International Journal of Astronomy and Astrophysics, 2023, 13, 39-60
https://www.scirp.org/journal/ijaa
ISSN Online: 2161-4725
ISSN Print: 2161-4717

The Evolving Absolute Magnitude of Type 1a Supernovae and Its Critical Impact on the Cosmological Parameters

Abraham P. Mahtessian[1], Garen S. Karapetian[1], Martik A. Hovhannisyan[2], Lazar A. Mahtessian[2]

[1]Byurakan Astrophysical Observatory after V. Ambartsumian, NAS of the Republic of Armenia, Byurakan, Republic of Armenia
[2]Institute of Applied Problems of Physics, NAS of the Republic of Armenia 25 Hrachya Nersissian Str., Yerevan, Republic of Armenia
Email: amahtes@gmail.com

How to cite this paper: Mahtessian, A.P., Karapetian, G.S., Hovhannisyan, M.A. and Mahtessian, L.A. (2023) The Evolving Absolute Magnitude of Type 1a Supernovae and Its Critical Impact on the Cosmological Parameters. *International Journal of Astronomy and Astrophysics*, **13**, 39-60.
https://doi.org/10.4236/ijaa.2023.132003

Received: February 18, 2023
Accepted: May 8, 2023
Published: May 11, 2023

Abstract

In this work, a computer optimization model has been developed that allows one to load the initial data of observations of supernovae 1a into a table and, in simple steps, by searching for the best fit between observations and theory, obtain the values of the parameters of cosmological models. The optimization is carried out assuming that the absolute magnitude of supernovae is not constant, but evolves with time. It is assumed that the dependence of the absolute magnitude on the redshift is linear: $M = M(z=0) + \varepsilon_c z$, where ε_c is the evolution coefficient of the absolute magnitude of type 1a supernovae. In the case of a flat universe ($\Omega_M + \Omega_\Lambda = 1$), the best fit between theory and observation is $\varepsilon_c = 0.304$. In this case, for the cosmological parameters we obtain $\Omega_\Lambda = 0.000$, $\Omega_M = 1.000$. Naturally, this result exactly coincides with the simulation result for the model with zero cosmological constant ($\varepsilon_c = 0.304$, $q_0 = 0.500$). Within the framework of the ΛCDM model, without restriction on space curvature ($\Omega_M + \Omega_\Lambda + \Omega_K = 1$), we obtain the following values: $\varepsilon_c = 0.304$, $\Omega_\Lambda = 0.000$, $\Omega_M = 1.000$, $\Omega_K = 0.000$. Those, this case also leads to a flat model of the Universe ($\Omega_K = 0.000$). In this work, the critical influence of the absolute magnitude *M* of type 1a supernovae on the cosmological parameters is also shown. In particular, it was found that a change in this value by only 0.4^m (from −19.11 to −18.71) leads to a change in the parameters from $\Omega_\Lambda = 0.7$ and $\Omega_M = 0.3$ to $\Omega_\Lambda = 0$ and $\Omega_M = 1$.

Keywords

Supernovae SNe1a, Cosmological Parameters, Cosmology, Acceleration

1. Introduction

It is believed that a type 1a supernova is formed when a white dwarf captures matter from its neighbor in a binary system, as a result of which its mass increases to a possible limit—the Chandrasekhar limit, when already degraded electrons cannot resist gravitational pressure and the star passes into an unstable stage. An increase in the temperature and density of the star makes it possible for carbon and oxygen to be converted into ^{56}Ni, which is accompanied by a thermonuclear explosion [1]. The brightness of the star increases so much that sometimes it exceeds the brightness of the host galaxy, and it can be seen for several thousand megaparsecs. The mass of the exploded star is always near the Chandrasekhar limit, so in the case of such explosions the absolute magnitude can vary within small limits. This allows these stars to be used as distance indicators [2].

This feature of type 1a supernovae makes it possible to study the behavior of the Universe at considerable distances and evaluate the validity of one or another cosmological model.

At the beginning of the 20th century, Hubble obtained a very interesting result, which led to the conclusion that galaxies are moving away from us, and the speed of receding is directly proportional to the distance from us. Hubble's work is based on the fact discovered by Slipher, that the spectral lines in the spectra of galaxies are shifted towards the long wavelength [3]. Hubble found that this shift increases with increasing distances to galaxies [4]. Another method for determining distance is based on the modulus of distance.

$$M = m - 5\lg D_L - 25, \tag{1}$$

where m is the apparent magnitude, M is the absolute magnitude, D_L is the Luminosity distance. When calculating the distance using this method, it is necessary to accurately estimate the value of the apparent magnitude of the object (take into account the galactic extinction, K-correction, spectral region, etc.). The absolute magnitude should be known either from theoretical approaches (for example, for type 1a supernova stars) or from empirical relationships (for example, in the case of Cepheids).

[5] and [6], in order to study the properties of the universe, made two assumptions:

1) Assume that type 1a supernovae are indicators of distances, that is, their absolute magnitudes can be considered constant.

2) That the Friedman-Robertson-Walker (FRW) cosmological model, for the case of a flat universe, accurately describes the Universe.

Taking into account quite accurately the phenomena that can influence the result, they calculated the apparent stellar magnitudes, and compared them with the values obtained from the cosmological model. It turned out that the apparent brightnesses were weaker than those obtained from the theory, that is, these objects are further away than they would be, based on Hubble's law. This led to the idea that the universe is expanding at an accelerating rate. In this regard, the idea of "dark energy" was introduced.

To avoid the idea of dark energy, various attempts have been made to explain the discrepancy between the theoretical and observed supernova luminosities by other phenomena. Let's list some of them.

1) The weakening of the apparent magnitude of a star occurs due to the absorption or scattering of light by matter in the path of light.

2) There is an evolution in the luminosity of a white dwarf, depending on the chemical composition of the host galaxy over time.

3) Gravity lenses.

4) The reason is the uneven distribution of matter in the Universe.

5) It is assumed that there are two types of supernovae 1a in nature. The second type is not numerous and is formed from the merger of two white dwarfs. As a result of the merger, the mass of the exploding star is no longer fixed.

6) Observational errors may also increase due to the fact that the brightness curves of various supernovae are recorded under different conditions (on Earth and in space).

The degree of influence of these phenomena has been discussed in various studies, showing that many of these inaccuracies cannot be considered satisfactory for refuting the results obtained by [5] and [6]. They can be found in [7].

However, in our opinion, there are two observational facts that cannot be ignored.

First, this is a rather large width of the distribution of the absolute magnitudes of type 1a supernovae (**Figure 1**). This issue was studied in the article by [8]. The average absolute magnitude of 115 studied stars was obtained $\langle M_B \rangle = -19.04 \pm 0.07$, standard deviation $\sigma M_B = 0.70$. 89 of them have late host galaxies (*Sa-Irr* or star-forming galaxies, *S-F*), for which $\langle M_B \rangle = -19.20 \pm 0.05$, $\sigma M_B = 0.49$, and 26 have early host galaxies (*E-SO* or passive galaxies), respectively $\langle M_B \rangle = -18.48 \pm 0.19$, $\sigma M_B = 0.98$.

Such large standard deviations in the absolute magnitude distributions of type 1a supernovae allow us to conclude that when estimating the values of cosmological parameters, it is wrong to take as a basis the absolute magnitude determined by several stars. In [9] (Paper I) showed that in this case the obtained cosmological parameters lead to a violation of the initial assumption that the absolute magnitudes of type 1a supernovae do not change with distance. This violation disappears when the absolute magnitude of supernovae is estimated in the course of estimating the cosmological parameters. Thus, when estimating cosmological parameters, the absolute magnitude of supernovae should also be an estimated parameter. The absence of such an approach can be considered a shortcoming in the works of other authors related to this topic. Note that this approach also improves the fit between the observational data and the theory. Assuming that the absolute magnitude of supernovae is constant with distance, we get that the share of dark energy in a flat universe does not exceed 50%. In [9]

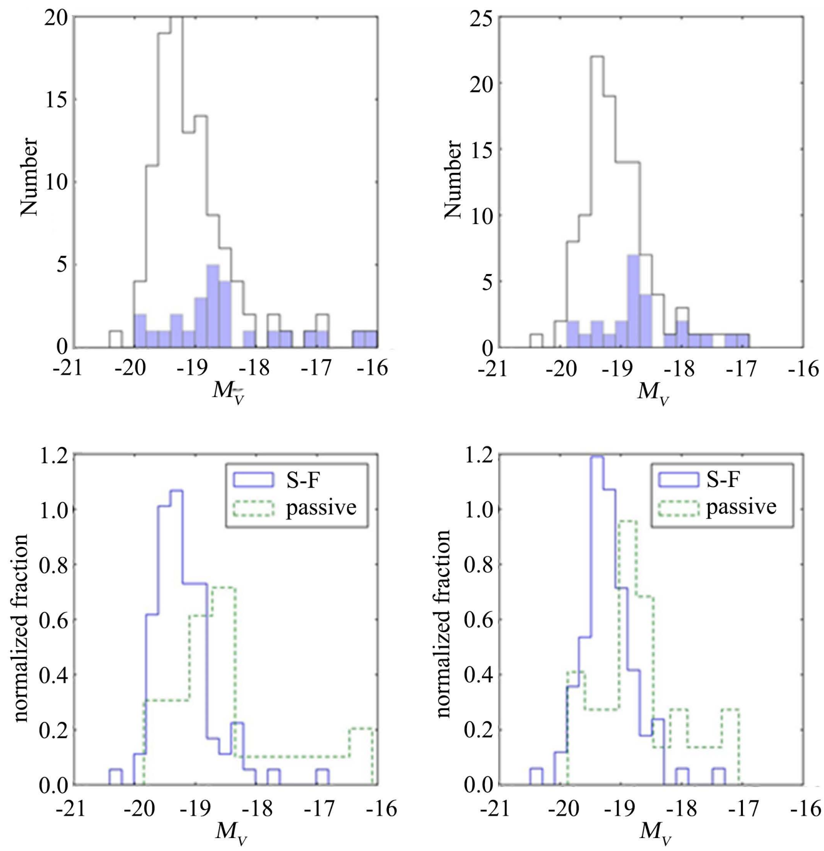

Figure 1. Distribution of absolute magnitudes of 115 type 1a supernovae. Graph copied from [8].

also obtained another important result that the cosmological model with zero cosmological parameter describes the universe no worse than the Friedmann-Robertson Walker model.

Second, the correlation between the absolute magnitude of supernovae and the age of the stellar population of host galaxies indicates that there is an evolution in the absolute magnitude of supernovae [10]. It is known that the absolute magnitude of type 1a supernovae correlates with the characteristics of the host galaxy. For example, in [11] found a systematic difference in the absolute magnitude of supernovae of ~0.14 magnitude between very early and very late galaxies. [12] and [13] found that SNe1a in less massive galaxies (by a factor of 10) by ~0.08 magnitudes are weaker than in more massive galaxies. [14] showed that SNe1a in environments with local star formation (higher local SFR) is about 0.16 magnitude weaker than in locally passive environments (lower local SFR). [10] noted features of the host galaxies (morphology, mass and local SFR) were converted to age differences with methods known in the literature. **Table 1** is taken from [10]. The table shows the correlation of the absolute magnitude of supernovae 1a with the properties of the parent galaxies. The last column of **Table 1** shows the estimated absolute magnitude evolution over 5.3 Gyr, which corresponds to the difference in age at $z = 0$ and $z = 1$ (see [10], for each of the

Table 1. Correlation of the absolute magnitude of supernovae 1a with the properties of host galaxies [10].

Host Property	References	Original Correlation	Direction	Converted to Age difference
Morphology	[11]	ΔHR/Δmorph ≈0.14 mag/ (Scd/Irr-E/S0)	Fainter in Later type galaxy	~0.19 mag/5.3Gyr Fainter in Younger galaxy
Mass	[12]	ΔHR/Δmass ≈0.08 mag/ ($\Delta \log M_* \sim 1$)	Fainter in Less massive galaxy	~0.21 mag/5.3Gyr Fainter in Younger galaxy
Local SFR	[14]	ΔHR/ΔlocalSFR ≈0.16 mag/ (ΔlogLsSFRstep ~2yr^{-1}·kpc^{-2})	Fainter in Higher SFR environments	~0.34 mag/5.3Gyr Fainter in Younger galaxy
Population Age	[10]	ΔHR/Δage ≈0.051 mag/Gyr (YEPS)	Fainter in Younger galaxy	~0.27 mag/5.3Gyr Fainter in Younger galaxy

four different studies. The average of these values is ~0.25 mag/5.3Gyr. In this range of redshifts, the observed decrease in supernova brightness in the Hubble diagram is approximately comparable to this value (see, for example, [5]. And so, this effect may be associated with the evolution of the luminosity of supernovae and has nothing to do with the accelerated receding of distant supernovae.

Thus, when estimating the cosmological parameters, it is also important to estimate the possible evolution of the absolute magnitudes of supernovae. In this article, we study models of the universe under the assumption of the existence of an evolution of the luminosities of type 1a supernovae and try to find those values of the cosmological parameters for which there will be the best fit between theory and observation.

2. Theory

In this paper, we will discuss two models: the ΛCDM model used by [5] and [6] for the case of a flat universe, and the model with a zero cosmological constant, which was widely used before these works (until 1999). The first model assumes the existence of dark energy; in the second model, such a hypothesis is not necessary.

In the case of the ΛCDM model, the dependence of the luminosity distance on redshift is given by the following formula:

$$D_L\left(z,\Omega_M,\Omega_\Lambda,\Omega_K\right) = CH_0^{-1}\left(1+z\right)\left|\Omega_K\right|^{-\frac{1}{2}} sinn\left\{\left|\Omega_K\right|^{\frac{1}{2}}\int_0^z \mathrm{d}z\left[\left(1+z\right)^2\left(1+\Omega_M z\right)-z\left(2+z\right)\Omega_\Lambda\right]^{-\frac{1}{2}}\right\} \quad (2)$$

where z is the redshift of the object. H_0 is the Hubble constant. Ω_K is related to the curvature of space and in the case of flat universe it is 0 [15]:

$\Omega_K = 1 - \Omega_M - \Omega_\Lambda$, $sinn = sinh$, when $\Omega_K \geq 0$ and $sinn = sin$, when $\Omega_K \leq 0$. In the case of $\Omega_K = 0$, we will have:

$$D_L(z, \Omega_M, \Omega_\Lambda) = \frac{C(1+z)}{H_0} \int_0^z dz \left[(1+z)^2 (1+\Omega_M z) - z(2+z)\Omega_\Lambda \right]^{-\frac{1}{2}} \quad (3)$$

or

$$D_L = \frac{C(1+z)}{H_0} \int_0^z dz \left[(1+z)^3 \Omega_M + \Omega_\Lambda \right]^{-\frac{1}{2}}$$

If we assume that $\Omega_\Lambda = 1$, and $\Omega_M = 0$, we will have [7]

$$D_L(z) = \frac{C}{H_0}(z + z^2) \quad (4)$$

If $\Omega_\Lambda = 0$, and $\Omega_M = 1$, we have

$$D_L(z) = \frac{2C}{H_0}\left[(1+z) - \sqrt{1+z} \right] \quad (5)$$

It should be noted that in 1998, prior to the work of [5] and [6] commonly used the equations of general relativity (GR) with zero cosmological constant ($\Lambda = 0$). Using this model, [16] integrated these equations exactly and obtained the luminosity distance as a function of redshift.

$$D_L(z, q_0) = \frac{C}{H_0 q_0^2}\left[q_0 z + (q_0 - 1)\left(\sqrt{1 + 2q_0 z} - 1 \right) \right] \quad (6)$$

where q_0 is the deceleration parameter, in this case:

$$q_0 = \frac{\Omega_M}{2}$$

(6) with $q_0 = 0.5$ coinciding with (5).

For the luminosity distance in the case of a flat universe we will use Formula (3), for the luminosity distance in the model with zero cosmological constant ($\Lambda = 0$) we will use Formula (6).

We will also discuss the general case (2) with nonzero space curvature. We also assume that the dependence of the absolute magnitude of the supernova on z is linear. Then we can assume $M_z = M_0 + \varepsilon_c z$ and Formula (1) can be written as follows:

$$M_0 + \varepsilon_c z = m - 5\log D_L - 25 \quad (7)$$

where M_0 is the absolute magnitude of the supernova at $z = 0$ ($M_0 = M(z=0)$), ε_c is the evolution coefficient of the absolute magnitude.

3. Procedure

Our approach was as follows: to develop a computer model where one can load the observed data of supernovae 1a and easily obtain cosmological parameters by achieving the best fit of observations with theory. As the search variables of the computer model, both the cosmological parameters of the Friedmann-Robertson-Walker model Ω_Λ, Ω_M, Ω_K and the parameters M_0 and ε_c were used.

The value of the Hubble constant H_0 is assumed to be 72.305. Solutions were sought in the ranges of variables:

$$0 \le \Omega_M \le 1,$$

$$0 \le \Omega_\Lambda \le 1,$$

$$0 \le \Omega_K \le 1,$$

$$-19.5 \le M_0 \le -18,$$

$$-1 \le \varepsilon_c \le 1,$$

under the condition of a flat universe ($\Omega_M + \Omega_\Lambda = 1$) and without it ($\Omega_M + \Omega_\Lambda + \Omega_K = 1$). A model with a zero cosmological constant is also investigated, in which the search parameters are q_0, M_0 and ε_c. The solution was searched in the ranges:

$$0 \le q_0 \le 0.5,$$

$$-19.5 \le M_0 \le -18,$$

$$-1 \le \varepsilon_c \le 1.$$

Solver Excell, Macroses and SciDAVIs were used as optimization decision tools.

4. Sample

For the study, we use a subsample from SNe1a "Union2" [17]. The sample consists of 719 supernovae identified in 17 papers [5] [6] [17]-[31]. Following several principles, the authors [17] cleared the sample and retained 557 supernovae for further study. We will also use the observational material of these 557 stars without making any changes.

5. Results

We analyze the Hubble diagram and find those values of the parameters present in the discussed model of the universe, which provide the best fit between the model and observation. On the Hubble diagram, the theoretical curve can be represented by the following relationship:

$$B_{mag}^{th}\left(z, \Omega_M, \Omega_\Lambda, \Omega_K\right) = M_0 + \varepsilon_c z + 5\log D_L\left(z, \Omega_M, \Omega_\Lambda, \Omega_K\right) + 25 \quad (8)$$

for the Friedmann-Robertson-Walker model, or

$$B_{mag}^{th}\left(z, q_0\right) = M_0 + \varepsilon_c z + 5\log D_L\left(z, q_0\right) + 25 \quad (9)$$

for the model with zero cosmological constant.

We need to find those values of the parameters Ω_M, Ω_Λ, Ω_K, M_0, ε_c in the first case and q_0, M_0, ε_c in the second case, so that the sum of squares ($B^{obs} - B_{mag}^{th}(z)$) would be minimal:

$$Chi^2 = \sum\left(B_{obs} - B_{mag}^{th}(z)\right)^2 = min.$$

For the luminosity distance D_L we use Formulas (2) and (3) for the Friedmann-Robertson-Walker model and Formula (6) for the model with zero cosmological constant. We will investigate the following cases:

1) The flat universe of Friedmann-Robertson-Walker ($\Omega_\Lambda + \Omega_M = 1$). We will look for Ω_Λ, Ω_M and M_0, ε_c for the best fit between theory and observation. We will discuss two cases:

a) There is no evolution of the absolute magnitudes of supernovae ($\varepsilon_c = 0$) and

b) There is an evolution of the absolute magnitudes of supernovae ($\varepsilon_c \neq 0$).

2) Friedmann-Robertson-Walker universe without restriction on space curvature ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$). We will look for $\Omega_\Lambda, \Omega_M, \Omega_K$, and M_0 for the best fit between theory and observation. We will also discuss two cases:

a) $\varepsilon_c = 0$ and

b) $\varepsilon_c \neq 0$.

3) Universe with zero cosmological constant. We will look for q_0 and M_0

For the best fit between theory and observation. We will also discuss two cases:

a) $\varepsilon_c = 0$ and

b) $\varepsilon_c \neq 0$.

All cases are tested by the "absolute magnitude test" we proposed in Paper I.

5.1. The Case $\Omega_K = 0$, $\Omega_\Lambda + \Omega_M = 1$, $\varepsilon_c = 0$. M_0, Ω_Λ, Ω_M Are Evaluated. Comparison with [17]

In Table 2, the case of a flat universe is considered without the assumption of the evolution of the absolute magnitude of type 1a supernovae. Here and in the tables that follow, the first column lists the parameters discussed, the second column is "Yes" when the parameter is evaluated in simulation, or "No" when the parameter is assumed to be constant. The third column shows the range of values for the parameter you are looking for. The fourth column shows the value of the desired parameter, at which the best fit between the observational data and the theory is observed (*i.e.*, the minimum value of *Chi*^{2} is obtained—the sum of the squared deviations of the observation points from the theoretical curve (Pearson's criteria) on the Hubble diagram). The fifth column shows the minimum value of *Chi*^{2}.

Table 2. The result of the search for the values of the parameters M_0, Ω_Λ, Ω_M for the Flat Universe ($\Omega_\Lambda + \Omega_M = 1$, $\Omega_K = 0$) without the assumption of the evolution of the absolute magnitude SNe1a.

Parameter	Variable	Search range	Parameter search result	*Chi*^{2}
M_0	Yes	−19.5 ÷ −18	−18.903	
ε_c	No	0	0	
Ω_Λ	Yes	0 ÷ 1	0.397	83.7439
Ω_M	Yes	0 ÷ 1	0.603	
Ω_K	No	0	0	

Table 2 shows that, assuming a constant absolute magnitude of supernovae 1a, we get $\Omega_\Lambda = 0.4$, $\Omega_M = 0.6$. This result is consistent with the result of Paper I [9], where only the case with a constant absolute magnitude of type 1a supernovae was studied. It was noted above that we use the same sample with the same observational data as used in [17]. But these authors obtained $\Omega_\Lambda = 0.73$, $\Omega_M = 0.27$.

How can such a large difference be explained? At the top, we noted that with a large spread in the absolute magnitudes of supernovae 1a, in our opinion, it is not correct to use the average absolute magnitude obtained by several stars in the simulation, and it is necessary that the absolute magnitude be obtained by the simulation method. Thus, with this approach, according to the studied sample, assuming the constancy of the absolute magnitude of supernovae, the fraction of dark energy is 0.4. Paper I studied different sub-samples of the Union [22] and Union 2 [17] compilations of type 1a supernovae. It has been found that, assuming a constant absolute magnitude of SNe1a, the fraction of dark energy does not exceed 0.5.

Now let's check the validity of our result with the test of the absolute magnitude proposed by us in Paper I. The meaning of the test is that after finding the values of the cosmological parameters, the dependence of the absolute magnitudes of SNe1a on the distance (on the redshift *z*) is plotted and its compliance with the initial assumption is checked. **That is, the adequacy of the model is checked**.

Figure 2 plots the absolute magnitudes of SNe1a calculated from the parameters $\Omega_\Lambda = 0.397$ and $\Omega_M = 0.603$ depending on the redshift.

As can be seen from **Figure 2**, there is no noticeable relationship between M and z.

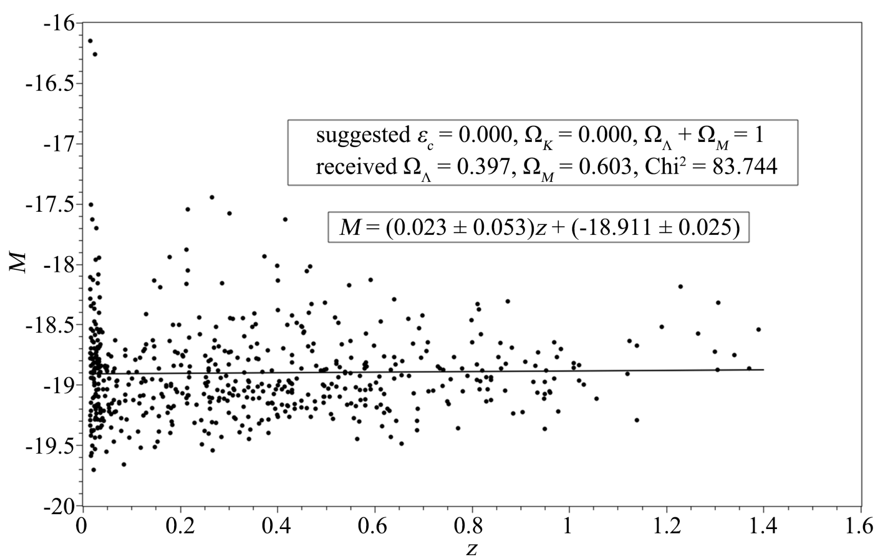

Figure 2. The absolute magnitudes of SNe1a calculated from the parameters $\Omega_\Lambda = 0.397$ and $\Omega_M = 0.603$ depending on the redshift.

To assess the significance of the correlation between the values of M and z, we use the parameter $t = \left(R\sqrt{n-2}\right) \div \sqrt{1-R^2}$, which is subject to t-statistics. Here R is the correlation coefficient and n is the number of observations. In this case, $n = 557$, $R = 0.018$. We get $t = 0.43$, which rejects the existence of a significant correlation between M and z.

I.e. the original assumption about the independence of the absolute magnitude of the redshift is observed.

In Paper I (where the case of distance-independent absolute magnitude of supernovae was investigated), different sub-samples from the Union [22] and Union 2 [17] compilations were studied, and in all cases, after simulation, we reach the original assumption about the independence of the absolute magnitude of supernovae from redshift.

Under the assumption that the absolute magnitude of supernova are constant, [17] obtained the value $\Omega_\Lambda = 0.73$ and $\Omega_M = 0.27$. Let's test the absolute magnitude. The dependence of the absolute magnitude of SNe1a on the redshift at $\Omega_\Lambda = 0.73$, $\Omega_M = 0.27$ is shown in **Figure 3** (see also Paper I).

As can be seen from **Figure 3**, there is a clear relationship between the quantities under consideration. In this case, $N = 557$, $R = 0.256$, $t = 6.25$, which shows a significant correlation between M and z at a significance level of $\ll 10^{-3}$.

Thus, in this case, the assumption that the absolute magnitudes of SNe1a are independent of the redshift is violated. This gives grounds to believe that the authors found incorrect values of Ω_Λ and Ω_M. This is also confirmed by the Chi^2 values. Their Chi^2 value is 94.85 (see **Figure 3**), while ours is 83.74 (**Table 2**, **Figure 2**).

It is noteworthy that the average absolute magnitude of SNe1a M_0 in **Figure 2** and **Figure 3** differ little. This means that the obtained values of the cosmological parameters depend very strongly on the previously accepted average absolute magnitude of supernovae. This issue will be explored in detail below.

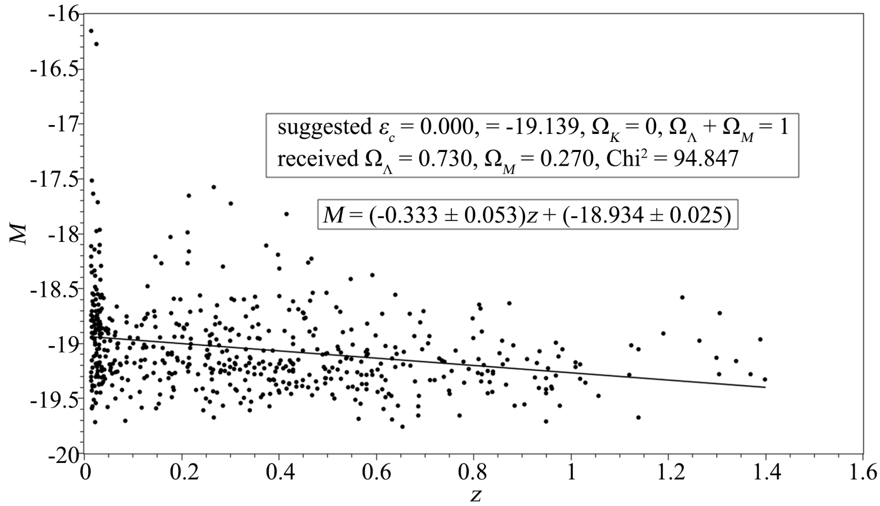

Figure 3. Dependence of the absolute magnitude of SNe1a on the redshift at $\Omega_\Lambda = 0.73$, $\Omega_M = 0.27$.

5.2. Case $\Omega_K = 0$, $\Omega_\Lambda + \Omega_M = 1$, Estimated M_0, ε_c, Ω_Λ, Ω_M

In Table 3, the case of a flat universe is considered with the assumption of the evolution of the absolute magnitude of type 1a supernovae.

It can be seen from the table that, assuming the evolution of SNe1a, the best fit (the smallest Chi^2) of the flat universe model ΛCDM with observational data is obtained at $\Omega_\Lambda = 0$. A comparison of Chi^2 in Table 2 and Table 3 shows that its value is smaller in Table 3, *i.e.*, assuming the evolution of the absolute magnitude of SNe1a, we obtain better fit between theory and observation. In this case, we need a change in the absolute magnitude of SNe1a of only 0.3^m for the time of the corresponding $z = 1$ (approximately 5.3 Gyr). This value is consistent with the value obtained in [10] (see Table 1).

Let's do an absolute magnitude test. The dependence of the absolute magnitudes of supernovae on the redshift at the values of the parameters $\varepsilon_c = 0.304$, $\Omega_\Lambda = 0.000$ and $\Omega_M = 1.000$ is shown in Figure 4.

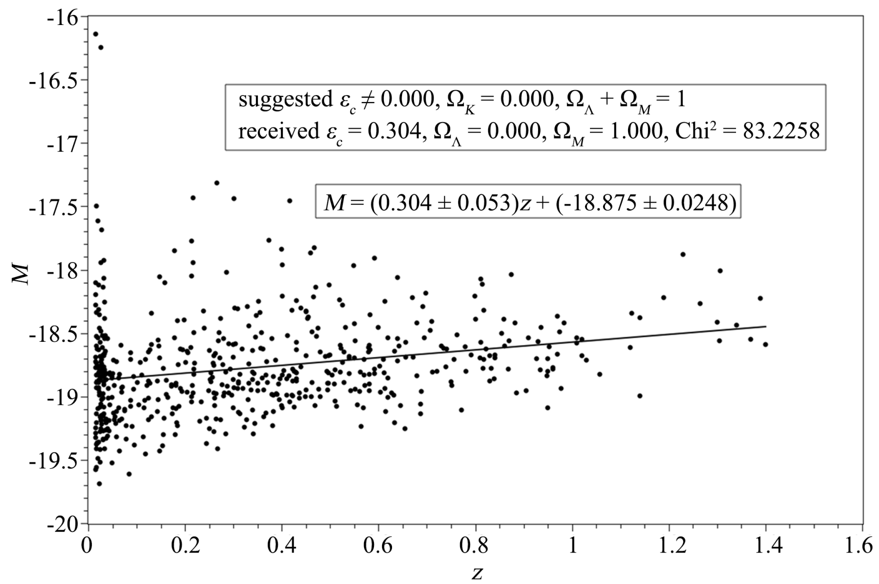

Figure 4. Dependence of the absolute magnitudes of supernovae on the redshift at the values of the parameters $\varepsilon_c = 0.304$, $\Omega_\Lambda = 0.000$ and $\Omega_M = 1.000$.

Table 3. The result of the search for the values of the parameters M_0, Ω_Λ, Ω_M and ε_c for the Flat Universe ($\Omega_\Lambda + \Omega_M = 1$, $\Omega_K = 0$) with the assumption of the evolution of the absolute magnitudes SNe1a.

Parameter	Variable	Search range	Parameter search result	Chi^2
M_0	Yes	$-19.5 \div -18$	-18.875	
ε_c	Yes	$-1 \div 1$	0.304	
Ω_Λ	Yes	$0 \div 1$	0.000	83.2258
Ω_M	Yes	$0 \div 1$	1.000	
Ω_K	No	0	0	

The Figure shows that the dependence of the absolute magnitudes of SNe1a repeats what is assumed in advance (there is an evolution of the absolute magnitude), the slope of the dependence is the same as in the simulation.

5.3. $\Omega_\Lambda + \Omega_M + \Omega_K = 1$, $\varepsilon_c = 0$, Estimated M_0, Ω_Λ, Ω_M, Ω_K

Table 4 shows the result of searching for the values of the parameters M_0, Ω_Λ, Ω_M, Ω_K without the assumption that the universe is flat ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$) and without taking into account the evolution of the absolute magnitude SNe1a.

As can be seen from the table, when in the general case ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$) we do not take into account the evolution of the absolute magnitude of SNe1a, the fraction of repulsive energy is 0, the fraction of gravitational material is approximately 0.37, which is consistent with the popular opinion about the fraction of dark + visible matter. The curvature of space is negative. Checking the test of the absolute magnitude is shown in Figure 5.

As can be seen in this case, the original assumption about the independence of the absolute magnitudes of the redshift is not violated.

It is worth comparing the value of Chi^2 in Table 3 (Chi^2 = 83.2258) and Table 4 (Chi^2 = 83.2808). Because they are close, further research into the existence of

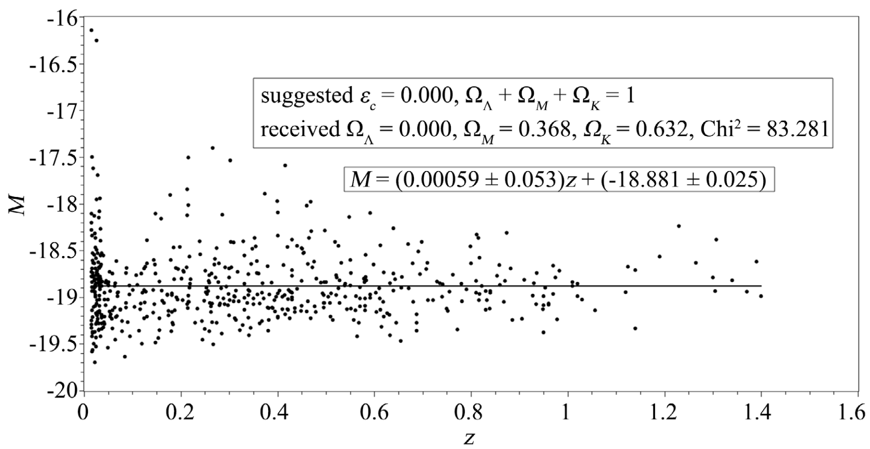

Figure 5. Absolute magnitude test for the case $\Omega_\Lambda + \Omega_M + \Omega_K = 1$, $\varepsilon_c = 0$.

Table 4. The result of the search for the values of the parameters M_0, Ω_Λ, Ω_M, Ω_K without restrictions on the curvature of the universe ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$) and without taking into account the evolution of the absolute magnitude of SNe1a.

Parameter	Variable	Search range	Parameter search result	Chi^2
M_0	Yes	−19.5 ÷ −18	−18.881	
ε_c	No	0	0	
Ω_Λ	Yes	0 ÷ 1	0.000	83.2808
Ω_M	Yes	0 ÷ 1	0.368	
Ω_K	Yes	0 ÷ 1	0.632	

the evolution of type 1a supernovae will be required, which we will do in the future. The important thing here is that in both cases we get $\Omega_\Lambda = 0$. The difference is that in the first case, this is caused by the assumption of a flat Universe and the existence of supernova evolution, and in the second case, the obtained lower value of the density of gravitational matter in the Universe without introducing restrictions on the curvature of the Universe, which leads to a negative curvature of the Universe.

5.4. $\Omega_\Lambda + \Omega_M + \Omega_K = 1$, Estimated M_0, ε_c, Ω_Λ, Ω_M, Ω_K

In Table 5, the result of the search for the values of the parameters M_0, Ω_Λ, Ω_M, Ω_K without restrictions on the curvature of the universe ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$) is given with the assumption of the evolution of the absolute magnitude SNe1a.

As can be seen from the table, the simulation gives the same result as Case $\Omega_K = 0$, $\Omega_\Lambda + \Omega_M = 1$ (see 5.2). That is, without initially assuming that the universe is flat, the largest probable estimate of cosmological parameters is obtained precisely with a flat universe.

As for the absolute magnitude test, it coincides with the graph in Figure 4.

Thus, we can say that, under the assumption of the evolution of type 1a supernovae, the ΛCDM model gives two important results:

1) The universe is flat.

2) Only gravitational material is present in it—the fraction of dark energy is equal to zero. Objects in the Universe move away from each other with deceleration.

5.5. $\Lambda = 0$, $\varepsilon_c = 0$, Estimated M_0, q_0

In Table 6 the result of the search for the values of the parameters M_0, q_0 for the model with zero cosmological constant ($\Lambda = 0$) is given without the assumption of the evolution of the absolute magnitude SNe1a.

In essence, this is similar to the case in 5.3. Those, space has a negative curvature and contains only gravitational matter (the sum of visible and invisible matter is 0.37, which is consistent with many other studies). Figure 6 shows the

Table 5. The result of searching for the values of the parameters M_0, Ω_Λ, Ω_M, Ω_K without restriction on the curvature of the universe $\Omega_\Lambda + \Omega_M + \Omega_K = 1$) with the assumption of the evolution of the absolute magnitude SNe1a.

Parameter	Variable	Search range	Parameter search result	*Chi²*
M_0	Yes	−19.5 ÷ −18	−18.875	
ε_c	Yes	−1 ÷ 1	0.304	
Ω_Λ	Yes	0 ÷ 1	0.000	83.2258
Ω_M	Yes	0 ÷ 1	1.000	
Ω_K	Yes	0 ÷ 1	0.000	

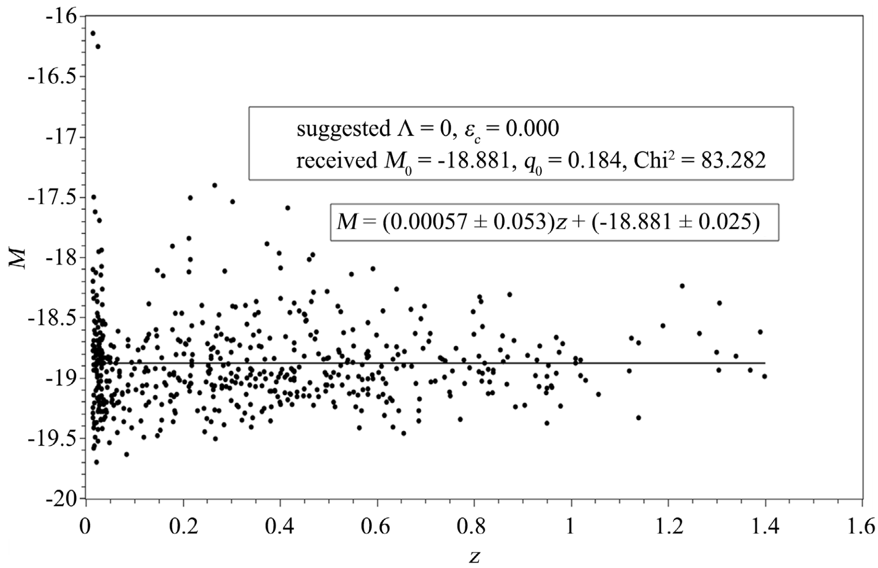

Figure 6. Dependence of the absolute magnitude of SNe1a on z at $M_0 = -18.881$, $q_0 = 0.184$ for a universe with zero cosmological constant.

Table 6. The result of the search for the best solutions for the model with zero cosmological constant ($\Lambda = 0$) without assuming the evolution of the absolute magnitude of SNe1a.

Parameter	Variable	Search range	Parameter search result	*Chi*²
M_0	Yes	$-19.5 \div -18$	-18.881	
ε_c	No	0	0	83.2808
q_0	Yes	$0 \div 0.5$	0.184	

absolute magnitude test. As can be seen from the Figure, with the results obtained ($M_0 = -18.881$, $q_0 = 0.184$), there is no dependence of the absolute magnitude of SNe1a on *z*; repeats the original assumption of independence between them.

5.6. $\Lambda = 0$, M_0, ε_c, q_0 Are Estimated

Table 7 shows the search result for the values of the parameters M_0, q_0 and ε_c for the model with zero cosmological constant ($\Lambda = 0$) with the assumption of the evolution of the absolute magnitude SNe1a. In fact, we get an analogy of the cases in 5.2 and 5.4: if we assume the evolution of SNe1a, we get a flat universe.

Absolute magnitude test, *i.e.* the dependence of the absolute magnitude of SNe1a on the redshift at the obtained values of the parameters $\varepsilon_c = 0.304$, $q_0 = 0.5$, is shown in **Figure 7**. As can be seen from **Figure 7** the absolute magnitude of supernovae SNe1a depends on the distance as obtained from the simulation, *i.e.* repeats the original assumption.

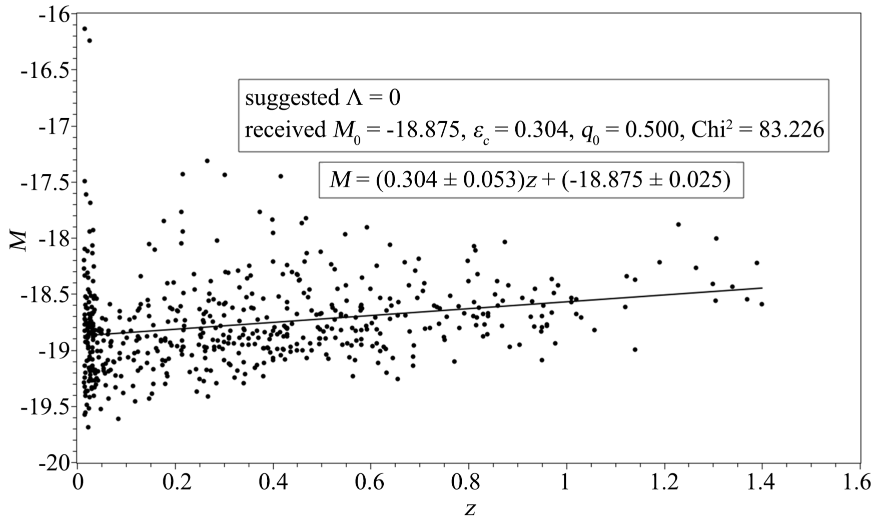

Figure 7. Dependence of the absolute magnitude of SNe1a on the redshift at the obtained values of the parameters $\varepsilon_c = 0.304$, $q_0 = 0.5$.

Table 7. The result of the search for the values of the parameters M_0, q_0 and ε_c for the model with zero cosmological constant ($\Lambda = 0$) with the assumption of the evolution of the absolute magnitude of SNe1a.

Parameter	Variable	Search range	Parameter search result	*Chi*²
M_0	Yes	−19.5 ÷ −18	−18.875	
ε_c	Yes	−1 ÷ 1	0.304	83.2258
q_0	Yes	0 ÷ 0.5	0.500	

This is essentially the analogy of the case in 5.3. We obtain that, under the assumption of the evolution of supernova magnitude 1a, both hypotheses give the same result, which consists in the fact that the universe is flat and consists only of gravitational material.

5.7. Verification of the Evolution of SNe1a at Different *z*

It is interesting to check the evolution of SNe1a at different *z*. To do this, we divided the Union 2 sample into two parts—a subsample with a redshift up to $z = 0.5$ and a subsample with $z \ge 0.5$.

5.7.1. The Case $\Lambda = 0$, $z = 0.00 \div 0.50$, $N = 403$ and the Case $\Lambda = 0$, $z = 0.50 \div 1.50$, $N = 154$ Are Evaluated M_0, ε_c, q_0

In **Table 8**, the result of the search for the values of the parameters M_0, q_0 and ε_c for a model with a zero cosmological constant ($\Lambda = 0$) is given with the assumption of the evolution of the absolute magnitude of SNe1a for "nearby" stars ($z = 0.00 \div 0.50$). In **Table 9**, the same is shown for distant stars ($z = 0.50 \div 1.50$). It can be seen from the tables that the smallest *Chi*² is obtained at $\varepsilon_c = 0.399$ and $\varepsilon_c = 0.403$, respectively. In these cases, we get the value 0.5 for q_0, *i.e.*, a flat universe.

Table 8. The result of the search for the values of the parameters, q_0 and ε_c for the model with zero cosmological constant ($\Lambda = 0$) with the assumption of the evolution of the absolute magnitude of SNe1a for "nearby" stars ($z = 0.00 \div 0.5$).

Parameter	Variable	Search range	Parameter search result	*Chi*²
M_0	Yes	−19.5 ÷ −18	−18.886	
ε_c	Yes	−1 ÷ 1	0.399	72.2283
q_0	Yes	0 ÷ 0.5	0.500	

Table 9. The same as in **Table 8**, for distant stars ($z = 0.5 \div 1.5$).

Parameter	Variable	Search range	Parameter search result	*Chi*²
M_0	Yes	−19.5 ÷ −18	−18.970	
ε_c	Yes	−1 ÷ 1	0.403	10.7607
q_0	Yes	0 ÷ 0.5	0.500	

5.7.2. The Case $\Omega_\Lambda + \Omega_M + \Omega_K = 1$, $z = 0.00 \div 0.50$, $N = 403$ and the Case $\Omega_\Lambda + \Omega_M + \Omega_K = 1$, $z = 0.50 \div 1.50$, $N = 154$ Are Evaluated M_0, ε_c, Ω_Λ, Ω_M, Ω_K

In **Table 10**, the result of the search for the values of the parameters M_0, Ω_Λ, Ω_M, Ω_K and ε_c for the universe without space curvature restrictions ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$) is given with the assumption of the evolution of the absolute magnitude of SNe1a for "nearby" stars ($z = 0.00 \div 0.50$). In **Table 11** the same is given for distant stars ($z = 0.5 \div 1.5$).

As can be seen from **Tables 8-11**, the simulation shows that the evolution of SNe1a is observed for both nearby and distant supernovae. The direction of evolution is also the same for nearby and distant supernovae—young supernovae are dimmer.

6. Significant Influence of the Value of the Absolute Magnitude of Type 1a Supernovae on the Cosmological Parameters

In this section, using the created computer model, we also study the influence of the absolute magnitude of supernovae 1a on the obtained cosmological parameters. It turned out that the cosmological parameters are very sensitive to even a small change in this value.

This issue is very important because, when estimating cosmological parameters, researchers use the absolute magnitude of supernovae obtained by only a few stars. As we noted above, the distribution of the absolute magnitude of type 1a supernovae is very wide, and at first glance it can be seen (you can compare **Figure 2** and **Figure 3**) that the value of the cosmological parameters strongly depends on the absolute magnitude of supernovae. Let's try to study this issue in more detail.

Table 10. The result of the search for the values of the parameters M_0, Ω_Λ, Ω_M, Ω_K and ε_c for the universe without space curvature restrictions ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$) with the assumption of the evolution of the absolute magnitude of SNe1a for "nearby" stars ($z = 0.00 \div 0.5$).

Parameter	Variable	Search range	Parameter search result	*Chi*^{2}
M_0	Yes	−19.5 ÷ −18	−18.886	
ε_c	Yes	−1 ÷ 1	0.399	
Ω_Λ	Yes	0 ÷ 1	0.000	72.2283
Ω_M	Yes	0 ÷ 1	1.000	
Ω_K	Yes	0 ÷ 1	0.000	

Table 11. The same as in **Table 10** for distant stars ($z = 0.5 \div 1.5$).

Parameter	Variable	Search range	Parameter search result	*Chi*^{2}
M_0	Yes	−19.5 ÷ −18	−18.970	
ε_c	Yes	−1 ÷ 1	0.403	
Ω_Λ	Yes	0 ÷ 1	0.000	10.7607
Ω_M	Yes	0 ÷ 1	1.000	
Ω_K	Yes	0 ÷ 1	0.000	

Figure 8 shows a graph of dependence Ω_Λ, Ω_M on *M*. This graph is constructed for the ΛCDM model for a flat universe ($\Omega_\Lambda + \Omega_M = 1$) and no evolution ($\varepsilon_c = 0$). The graph shows the values of *M* corresponding to three combinations of cosmological parameters:

1) $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$ obtained at $M = -19.11$;
2) $\Omega_\Lambda = 0$, $\Omega_M = 1$ obtained at $M = -18.71$;
3) $\Omega_\Lambda = 0.397$, $\Omega_M = 0.603$ obtained at $M = -18.90$;

As shown above, the best solution for a flat universe, without taking into account evolution, was obtained in the latter case (see **Table 2**).

The difference in the absolute magnitude of supernovae 1a with combinations of 1) and 2) is: $19.11 - 18.71 = 0.4$ magnitudes, while the standard deviation of the distribution of absolute magnitudes of SNe1a is 0.7 magnitudes. At the same time, the difference in magnitude between the combinations, 1) and 3) is only 0.2.

Table 12 shows an excerpt from the table from the simulation. The first line shows the case of the best fit between theory and observation. The rest of the lines show the results in the vicinity of the point $\Omega_\Lambda = 0.7$ and $\Omega_M = 0.3$. It can be seen that when *M* changes by only 0.1 in one direction or another, we obtain the value of Ω_Λ from 0.6 to 0.8. Such a difference in the values of Ω_Λ significantly changes the idea of cosmology. The change in *M* by 0.1 is well below the range of this value used in various papers.

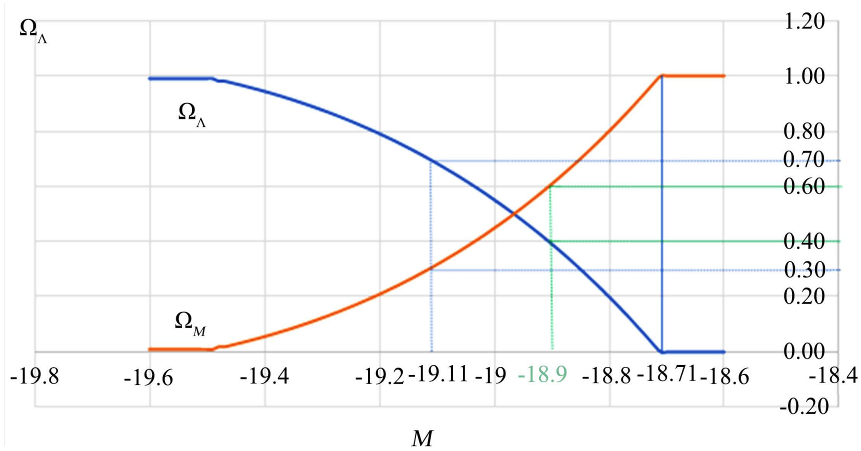

Figure 8. Plot, Ω_Λ, Ω_M versus M calculated for the ΛCDM model for a flat universe ($\Omega_\Lambda + \Omega_M = 1$). As can be seen from the Figure, a change in M by only 0.4^m (from −19.11 to −18.71) leads to a change in parameters from $\Omega_\Lambda = 0.7$ and $\Omega_M = 0.3$ to $\Omega_\Lambda = 0$ and $\Omega_M = 1$.

Table 12. Excerpt from the table from the simulation.

M	Ω_Λ	Ω_M	Chi^2
−18.9	0.39	0.61	83.74
−19.03	0.60	0.40	86.85
−19.04	0.61	0.39	87.36
−19.05	0.62	0.38	87.92
−19.06	0.64	0.36	88.53
−19.07	0.65	0.35	89.17
−19.08	0.66	0.34	89.86
−19.09	0.67	0.33	90.60
−19.10	0.69	0.31	91.38
−19.11	0.70	0.30	92.20
−19.12	0.71	0.29	93.07
−19.13	0.72	0.28	93.98
−19.14	0.73	0.27	94.95
−19.15	0.74	0.26	95.95
−19.16	0.75	0.25	97.01
−19.17	0.76	0.24	98.11
−19.18	0.77	0.23	99.26
−19.19	0.78	0.22	100.46
−19.20	0.79	0.21	101.71
−19.21	0.80	0.20	103.01

Thus, the dependence of the values of the parameters Ω_Λ and Ω_M on the accepted absolute magnitude *M* SNe1a is very strong, and therefore we must be extremely careful when determining *M*. According to the authors of this article, the determination of the absolute magnitude of supernovae should be the subject of a simulation using the entire sample of type 1a supernovae. As we saw above, this approach does not violate the original assumption that the absolute magnitudes of supernovae depend on the redshift and, therefore, gives the correct value of the parameters Ω_Λ and Ω_M.

7. Conclusions

In the previous article, we studied the value of cosmological parameters in the ΛCDM and CDM models with the assumption that the absolute magnitude of type 1a supernovae is independent of distance. The values of the cosmological parameters were estimated on the basis of the Hubble diagram. The values of these parameters were determined for which the best fit between the theoretical curve of the Hubble diagram and observational data was obtained. Pearson's goodness-of-fit test or Chi^2 (Chi-square) test was used.

In this article, we study the case with the assumption of the evolution of the absolute magnitude. We accept that the dependence of the absolute magnitude on the redshift is linear.

It turns out that when the evolution of the absolute magnitudes of supernovae is taken into account, a better fit between theory and observation is obtained.

The main difference between the approaches in our work and the works of other authors is that we estimate the average absolute magnitude of supernovae in the course of simulation, while the authors of other works take into account the average absolute magnitude of these stars, previously obtained by several well-studied stars. We pay attention to the fact that the distribution of the absolute magnitude of supernovae 1a is very wide [8]. This makes it incorrect to use the latter approach. In addition, as the simulation shows, the result strongly depends on the assumed absolute magnitude of the stars.

In the case of a flat universe ($\Omega_\Lambda + \Omega_M = 1$), the best fit between theory and observation is given by the value of the evolution coefficient $\varepsilon_c = 0.304$. In this case, for the cosmological parameters we obtain $\Omega_\Lambda = 0.000$, $\Omega_M = 1.000$. And for the absolute magnitude of supernovae 1a, −18.875 was obtained. Naturally, this result exactly matches the simulation result for the model with zero cosmological constant ($\varepsilon_c = 0.304$, $q_0 = 0.500$, $M_0 = -18.875$).

The Friedmann-Robertson-Walker model is also studied, without restriction on space curvature ($\Omega_\Lambda + \Omega_M + \Omega_K = 1$). Within the framework of this model, we obtain the following values $\varepsilon_c = 0.304$, $\Omega_\Lambda = 0.000$, $\Omega_M = 1.000$, $\Omega_K = 0.000$, $M_0 = -18.875$. Those, the case also leads to a flat Universe model ($\Omega_K = 0.000$).

In the framework of this work, the degree of influence of the absolute magnitude *M* of supernovae of type 1a on the cosmological parameters is also investigated. In particular, it was found that a change in this value by only 0.4^m (from −19.11 to −18.71) leads to a change in the parameters from $\Omega_\Lambda = 0.7$ and $\Omega_M = 0.3$ to $\Omega_\Lambda = 0$ and $\Omega_M = 1$. As we noted above, the distribution of the absolute magnitudes of supernovae 1a has a rather large width, which leads us to think that we must be very careful when accepting the absolute magnitude of supernovae. We have come to the conclusion that the absolute magnitude of supernovae must also be subject to simulation. This is the main reason that leads to a discrepancy between our results and the results obtained by other authors, who took as a basis the value of the absolute magnitude of supernovae calculated from a small number of stars.

The validity of our results is substantiated by the absolute magnitude test we proposed in the previous article (Paper I). In essence, this is a test proving that the simulation does not violate the initially accepted dependence of the absolute magnitudes of supernovae on the redshift. This test can also be called a model adequacy test.

The main results of this work are the following:

1) Under the assumption of the evolution of supernovae SNe1a, the ΛCDM model describes the observational data better than under the assumption that the absolute magnitudes of SNe1a are independent of redshift. In this case, a small evolution is obtained ($\Delta M = 0.304$ during the time of the corresponding z = 1). Young supernovae are dimmer. Evolution is observed for both nearby and distant stars.

2) The universe turns out to be flat, even if this constraint is not initially introduced.

3) There is only gravitational matter in the universe.

4) Objects in the Universe move away from each other with deceleration.

The main difference between our and other authors' approaches is that we pay attention to two facts in the nature of SNe1a. The first is that there is a critical dependence of the values of cosmological parameters obtained during the simulation on the absolute magnitude of SNe1a. The second is that the distribution of absolute magnitudes of supernovae is very wide.

These facts lead us to the conclusion that the found cosmological parameters based on the absolute magnitude of supernovae 1a predetermined from several stars are not accurate and that it is necessary to find the cosmological parameters and absolute magnitude of supernovae simultaneously in the simulation process using the full sample of supernovae 1a.

Data Availability

The observational data underlying this article are taken from [17] (link below).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Hoyle, F. and Fowler, W.A. (1960) Nucleosynthesis in Supernovae. *The Astrophysical Journal*, **132**, 565-590. https://doi.org/10.1086/146963

[2] Sandage, A. and Tammann, G. (1982) Steps toward the Hubble Constant. VIII—The Global Value. *The Astrophysical Journal*, **256**, 339-345. https://doi.org/10.1086/159911

[3] Eddington, A.S. (1923) The Mathematical Theory of Relativity. 2nd Edition, Cambridge University Press, London, 288. https://www.gutenberg.org/files/59248/59248-pdf.pdf

[4] Hubble, E.P. (1929) A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae. *Proceedings of the National Academy of Sciences of the United States of America*, **15**, 168-173. https://doi.org/10.1073/pnas.15.3.168

[5] Riess, A., *et al.* (1998) Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*, **116**, 1009-1038. https://doi.org/10.1086/300499

[6] Perlmutter, S., *et al.* (1999) Measurements of Ω and Λ from 42 High-Redshift Supernovae. *The Astrophysical Journal*, **517**, 565-586. https://doi.org/10.1086/307221

[7] Weinberg, S. (2008) Cosmology. Oxford University Press, Oxford. https://doi.org/10.1007/s10714-008-0728-z

[8] Ashall, C., *et al.* (2016) Luminosity Distributions of Type Ia Supernovae. *Monthly Notices of the Royal Astronomical Society*, **460**, 3529-3544. https://doi.org/10.1093/mnras/stw1214

[9] Mahtessian, A.P., *et al.* (2020) Absolute Magnitude Test: Testing Cosmological Models Based on Compilations of Supernovae SNe Ia "Union" and "Union2". *Advances in Astrophysics*, **5**, 18-36. https://doi.org/10.22606/adap.2020.51003 http://www.isaacpub.org/images/PaperPDF/AdAp_100138_2019122514411303809.pdf

[10] Kang, J., *et al.* (2020) Early-Type Host Galaxies of Type Ia Supernovae. II. Evidence for Luminosity Evolution in Supernova Cosmology. *The Astrophysical Journal*, **889**, 8-23. https://doi.org/10.3847/1538-4357/ab5afc

[11] Hicken, M., *et al.* (2009) Improved Dark Energy Constraints from ~100 New CfA Supernova Type Ia Light Curves. *The Astrophysical Journal*, **700**, 1097-1140. https://doi.org/10.1088/0004-637X/700/2/1097

[12] Sullivan, M., *et al.* (2010) The Dependence of Type Ia Supernovae luminosities on Their Host Galaxies. *Monthly Notices of the Royal Astronomical Society*, **406**, 782-802. https://doi.org/10.1111/j.1365-2966.2010.16731.x

[13] Kelly, P.L., *et al.* (2010) Hubble Residuals of Nearby Type Ia Supernovae Are Correlated with Host Galaxy Masses. *The Astrophysical Journal*, **715**, 743-756. https://doi.org/10.1088/0004-637X/715/2/743

[14] Rigault, M., *et al.* (2018) Strong Dependence of Type Ia Supernova Standardization on the Local Specific Star Formation Rate. https://doi.org/10.48550/arXiv.1806.03849

[15] Carroll, S.M. *et al.* (1992) The Cosmological Constant. *Annual Review of Astronomy and Astrophysics*, **30**, 499-542. https://doi.org/10.1146/annurev.aa.30.090192.002435

[16] Mattig, W. (1958) Über den Zusammenhang zwischen Rotverschiebung und scheinbarer Helligkeit. *Astronomische Nachrichten*, **284**, 108-111. https://doi.org/10.1002/asna.19572840303

[17] Amanullah, R., *et al.* (2010) Spectra and Hubble Space Telescope Light Curves of Six Type Ia Supernovae at $0.511 < z < 1.12$ and the Union2 Compilation. *The Astrophysical Journal*, **716**, 712-738. https://doi.org/10.1088/0004-637X/716/1/712

[18] Hamuy, M., *et al.* (1996) The Hubble Diagram of the Calan/Tololo Type IA Supernovae and the Value of HO. *The Astronomical Journal*, **112**, 2398-2407. https://doi.org/10.1086/118191

[19] Krisciunas, K., *et al.* (2005) Hubble Space Telescope Observations of Nine High-Redshift ESSENCE Supernovae. *The Astronomical Journal*, **130**, 2453-2472. https://doi.org/10.1086/497640

[20] Riess, A., *et al.* (1999) BVRI Light Curves for 22 Type IA Supernovae. *The Astronomical Journal*, **117**, 707-724. https://doi.org/10.1086/300738

[21] Jha, S., *et al.* (2006) UBVRI Light Curves of 44 Type Ia Supernovae. *The Astronomical Journal*, **131**, 527-554. https://doi.org/10.1086/497989

[22] Kowalski, M., *et al.* (2008) Improved Cosmological Constraints from New, Old, and Combined Supernova Data Sets. *The Astrophysical Journal*, **686**, 749-778. https://doi.org/10.1086/589937

[23] Schmidt, B.P., *et al.* (1998) The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type IA Supernovae. *The Astrophysical Journal*, **507**, 46-63. https://doi.org/10.1086/306308

[24] Holtzman, J.A., *et al.* (2008) The Sloan Digital Sky Survey-II: Photometry and Supernova IA Light Curves from the 2005 Data. *The Astronomical Journal*, **136**, 2306-2320. https://doi.org/10.1088/0004-6256/136/6/2306

[25] Barris, B., *et al.* (2004) Twenty-Three High-Redshift Supernovae from the Institute for Astronomy Deep Survey: Doubling the Supernova Sample at $z > 0.7$. *The Astrophysical Journal*, **602**, 571-594. https://doi.org/10.1086/381122

[26] Amanullah, R., *et al.* (2008) Light Curves of Five Type Ia Supernovae at Intermediate Redshift. *Astronomy and Astrophysics*, **486**, 375-382. https://doi.org/10.1051/0004-6361:20079070

[27] Knop, R., *et al.* (2003) New Constraints on Ω_M, Ω_Λ and w from an Independent Set of 11 High-Redshift Supernovae Observed with the Hubble Space Telescope. *The Astrophysical Journal*, **598**, 102-137. https://doi.org/10.1086/378560

[28] Astier, P., *et al.* (2006) The Supernova Legacy Survey: Measurement of Ω_M, Ω_Λ and w from the First Year Data Set. *Astronomy and Astrophysics*, **447**, 31-48. https://doi.org/10.1051/0004-6361:20054185

[29] Miknaitis, G., *et al.* (2007) The ESSENCE Supernova Survey: Survey Optimization, Observations, and Supernova Photometry. *The Astrophysical Journal*, **666**, 674-693. https://doi.org/10.1086/519986

[30] Tonry, J., *et al.* (2003) Cosmological Results from High-z Supernovae. *The Astrophysical Journal*, **594**, 1-24. https://doi.org/10.1086/376865

[31] Riess, A., *et al.* (2007) New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z \geq 1$: Narrowing Constraints on the Early Behavior of Dark Energy. *The Astrophysical Journal*, **659**, 98-121. https://doi.org/10.1086/510378