1.8) \times 10^{-11}$ |

The dark matter particle mass is $M_S = \sqrt{\bar{m}_S^2 + \frac{1}{2}\lambda_{hS} v_h^2} = 150 \pm 2\ \text{eV}$ [4], with $\bar{m}_S^2$ assumed positive, while $\lambda_{hS}$ is negative. The Higgs boson mass is $m_H = \sqrt{2\lambda_h} v_h \approx 125\ \text{GeV}$. These masses play no significant role in inflation.

In this section we will assume that the field $h$ drives inflation, and therefore set $S = 0$ initially, and so neglect the term with $\xi_S$.

The Lagrangian can be brought to the "Einstein form", without the non-minimal couplings to $R$, with a transformation of the metric [15]

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \xi_h h^2 \frac{8\pi}{M_P^2}, \tag{35}$$

and a redefinition of the field. We distinguish two versions of General Relativity: the "metric" version, and the "Palatini" version. In the "Palatini" version of General Relativity the metric $g_{\mu\nu}$ and the affine connection $\Gamma^{\alpha}_{\beta\gamma}$ are treated as independent variables. The metric and Palatini versions differ if the non-minimal couplings to gravity, $\xi_h$ or $\xi_S$, are different from zero. In the metric version, the needed field redefinition and the transformed potential, for $\chi \gg \sqrt{6/(8\pi)} M_P$, are [1] [15]:

$$h \approx \frac{M_P}{\sqrt{8\pi\xi_h}} \exp\left(\frac{\chi\sqrt{8\pi}}{\sqrt{6}M_p}\right), \tag{36}$$

$$U(\chi) \approx \frac{\lambda_h}{4\xi_h^2}\left(\frac{M_P^2}{8\pi}\right)^2\left[1+\exp\left(-\frac{2\chi\sqrt{8\pi}}{\sqrt{6}M_P}\right)\right]^{-2} \tag{37}$$

In the Palatini version, the needed field redefinition and the transformed potential are [16]:

$$h = \frac{M_P}{\sqrt{8\pi\xi_h}} \sinh\left(\frac{\sqrt{8\pi\xi_h}\chi}{M_P}\right). \tag{38}$$

$$U(\chi) = \frac{\lambda_h}{4\xi_h^2}\left(\frac{M_P^2}{8\pi}\right)^2\left[\tanh\left(\frac{\sqrt{8\pi\xi_h}\chi}{M_P}\right)\right]^4. \tag{39}$$

The slow-roll parameters are defined as

$$\varepsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{\mathrm{d}U/\mathrm{d}\chi}{U}\right)^2 \approx -\frac{\mathrm{d}H/\mathrm{d}t}{H^2}, \tag{40}$$

$$\eta \equiv \frac{M_P^2}{8\pi}\frac{\mathrm{d}^2U/\mathrm{d}\chi^2}{U}. \tag{41}$$

In the slow-roll approximation, while $\varepsilon \ll 1$ and $\eta \ll 1$, in the metric [15] or Palatini [16] versions of General Relativity, we obtain

$$\varepsilon \approx \frac{4}{3\xi_h^2 h^4}\left(\frac{M_P^2}{8\pi}\right)^2 \quad \text{or} \quad \approx \frac{8}{\xi_h h^4}\left(\frac{M_P^2}{8\pi}\right)^2, \tag{42}$$

$$\eta \approx -\frac{4}{3\xi_h h^2}\left(\frac{M_P^2}{8\pi}\right) \quad \text{or} \quad \approx -\frac{8}{h^2}\frac{M_P^2}{8\pi}, \tag{43}$$

$$N \approx \frac{3}{4}\xi_h\left(h_i^2 - h_f^2\right)\left(\frac{8\pi}{M_P^2}\right) \quad \text{or} \quad \approx \frac{\pi}{M_P^2}\left(h_i^2 - h_f^2\right), \tag{44}$$

$$n_s - 1 \equiv \frac{\mathrm{d}\ln P_\zeta}{\mathrm{d}\ln k} \approx -\frac{2}{N} \quad \text{or} \quad \approx -\frac{2}{N}, \tag{45}$$

$$r \equiv \frac{P_t(k_{\text{ref}})}{P_\zeta(k_{\text{ref}})} \approx \frac{12}{N^2} \quad \text{or} \quad \approx \frac{2}{\xi_h N^2}, \tag{46}$$

$$\rho_i = U_i = \frac{\lambda_h}{4\xi_h^2}\left(\frac{M_P^2}{8\pi}\right)^2 \quad \text{or} \quad = \frac{\lambda_h}{4\xi_h^2}\left(\frac{M_P^2}{8\pi}\right)^2, \tag{47}$$

$$\rho_f \approx U_f \approx 0.134 U_i \quad \text{or} \quad \approx \left[\tanh\left(\sqrt{8\xi_h}\right)\right]^4 U_i, \tag{48}$$

$$P_\zeta\left(k_{\text{ref}}\right) = \frac{H_i^2}{\pi \varepsilon_i M_P^2} \approx \frac{\lambda_h}{2}\left(\frac{h_i}{M_P}\right)^4 \quad \text{or} \quad \approx \frac{\lambda_h}{12\xi_h}\left(\frac{h_i}{M_P}\right)^4. \tag{49}$$

Let us consider the metric version of General Relativity. Note that the measured $n_s$ obtains $N$, $r$, $\sqrt{\xi_h} h_i$, $\varepsilon_i$, and $\eta_i$, see **Table 2**. The measured $P_\zeta(k)$ obtains $\lambda_h^{1/4} h_i$. Together they determine $\lambda_h / \xi_h^2$, $\rho_i$, $\rho_f$, $H_i$, and $H_f$. Finally, $H_i$ and $k$ obtains $a_i$, and, with $N$, obtains $a_f$. The resulting $a_i$ and $N$, with their uncertainties, obtains the green closure box in **Figure 1**. Note that a different measured $P_\zeta(k_{\text{ref}})$ would shift the green box. Results for the Palatini version of General Relativity are listed in **Table 3**.

The preceding results, with the interchange $S \leftrightarrow h$, are also valid for inflation driven by $S$ instead of $\phi$. We note that $v_h$ and $M_S$ have no significant effect on inflation.

## 5. Comments on Higgs Inflation

The Higgs quartic coupling $\lambda_h$ runs from $\lambda_h(m_t) \approx 0.126$ at $m_t$ to

$$\begin{aligned} \lambda_h\left(M_P\right) = & -0.0143 - 0.0066\left(\frac{m_t}{\text{GeV}} - 173.34\right) \\ & + 0.0018\frac{\alpha_3\left(M_Z\right) - 0.1184}{0.0007} + 0.0029\left(\frac{m_H}{\text{GeV}} - 125.15\right), \end{aligned} \tag{50}$$

at $M_P$ [17]. Note that, within $3\sigma$ uncertainties, $\lambda_h(M_P)$ may be negative, zero or positive! The relation $\lambda_h(M_P) \approx 0$ is not a prediction of the Standard Model since it depends on input parameters to the model, e.g. $\lambda_h$, $v_h$ and $y_t$. Either $\lambda_h(M_P) \approx 0$ is a coincidence, or some unknown high energy extension of the Standard Model relates these parameters. Since $\lambda_h(M_P)$ may be positive up to $M_P$, the possible need for new physics is pushed to very high energies. Note that the Higgs mass $m_H = \sqrt{2\lambda_h} v_h$ has relatively little running [17] [18], so $v_h^2$ running compensates $\lambda_h$ running. $\beta_\lambda$ also vanishes around the Planck mass. "Near-criticality holds for both the Higgs quartic and the top Yukawa coupling" [17].

From [17] and [19] we conclude that Electro Weak Symmetry Breaking (EWSB) at $T_{\text{EWSB}} = 159.5 \pm 1.5\,\text{GeV}$ [19] is not due to $v_h^2$ running to zero at this temperature, but to a fraction of the Higgs bosons excited to above the potential barrier $\left(\frac{1}{4}\lambda_h\right)^{1/4} v_h = m_H / \left(16\lambda_h\right)^{1/4}$. We note that this potential barrier grows with the renormalization energy scale $\mu$ slower than the temperature $T = \mu$, which allows the restoration of the electroweak symmetry $U(1)_Y \times SU(2)_L$ at $T > T_{\text{EWSB}}$.

The renormalization energy scale at which $\lambda_h$ crosses zero depends critically on the Yukawa coupling $y_t$, on the SU(3) gauge coupling $g_3$, and, to a lesser extent, on $g_2$. During inflation the Higgs boson does not have time to become "dressed", so we should use $\lambda_h(M_P)$ (not $\lambda_h(m_t)$) which depends critically on $m_t$, $g_3$, and $m_H$, and may even be zero or negative, and so, Higgs inflation may break down.

The renormalization group equations for the Standard Model extended with a scalar field coupled to the Higgs boson, have been studied in [20] [21] and [22]. Our conclusions, for the warm dark matter extension of the Standard Model studied in this article, with the expected $\lambda_{hS}$ of order $10^{-8}$ to $10^{-7}$, is that the running of Standard Model parameters is not significantly changed, while $\lambda_S(\mu)$ increases with $\mu$, and so does not become zero. For example, if we set $\lambda_S(M_Z) = 0.126$, we obtain $\lambda_S(M_P) \approx 0.5$.

## 6. Preheating after Inflation

If decays of the inflaton field can be neglected during inflation, then $T^3 a^3$ is constant, and the temperature is exponentially small, or zero, at the end of this "cold" inflation. A "reheating" period is needed after the end of cold inflation at $a_f$ in order to start the radiation-dominated hot big-bang cosmology. After inflation "the field oscillates about a potential minimum and decays. How this occurs is unknown in any detail." [23] Steven Weinberg makes a similar comment [14]. This oscillation and decay is known as "preheating". For a potential $V(h) \propto h^n$, the equation of state is $w \equiv p/\rho = (n-2)/(n+2)$ [24]. So, while the oscillation is dominated by the quartic potential, the equation of state is $w = 1/3$, as for radiation. If the potential becomes quadratic, $w = 0$, and the inflaton field expands as matter (but this will not be the case in the present scenario).

Assume $S$ is the inflaton field. During preheating $S$ oscillates anharmonically with angular frequency $\omega(t)$ [25] (defined as 2π divided by the period), and

$$\rho = H^2 \frac{3M_P^2}{8\pi} = \frac{1}{(2t)^2} \frac{3M_P^2}{8\pi} = \rho_f \left( \frac{a_f}{a} \right)^4 \approx \frac{\omega^2 S_{\text{peak}}^2}{2}, \tag{51}$$

$$\rho \equiv \frac{\pi^2}{30} g_{\text{reh}} T^4 \approx \frac{\pi^2}{30} 107.75 \left( \frac{0.332 T_0}{a} \right)^4, \tag{52}$$

where $T$ is a pseudo-temperature since thermal equilibrium has not yet been reached. Note that in the metric version of General Relativity $a_f$ and $\rho_f$ are obtained from measurements of $n_s$ and $P_\zeta(k)$, and so all parameters are known during preheating. Somehow we need to create particle pairs with opposite momenta $\boldsymbol{\omega}$ and $-\boldsymbol{\omega}$. To this end we need density and/or curvature perturbations that couple to the oscillating inflaton field.

Local thermal and chemical equilibrium needs to be achieved before the temperature $T$ drops below $T_{\text{BBN}} \approx 1$ MeV to not upset the predictions of Big Bang Nucleosynthesis (BBN) [1]. Thermal and chemical equilibrium between $S$ and $h$ needs to be achieved before $T$ drops below $m_H$, so that dark matter agrees with observations [4].

## 6.1. Preheating after Higgs Inflation

This subsection is presented to illustrate issues with preheating. The evolution of the coherent inflaton field is given by (16) (where in the case of Higgs inflation we need to replace $S$ by $h$). When the expansion rate $H \propto \sqrt{\rho} \propto a^{-2} \propto t^{-1}$ drops below the decay rate Γ, $h$ decays effectively and, with (9), defines a preheating pseudo-temperature $T_{\text{pre}}$ ("pseudo" because thermal equilibrium has not yet been reached) [1]:

$$T_{\text{pre}} \approx 0.2 \left( \frac{100}{g_{\text{reh}}} \right)^{1/4} \sqrt{M_P \Gamma_{\text{tot}}}. \tag{53}$$

The theoretical total width of the Higgs boson is $\Gamma_{\text{tot}} = 4.07 \times 10^3\ \text{GeV}$ [1] (neglecting decays to $SS$), so a very rough first estimate is

$$T_{\text{pre}} \approx 10^8\ \text{GeV}. \tag{54}$$

However, it is not a Higgs boson $h$ with mass $m_H$ (defined by a quadratic potential) that decays, it is a coherent Higgs boson field ($h$), oscillating in a quartic potential with a frequency $\omega \gg m_H$, that (perhaps?) decays. Furthermore, $\Gamma_{\text{tot}}$ does not apply. The Lagrangian interaction terms containing $h$ have the forms $Y\tilde{L}hR$, $gg'hv_hVV'$, $gg'h^2VV'$, $\frac{1}{2}\lambda_{hS}hv_hS^2$, $\frac{1}{4}\lambda_{hS}h^2S^2$, and $\frac{1}{4}\lambda_h h^4$, where $L$ ($R$) are left-handed (right-handed) quarks and leptons, and $V$ and $V'$ are gauge bosons $\boldsymbol{W}_\mu$, $B_\mu$. At $T > T_{\text{EWSB}}$ the electroweak symmetry $SU(2)_L \times U(1)_Y$ is restored, so $v_h$ becomes effectively zero, and several decay channels are closed. The terms $Y\tilde{L}hR$ can be neglected because the Yukawa couplings $Y \ll T_{\text{pre}}/v_h$. The surviving terms are quadratic in the oscillating inflaton field ($h$), and need a special treatment.

There are several other issues [26]. For example, if there are already $n$ bosons in some final state, then the decay rate to this state needs to be multiplied by $n+1$. Also, since $\lambda_h(M_P)$ at Planck energies is very small (or even negative!) Higgs inflation may break down, and $S$-inflation may need to come to the rescue.

## 6.2. Preheating by Parametric Resonance

Consider inflation driven by the field $S$. Inflation ends, *i.e.* $\ddot{a} = 0$, when $\varepsilon \approx -\dot{H}/H^2 = 1$. Thereafter the homogeneous classical field $S$ oscillates coherently around the minimum of the potential $V(S)$. We assume that the quartic term dominates the potential $V(S)$. The problem of preheating is how to convert the homogeneous classically oscillating coherent field into quantum mechanical particles. The classical equations to be solved, while $\Gamma < H$, are (1) with $\rho = \frac{1}{2}\dot{S}^2 + \frac{1}{4}\lambda_S S^4$, and (16). The term in the Lagrangian (30) with $\xi_S$, that is important early on during inflation, can be neglected when inflation ends and $S$ oscillates. In our case of interest of warm dark matter, we also neglect the term with $\bar{m}_S^2$. Then the potential of interest is $\frac{1}{4}\lambda_S S^4 + \frac{1}{4}\lambda_{hS}h^2S^2$. This "conformally invariant" potential has been studied in detail in [25]. The analytical solution obtains $\rho \propto a^{-4}$, as in a radiation-dominated universe. The oscillation has an amplitude that decays as $S_{\max} \propto a^{-1} \propto t^{-1/2}$ due to the expansion of the universe, and the angular frequency of the non-sinusoidal oscillation $\omega \equiv 2\pi/\tau$, where $\tau$ is the period, becomes redshifted:

$$\omega = \frac{2\pi}{7.416}\left(\frac{\pi}{6\lambda_S M_P^2}\right)^{1/4} t^{-1/2}\sqrt{\lambda_S} M_P. \tag{55}$$

The classically oscillating field ($S$) decays into $S$ and $h$ particles that are treated quantum-mechanically. Let $S_k$ and $h_k$ be the quantum mechanical amplitudes of the particles $S$ and $h$ in mode $k$ in the Heisenberg representation. Then

$$\ddot{h}_k + 3\frac{\dot{a}}{a}\dot{h}_k + \left(\frac{k^2}{a^2} + \frac{1}{2}\lambda_{hS} S^2\right) h_k = 0, \tag{56}$$

$$\ddot{S}_k + 3\frac{\dot{a}}{a}\dot{S}_k + \left(\frac{k^2}{a^2} + 3\lambda_S S^2\right) S_k = 0, \tag{57}$$

where $k$ is the comoving momentum, and $k/a$ is the proper (or physical) momentum, of the mode. Dots indicate derivatives with respect to time $t$. The terms proportional to $S^2$ in (56) and (57) come from $\mathrm{d}^2V/\mathrm{d}h^2$ or $\mathrm{d}^2V/\mathrm{d}S^2$, respectively. These terms are the source of the parametric resonance. (Parametric resonance is understood by every child on a swing: the child stands up every time the swing passes its lowest point!) Note that both terms inside the parentheses are proportional to $a^{-2}$, which allows an elimination of $a$ by changes of variables. The solutions of these equations depend on the parameter combination $\lambda_{hS}/\lambda_S$, and on the time independent comoving momentum $k$ of the modes. The oscillations of $S$ cause parametric resonances with numbers of particles $n_S$ and $n_h$ per mode $k$ that grow exponentially with $\sqrt{t}$ (in instability bands in the $(\lambda_{hS}/\lambda_S, k)$ parameter space). We note that, for the conformally invariant potential, a mode $k$ inside a parametric resonance band stays fixed in the $(\lambda_{hS}/\lambda_S, k)$ space. (If the $\bar{m}_S^2$ were non-negligible, the resonances enter and leave instability bands, and the parametric resonances becomes chaotic [25].) An estimate of the number $n_h$ of Higgs bosons in mode $k$, obtained from Figure 6 of [25], is

$$n_h \approx \frac{1}{2}\mathrm{e}^{2\mu_h x} \quad \text{with } x = \left(\frac{6\lambda_S M_P^2}{\pi}\right)^{1/4} t^{1/2}, \tag{58}$$

and $0 < \mu_h < 0.2377$. Note that the exponential growth of $n_h$ starts with the vacuum initial quantum field amplitude for the field $h$, corresponding to $n_h = 1/2$.

This is only half of the story: we have reviewed the increase of $n_h$, but not the decrease of the coherent inflaton field ($S$). This "backreaction" is studied in [25].

## 7. Reheating

For more general presentations see [27] and [26]. Here we will study two channels that need to bring warm dark matter $S$ into thermal and diffusive equilibrium with the Higgs boson before the temperature drops below $m_H$. This is a requirement of cosmological observations [4].

### 7.1. Thermalization of $S$ by Higgs Boson Annihilations $hh \to SS$

Consider Higgs inflation followed by Standard Model reheating to a temperature $T_{\rm reh} \equiv T_1$. Here we study thermalization of $S$ by Higgs boson annihilation $hh \to SS$, which must occur before the temperature drops below $T_2 = m_H$ to satisfy the "no freeze-in and no freeze-out" warm dark matter scenario. The Higgs boson is assumed to remain in thermal equilibrium. The universe evolves as (1) with density (9). The expansion parameter $a(t)$, before $T$ drops below $m_P$, is related to temperature $T$ by (8). The increase of the number of dark matter particles $S$ per unit time and unit comoving volume is

$$\frac{\mathrm{d}\left(n_S a^3\right)}{\mathrm{d}t} = 2\sigma n_h^2 a^3, \text{ with} \tag{59}$$

$$\sigma\left(hh \to SS\right) \approx \frac{\lambda_{hS}^2}{256\pi\left(2 \cdot 2.7T\right)^2}, \text{ and} \tag{60}$$

$$n_h = 0.1218 \cdot T^3. \tag{61}$$

From the above equations, integrating from $n_{S1}a_1^3$ and $T_1$, to $n_{S2}a_2^3$ and $T_2$, neglected $n_{S1}a_1^3$ (to be conservative) and $1/T_1$, and setting $n_{S2} = n_{h2}$ for thermal and diffusive equilibrium, we obtain the coupling constant $\lambda_{hS}$ needed to obtain thermal and diffusive equilibrium between $S$ and $h$ at temperature $T_2$ with this annihilation channel only:

$$\lambda_{hS} = -1.3\times10^3\left(\frac{T_2}{M_P}\right)^{1/2}. \tag{62}$$

For example, for $T_2 = 246\,\text{GeV}$, we need $\lambda_{hS} \approx -5.8\times10^{-6}$. For $T_2 = 125\,\text{GeV}$, we need $\lambda_{hS} \approx -4.1\times10^{-6}$. A similar result is obtained for the channel $\sigma\left(W^+W^- \to h^* \to SS\right)$ [10].

### 7.2. Thermalization of $S$ by Higgs Boson Decays $h \to SS$

For Higgs boson decay to $SS$, the decay probability per unit time below $T_{\rm EWSB} = 160\,\text{GeV}$, is

$$\Gamma\left(h \to SS\right) = \frac{\lambda_{hS}^2 v_h^2}{64\pi m_H}. \tag{63}$$

We set $\Gamma \approx H$ to estimate the temperature of equilibrium. The coupling $\lambda_{hS}$ required to obtain thermal and diffusive equilibrium of $S$ with $h$ at $T_2 = m_H$, assuming this is the only channel, is $\lambda_{hS} \approx -1\times10^{-7}$.

### 7.3. Self-Interacting Dark Matter $S$

There is mounting evidence that dark matter is self-interacting: see [28] and the extensive list of references therein. From observations of galaxy NGC5044, the following estimate of the dark matter-dark matter self-interaction cross-section per unit mass is obtained: $\sigma_{\text{DM-DM}}/m_{\text{DM}} = 0.165 \pm 0.025\ \text{cm}^2/\text{g}$ (for a dark matter velocity dispersion of about 300 km/s) [28]. We consider two interaction channels, $SS \to SS$ (that depends on $\lambda_S$), and $SS \to h^* \to SS$ (that depends on $\lambda_{hS}$), and assume one dominates the other, so we neglect their interference. If $SS \to SS$ dominates, we obtain $\lambda_S \approx 4.5 \times 10^{-8}$. If $SS \to h^* \to SS$ dominates, we obtain $\lambda_{hS} \approx -8.8 \times 10^{-5}$. The coupling $|\lambda_{hS}|$ needs to be less than 0.03 to not exceed the limit on the Higgs boson invisible decay width [10]. Calculating $\sigma\left(SS \to h^* \to \tilde{\nu}\nu\right)$, and $n = 0.1218 \cdot m_{\text{DM}}^3$, we verify that $t = 1/(\sigma c n)$ is much greater than the age of the universe.

## 8. Conclusions

We have extended the Standard Model with a scalar warm dark matter field $S$ that couples to the Higgs boson $\phi$. This scenario is in agreement with recent cosmological observations if $S$ and $\phi$ come into thermal and diffusive equilibrium before the temperature drops below $m_H$ [4]. The extended model has two scalar fields, $S$ and $\phi$. Either of these fields may drive inflation. We study several inflation options, and arrive at the following conclusions:

1) $S$-inflation with quadratic potential: If the field that drives inflation is a scalar $S$ with a quadratic potential $\frac{1}{2}\bar{m}_S^2 S^2$, we can satisfy the observed inflation constraints $n_s$ and $P_\zeta(k)$ with a mass $\bar{m}_S \approx 1.2 \times 10^{-6} M_P$. This mass corresponds to cold dark matter, and so $S$ would not be the warm dark matter favored by observations.

2) $S$-inflation with quartic potential: If the field that drives inflation is a scalar $S$ with a dominating quartic potential $\frac{1}{4}\lambda_S S^4$, and a non-minimal coupling $-\xi_S S^2 R$ to the curvature scalar $R$, we can satisfy the observed inflation constraints $n_s$ and $P_\zeta(k)$ if $\lambda_S/\xi_S^2 = (4.6 \pm 1.1) \times 10^{-10}$ for the metric version of General Relativity (see **Table 2**), or $\lambda_S/\xi_S = (7.6 \pm 1.8) \times 10^{-11}$ for the Palatini version (see **Table 3**). Thermal and diffusive equilibrium between $S$ and $h$ is achieved at $T = m_H$ if $|\lambda_{hS}| \gtrsim 10^{-7}$. $S$-inflation does not suffer from critical $\lambda_S$ running at Planck energies. For the metric version of General Relativity, the predicted tensor-to-scalar ratio is $r = 0.0037 \pm 0.0009$, while it is unconstrained in the Palatini version. In summary, this scenario passes current observational tests.

3) Complex field $S$: An alternative to the real dark matter field $S$ with $Z_2$ symmetry, is a complex $S$ with a Lagrangian with $U(1)_S$ symmetry, with the ground state of $S$ and $\phi$ breaking the $U(1)_S$ and $U(1)_Y \times SU(2)_L$ symmetries [10]. In this scenario, $S$ decays to two massive gauge bosons $V$ that are the dark matter. The present analysis is also valid for this complex $S$ field with minor notational modifications, e.g. $S^2 \to S^* S$.

4) Higgs inflation: Higgs inflation is similar to $S$-inflation with quartic potential. The main difference is an uncertain quartic self-coupling $\lambda_h(M_P)$ at the Planck energy scale, that, within $3\sigma$ uncertainties, may even be zero or negative,

in which case Higgs inflation breaks down!

5) The mass $M_S$ of $S$, and the Higgs vacuum expectation value $v_h$ do not play a significant role in inflation.

The closure test presented in **Figure 1**, obtained from observations of $n_s$ and $P_\zeta(k)$ *with no free parameters*, is truly amazing!

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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## Appendix

### A Pedestrian Derivation of $P_\zeta(k)$

The purpose of this appendix is to try to understand the quantum origin of the density perturbations that are observed in the Cosmic Microwave Background radiation, which are the seeds of galaxy formation and of the large-scale structure of the universe. This derivation is outlined in [23]. For rigorous derivations see [1] [13], or [14].

Consider the comoving length scale $1/k$ that exits the horizon at $a_i$, and re-enters the horizon at $a_r$: $k = a_i H_i = a_r H_r$. See **Figure 1**. We assume that during inflation, and well within the horizon, the field $S$ has a homogeneous component, plus random quantum fluctuations $\delta S$. We apply periodic boundary conditions to a cube of volume $L^3$, and consider the lowest non-zero mode with $\omega = 2\pi/L$, and assume that the energy in the fluctuating field in this mode is $\frac{1}{2}\omega$, as in the quantum harmonic oscillator. We neglect masses, so take $\omega = k$. This energy is $L^3 \left\langle \dot{S}^2 \right\rangle_k = L^3 \omega^2 \left\langle \delta S^2 \right\rangle_k$, so $\left\langle \delta S^2 \right\rangle_k = 1/\left(4\pi L^2\right)$. As the universe expands, the box expands as $L \propto a$, $\omega = k \propto a^{-1}$ and $\left\langle \delta S^2 \right\rangle \propto a^{-2}$. When the length scale $1/k$ exits the horizon, $L = 2\pi/k$, and $H_i \approx 1/L$, so, within a factor of $O(1)$, we obtain the result of [1] for the power spectrum of vacuum field fluctuations at horizon exit $a_i$:

$$\left\langle \delta S_i^2 \right\rangle_k = \left( \frac{H_i}{2\pi} \right)_k^2 . \tag{64}$$

We now consider a "small" volume well within the horizon. The density and field evolve as (5) and (16), which neglect spatial derivatives. Inflation ends at $\varepsilon_f = 1$, so the density $\rho_f$ is independent of $\delta S$. The fluctuation $\left\langle \delta S_i^2 \right\rangle_k$, at $a_i$ during slow-roll inflation, causes a spread $\left\langle \delta t_f^2 \right\rangle_k$ of the time $t_f$ of the end of inflation:

$$\left\langle \delta t_f^2 \right\rangle_k = \frac{\left\langle \delta S_i^2 \right\rangle_k}{\left(\mathrm{d}S/\mathrm{d}t\right)_i^2} = \left\langle \delta S_i^2 \right\rangle_k \left( \frac{3H_i}{-\mathrm{d}V/\mathrm{d}S} \right)^2 \tag{65}$$

$$= \left( \frac{H_i}{2\pi} \right)_k^2 \left( \frac{3H_i}{\mathrm{d}V/\mathrm{d}S} \right)^2 = \frac{1}{\pi \varepsilon_i M_P^2} . \tag{66}$$

(Drawing a graph $V(t)$ helps.)

We must now pass from slow roll inflation to expansion with $\rho \propto t^{-2}$ characteristic of radiation (or matter), or of oscillations of $S$ in a quartic potential at the end of inflation. The time fluctuation $\left\langle \delta t_f^2 \right\rangle_k$ causes a density perturbation at a fixed time $t_{f'}$:

$$\frac{\delta \rho_{f'}}{\rho_{f'}} = \frac{\mathrm{d}\rho_{f'}/\mathrm{d}t}{\rho_{f'}} \sqrt{\left\langle \delta t_f^2 \right\rangle_k} = -\frac{2}{t_{f'}} \sqrt{\left\langle \delta t_f^2 \right\rangle_k} = -4H_f \sqrt{\left\langle \delta t_f^2 \right\rangle_k} . \tag{67}$$

For $\rho \propto t^{-2}$, and $\delta\rho \propto t^{-2}$, $\delta\rho/\rho$ is independent of $t$, so $\delta\rho_f/\rho_f = \delta\rho_r/\rho_r$.

Also $3H_f \approx H_i$, see **Table 2**, so, within a factor $O(1)$, we obtain the result of [1]:

$$P_\zeta(k) \equiv \left( \frac{\left\langle \delta\rho_r^2 \right\rangle}{\rho_r^2} \right)_k = \left( \frac{H_i^2}{\pi \varepsilon_i M_P^2} \right)_k . \tag{68}$$