

# A Comment on the Hubble Expansion Parameter Tension

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## Abstract

We point out that the recent baryon acoustic oscillation measurement by the Dark Energy Survey collaboration relieves the Hubble expansion parameter tension.

## Keywords

Hubble Parameter,  $H_0$  Tension

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## 1. Introduction

The local “distance ladder” measurement of the Hubble expansion parameter by the SHOES team is  $H_0 = 73.0 \pm 1.0 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$  [1], while the early universe measurement by the Planck collaboration is  $H_0 = 67.37 \pm 0.54 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$  [2]. Other measurements of  $H_0$ , and attempts to modify the  $\Lambda$ CDM cosmology to relieve this tension, are described, for example, in [3] [4] [5]. The tension is well illustrated in Figure 1 of [3]. Given the success of the  $\Lambda$ CDM cosmology, and the many experimental constraints, none of the attempted modifications of the theory is entirely successful [3] [4]. In this article we suggest a data driven solution, based on outlying measurements of Baryon Acoustic Oscillations (BAO), and on the Planck internal tension between data for spherical harmonic multipoles  $l < 800$  and  $l > 800$ , as shown in Figure 1 of [3].

## 2. Baryon Acoustic Oscillation Measurements

The Dark Energy Survey (DES) collaboration has released a measurement of the uncalibrated BAO parameter:

$$\frac{D_M(0.85)}{r_d} \equiv \frac{c\chi(0.85)}{r_d H_0} = 19.51 \pm 0.41 \quad (1)$$

(at 68.3% confidence interval) at the effective redshift  $z = 0.85$  [6]. This measurement, relative to the Planck reference, is [6]

$$\alpha \equiv \frac{D_M/r_d}{(D_M/r_d)_{\text{Planck}}} = 0.957 \pm 0.020. \quad (2)$$

The Baryon Oscillation Spectroscopy Survey (BOSS) and eBOSS BAO measurements have  $\alpha$  generally between (2) and 1.0, as summarized in Figure 18 of [7]. The main challenge of BAO measurements is the low signal (data minus background) significance, so the results can benefit from different data and analysis methods.

We consider the case of zero space curvature ( $\Omega_k = 0$ ), and a cosmological constant ( $\Omega_{DE} = \Omega_\Lambda = \text{constant}$ ). Then, for  $z \lesssim 1$ ,

$$\chi(z) \equiv \int_0^z \frac{dz'}{E(z')} \quad \text{and} \quad E(z) = \sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}. \quad (3)$$

$\Omega_m$  is the present time total (dark plus baryonic) matter density in units of the critical density (throughout we use the standard notation of [5]).

$$r_s \equiv r_d = (1.0184 \pm 0.0004)d_* \quad (4)$$

is the comoving sound horizon at the drag epoch (early versions of [2]), and

$$d_* = \frac{c}{H_0} \theta_* \chi(z_*) = \frac{c}{H_0} 0.03401 \left( \frac{0.28}{\Omega_m} \right)^{0.4} \quad (5)$$

is the comoving sound horizon at decoupling measured by the Planck collaboration (early version of [2]). Equations (4) and (5) can be summarized as  $(r_s h/\text{Mpc})(\Omega_m/0.3)^{0.4} = 101.056 \pm 0.036$  [2].

From (1) to (5) we obtain

$$\Omega_m = 0.260 \pm 0.022, \quad r_s H_0/c = 0.0358 \pm 0.0012. \quad (6)$$

These results are in agreement with 18 BAO measurements with Sloan Digital Sky Survey (SDSS) DR13 data [8]:

$$\Omega_m = 0.284 \pm 0.014, \quad r_s H_0/c = 0.0339 \pm 0.0002, \quad (7)$$

or

$$\Omega_m = 0.281 \pm 0.003, \quad r_s H_0/c = 0.0340 \pm 0.0002, \quad (8)$$

when combined with Planck  $\theta_{\text{MC}}$  (closely related to  $\theta_*$ ). The result (6) is also in agreement with 6 BAO measurements with SDSS DR14 data [9]:

$$\Omega_m = 0.288 \pm 0.037, \quad r_s H_0/c = 0.03487 \pm 0.00052, \quad (9)$$

or

$$\Omega_m = 0.2724 \pm 0.0047, \quad r_s H_0/c = 0.03506 \pm 0.00024, \quad (10)$$

when combined with Planck  $\theta_{\text{MC}}$ . For comparison, the Planck only analysis obtains [2]

$$\Omega_m = 0.3147 \pm 0.0074, \quad r_s H_0/c = 0.03307 \pm 0.00031. \quad (11)$$

A novel measurement of  $\Omega_m$ , that exploits the non-linear filamentary nature

of galaxy clustering, based on density-marked correlation functions of SDSS BOSS DR12 CMASS data compared with simulations, has recently been obtained in [10]:

$$\Omega_m = 0.293 \pm 0.006. \quad (12)$$

This competitive measurement is independent of BAO.

### 3. Discussion

The Cosmic Microwave Background (CMB) anisotropies, measured by the Planck collaboration, determine all six primary parameters of the  $\Lambda$ CDM cosmology (assuming flat space and a cosmological constant). The parameter  $\theta_{\text{MC}}$  is constrained with extreme precision:  $\theta_{\text{MC}} = 0.0104089 \pm 0.0000031$ . On the other hand, the derived parameter  $\Omega_m$  is constrained with a relatively large uncertainty,  $\Omega_m = 0.3153 \pm 0.0073$  (early version of [2]) as discussed in [9] ((11) has been slightly updated since then). So it makes sense to combine the Planck analysis [2] with a BAO plus  $\theta_*$  measurement of  $\Omega_m$ . The measurement  $\Omega_m = 0.2724 \pm 0.0047$  [9] has been combined with the Planck analysis [2] (this combination is preliminary due to the sparseness of the Planck MC chains at such low values of  $\Omega_m$ ). The results of this combination, for primary and derived parameters, are presented in Table 10 of [9]. The combination obtains, in particular,  $\Omega_m = 0.2853 \pm 0.0040$  and  $H_0 = 69.90 \pm 0.30 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ . So the tensions of  $H_0$  are relieved, as well as the tensions of  $\Omega_m$ ,  $\sigma_8$  and  $S_8 \equiv \sigma_8 (\Omega_m/0.3)^{0.5}$  [5] [9]. The fit reduces the total  $\chi^2$  from  $13040.09 = 12956.78 + 83.31$  to  $12976.17 = 12968.64 + 7.53$ , where the first term comes from Planck and the second term comes from  $\Omega_m$ . As a cross-check of the fit, the ‘‘standard ruler’’ equation that calibrates the BAO measurements,  $\Omega_m h^3 = 0.09633 \pm 0.00029$  [2], is satisfied.

The fit can be understood by studying Figure 1 of [3]. The green hyperbola  $r_s H_0 = \text{constant}$ , representing the uncalibrated BAO measurement, becomes shifted upwards due to the low value of  $\alpha$  in (2), compare (6) with, for example, (11). The intercept with the Planck confidence contours is shifted to larger  $r_s$ , larger  $H_0$  and smaller  $\Omega_m h^2$ , *i.e.* smaller  $\Omega_m$ , in agreement with the fit indicated above. The fit agrees with the Planck analysis for  $l < 800$ , but is in tension with the Planck data for  $l > 800$ , see Figure 1 of [3].

### 4. Conclusion

We have presented a simple, well-motivated, and plausible, data driven solution to the  $H_0$  tension. The Planck determination of the cosmological parameters can benefit from a combination with a BAO plus  $\theta_{\text{MC}}$  measurement of  $\Omega_m$ . This solution needs to be tested. To this end, three areas of research are: 1) Understand the dependence of BAO measurements on different data sets and analysis methods; 2) Understand the internal tensions between the Planck low- $l$  acoustic peaks and the high- $l$  damping tail presented in Fig. 1 of [3] (dedicated comparisons between Planck, Atacama Cosmology Telescope (ACT), and South Pole

Telescope (SPT) data will help); 3) More local measurements of  $H_0$  with complementary data and methods, e.g. Figure 2 of [4].

### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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