Intelligent Information Management, 2023, 15, 373-389
https://www.scirp.org/journal/iim
ISSN Online: 2160-5920
ISSN Print: 2160-5912

Comparison of Defuzzification Operators on Geographic Data of Ratio Scale

Thomas Hatzichristos[1], Dimitrios Ntzanis[2]

[1]Department of Geography and Regional Planning, School of Surveying & Geoinformatics Engineering, National Technical University of Athens, Athens, Greece
[2]Department of Construction and Urban Planning, Municipality of Agrinio, Agrinio, Greece
Email: thomasx@survey.ntua.gr, dim.ntzanis@gmail.com

How to cite this paper: Hatzichristos, T. and Ntzanis, D. (2023) Comparison of Defuzzification Operators on Geographic Data of Ratio Scale. *Intelligent Information Management*, **15**, 373-389.
https://doi.org/10.4236/iim.2023.155018

Received: August 24, 2023
Accepted: September 23, 2023
Published: September 26, 2023

Abstract

Fuzzy logic is a contemporary theory that has found numerous applications in Geographic Information Systems (GIS). Fuzzy logic allows for the representation of uncertainty and imprecision in spatial data, making it a valuable tool for dealing with the inherent ambiguity present in many geographic datasets. To solve a problem using a knowledge-based fuzzy system, the description and processing of the influencing factors or variables in fuzzy terms is required. The key components of a knowledge-based fuzzy system within the context of GIS are: Fuzzification, definition of the knowledge base, processing of the rules and finally defuzzification. Defuzzification is an important aspect of fuzzy logic and fuzzy set theory, as it helps convert fuzzy linguistic terms or fuzzy sets into crisp values that can be used in decision-making or analysis. Moreover, this might seem contradictory to the primary objective of fuzzy set theory, which is to model and work with uncertainty and imprecision. The aim of this paper is, first, to review defuzzification operators that are suitable for handling geographic data of ratio scale and second to compare these defuzzification operators by applying them to actual geographic data sets. For this reason, a case study based on pollution data of the municipality of Athens, Greece, was carried out to estimate pollution produced by SO_2. The results of the application of defuzzification operators for the above geographic data set are compared and final conclusions are presented.

Keywords

Fuzzy Logic, Defuzzification, GIS

1. Introduction

Fuzzy logic is a generalization of the classic Boolean logic, which has been extended to be able to handle truth values from "strictly true" to "strictly false". It originates from the fuzzy set theory that was proposed by Lofti Zadeh [1] in the 60's and permits the notion of nuance. According to him: "Fuzzy logic is the methodology for calculations with linguistic terms." In fuzzy logic, propositions are not limited to just being "true" or "false" as in Boolean logic. Instead, fuzzy logic allows for a continuum of truth values between 0 and 1, where 0 represents "completely false" and 1 represents "completely true". In between, you can have values that indicate varying degrees of truth, such as "almost true", "partially true", "more true than false", and so on [2]. Another fundamental idea of this theory is that the fuzzification procedure allows the generalization of a distinct theory to a continuous one. Fuzzy sets differ from Boolean sets (also known as crisp sets) in that they do not have sharply defined boundaries. Fuzzy sets allow for more nuanced and flexible modeling of uncertainty and imprecision, making them valuable in various real-world applications.

Fuzzy degrees are not the same as probability percentages. Probabilities measure whether something will occur or not. Fuzziness measures the degree to which something occurs, or some condition exists. Crisp sets are a subset of fuzzy sets [3]. Only when an object belongs 100% to a group, are fuzzy sets identical to crisp sets, however.

The analytical functions in the most contemporary Geographic Information software packages are based on binary logic which is by nature exact and absolute and therefore does not handle properly the imprecision of geographical data. The ineffectiveness of traditional logic in matters of planning has become clear in the last decade [4] [5] [6]. Fuzzy logic is a contemporary theory that is applied in the field of GIS [7] [8] [9]. There are several applications that relate to all the phases of a GIS project, namely the input, the management, the analysis, and the spatial data representation while compromising the information fuzziness, human knowledge, perception, and thinking. This characteristic renders the fuzzy logic more appropriate for the problems confrontation of the real world since the bigger part of human thinking is inaccurate. To mention a few of the most important papers, Robinson and Strahler [10] utilized some of the operations on fuzzy sets in a geographic database, while Wang [11] developed a natural query language in a GIS. Kollia and Voliotis [12] employing the relational database INGRES presented a fuzzy GIS for soils.

A knowledge-based fuzzy system is a framework used to solve problems by representing and processing information in fuzzy terms. The key components of a knowledge-based fuzzy system within the context of GIS are: Fuzzification, definition of the knowledge base, processing of the rules and finally defuzzification. Fuzzy systems are computational models used to handle and process imprecise and uncertain information. They are particularly useful when dealing with data that may not have clear boundaries or precise values. Fuzzy systems are based on the principles of fuzzy logic, which involves reasoning with linguistic variables rather than precise numerical values. The core of a fuzzy system typically consists of rules that describe how inputs, often represented as fuzzy sets, should be combined to produce fuzzy outputs. These rules are often expressed in the form of IF-THEN statements using linguistic variables. The combination of these rules results in a fuzzy set as the system's output.

Defuzzification is the process of converting a fuzzy output (a fuzzy set) into a single, crisp value. This step is essential when the output of a fuzzy system needs to be used for decision-making as many practical applications require numerical values. The choice of defuzzification method depends on the problem at hand and the desired characteristics of the output. Since it's the last step and primarily serves to bridge the gap between fuzzy and crisp representations, it may be viewed as less central to the core concepts of fuzzy sets. This synthesis process goes against the main purpose of fuzzy set theory, which is to extend crisp concepts and theories by capturing imprecision and uncertainty. In a way, defuzzification reduces the richness of the fuzzy set to a simpler, crisp value, which can be seen as contrary to the essence of fuzzy logic [2]. Despite these reasons, defuzzification remains an important aspect of applying fuzzy logic in practice.

The aim of this paper is the comparison of defuzzification operators on geographical data of ratio scale. More specifically, it examines and analyzes the application of certain methods in raster geographical data of ratio scale, concerning the estimation of pollution produced by SO_2. The results of the application of defuzzification operators are compared and final conclusions are presented.

In the next chapter of the paper, the basic elements of the fuzzy system are presented in detail, while thereafter the basic defuzzification operators used in the analysis of environmental geographic data of ratio scale are briefly overviewed. In the chapter "Case study", the comparison of these operators is presented. In the last chapter there are the conclusions.

2. Fuzzy Systems

A knowledge-based fuzzy system is a type of artificial intelligence system that uses fuzzy logic to handle uncertainty and imprecision in decision-making. The basic elements of a knowledge-based fuzzy system, are:

1) Fuzzification;
2) Knowledge base;
3) Processing;
4) Defuzzification.

These elements are briefly described. Several types of membership functions can be utilized [13]. The membership function reflects the knowledge of the specific object or event. Every continuous mathematical function can be approximated by a fuzzy set. For example, the criterion "distance from a river" can be approximated from the membership function illustrated in **Figure 1**.

The assignment of a membership function to every variable of the problem is called "fuzzification". During this process, crisp subsets are transformed into linguistic subsets, such as small or great distance (**Figure 1**). The concept of the

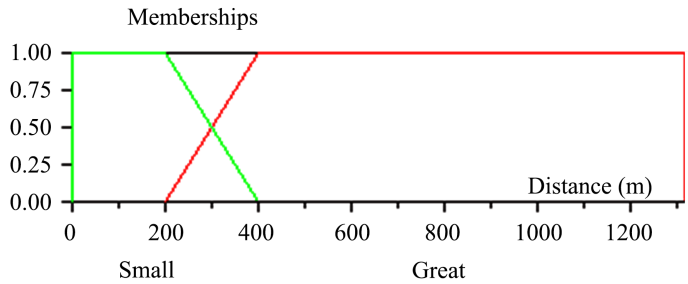


Figure 1. Membership function for the criterion "distance from a river".

linguistic variable illustrates particularly clearly how fuzzy sets can form the bridge between linguistic expression and numerical information. The most widely used form of membership function is the triangular. Its maximum is the most representative value. Other forms also exist, however, such as the trapezoid, the Gaussian etc. The definition of the membership function becomes available with a variety of "objective" methods.

The second step in the Fuzzy systems methodological approach is the knowledge base. A knowledge-based fuzzy system contains the rules and domain-specific information needed for decision-making. These rules are typically expressed in the form of fuzzy IF-THEN rules. Each rule consists of antecedents (IF part) and consequents (THEN part), that specify how the system should behave based on the fuzzy inputs. These rules are often defined by experts in the field and encode their domain knowledge. There is no need to assign weights to the criteria used. The weights are implicitly considered through the rules defined. For example, if the output set "suitability" is comprised of two subsets called: "poor" and "appropriate", the rules could be:

If the distance is small, then suitability is poor.

If the distance is large, then suitability is appropriate.

The next step is the processing of the rules. This step is also called inference. It consists of the three stages, aggregation, implication, and accumulation. Aggregation provides the degree of fulfillment for the entire rule concerned. All the Boolean algebra operations (like intersection, union, negation, etc.) can be easily extended to fuzzy set operations and can be used in this stage. In implication, the degree of fulfilment of the conclusion is determined. Accumulation brings together the individual results of the variables used. Details of this process can be found in Zadeh [14].

After processing, the system needs to convert the fuzzy output values, which represent linguistic terms, back into a crisp or numerical result that can be used for decision-making. Defuzzification is the process of finding a single, real-valued output from the fuzzy set of outputs. Various methods, such as the center of gravity or maximum membership, can be used for defuzzification [15]. The most important of them, which, in our opinion, can be utilized in geographic data of ratio scale, are presented briefly in the next chapter. Defuzzification operators for geographic data of nominal scale have been discussed by Hatzichristos and

Potamias [16]. Also, defuzzification strategies on remote sensing data is discussed by Hofmann P. [17].

3. Defuzzification Operators

The most important and widespread defuzzification methods suitable for geographical data of ratio scale of measurement are Maxima methods and more specifically, first of maxima (FOM), middle of maxima (MOM) and last of maxima (LOM), the center of gravity (COG) method, the indexed center of gravity (ICOG) method and finally the mean of maxima (MeOM) method. These methods are discussed in detail in Driankov *et al.* [18], Runkler and Glesner [19], in Jager and Filev [20] and in Užga-Rebrovs and Kuļešova [21]. In the following paragraphs some examples of these defuzzification operators are presented.

Let us suppose that the results of the fuzzy system that estimates the pollution of SO_2 are given in **Table 1**. To apply the above defuzzification methods, membership functions have to be defined to connect the linguistic output of the fuzzy system with crisp values. The membership function used is illustrated in **Figure 2**. It concerns values of the pollutant SO_2 in the range 0 to 44 μg/m^3 and three subclasses.

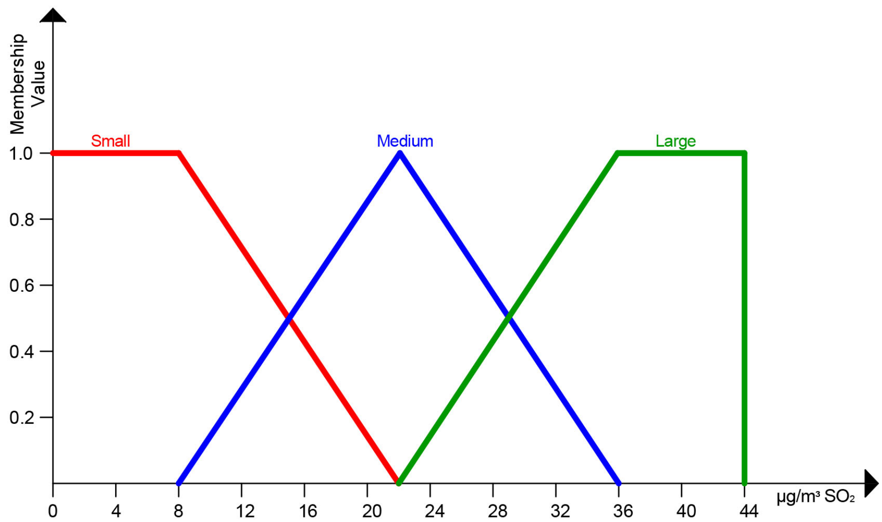


Figure 2. Membership function for the pollution caused from SO_2.

Table 1. Membership values for the pollution caused by SO_2.

Pixel Number	Membership Value		
	Small	Medium	Large
1	0.6	0.8	0.4
2	0.6	0.4	0.6
3	0.7	0.7	0.3
4	0.8	0.6	0.1

3.1. Maxima (FOM, MOM, LOM)

In general, to calculate the defuzzified value D(A), when the Maxima methods are employed, the following procedure is applied.

- The classes that display the greatest membership value are chosen.
- On the graph, the line parallel to the X axis is plotted, commencing at the point (0, the greatest membership value).
- The sectors of this line are connected with the class that constitutes the greatest membership value.
- The defuzzified value D(A) is the value that arises, when the appropriate point is projected onto the X axis (**Figure 3**). In particular, for each method the appropriate point is as follows:
- For the FOM method, the first point of the sector.
- For the LOM the last point of the sector and,
- For the MOM method, the midpoint of the straight line uniting the first and the last points of the sector.

3.2. Center of Gravity (COG)

To calculate the defuzzified value D(A) regarding the centre of gravity method (COG) the following procedure is used:

- On the graph are plotted the lines parallel to the X axis, each of the lines starting at the point (0, membership value degree).
- Then we locate the sections of each straight line by means of the branch of the membership function that corresponds to the subclass that contains the membership value defining the straight line in question.
- Then we plot the polygon, or polygons, whose points are defined by the points of intersection of the straight lines by means of the membership function.

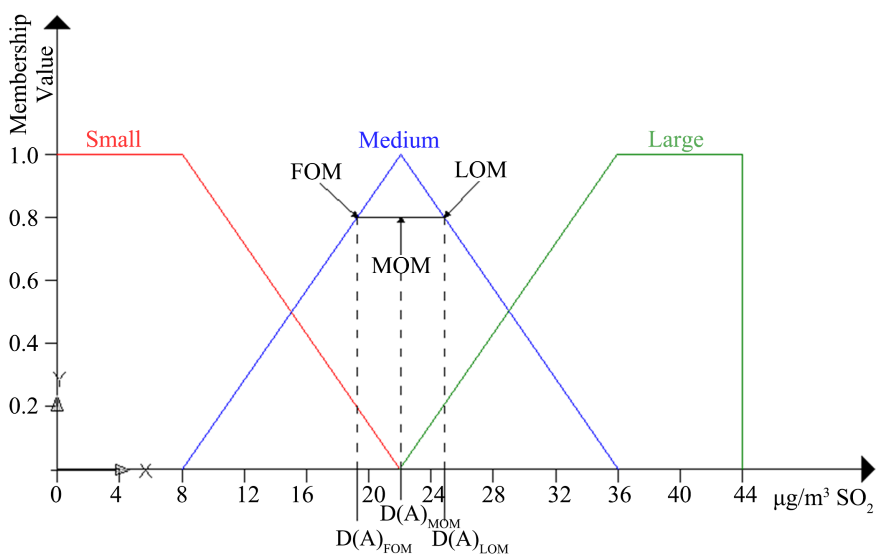


Figure 3. Maxima operators (Pixel number 1).

➢ The centre of gravity of the polygon, or polygons, is located and the defuzzified value D(A) is the value that occurs when the centre of gravity of the polygon is projected on the X axis (Figure 4).

3.3. Indexed Center of Gravity (ICOG)

To calculate the defuzzified value D(A) regarding the indexed centre of gravity method (ICOG) a method similar to that employed for the centre of gravity method is used. The sole difference in the present case lies in the fact that we now locate the centre of gravity of the polygon, or polygons, which are redefined, so that their lower base consists of the straight line defined as beginning from the point (0, α). The defuzzified value D(A) is the value that occurs when the centre of gravity of the polygon, or polygons, is projected onto the X axis (Figure 5). The user of the fuzzy system judges the threshold value α to be selected, with the

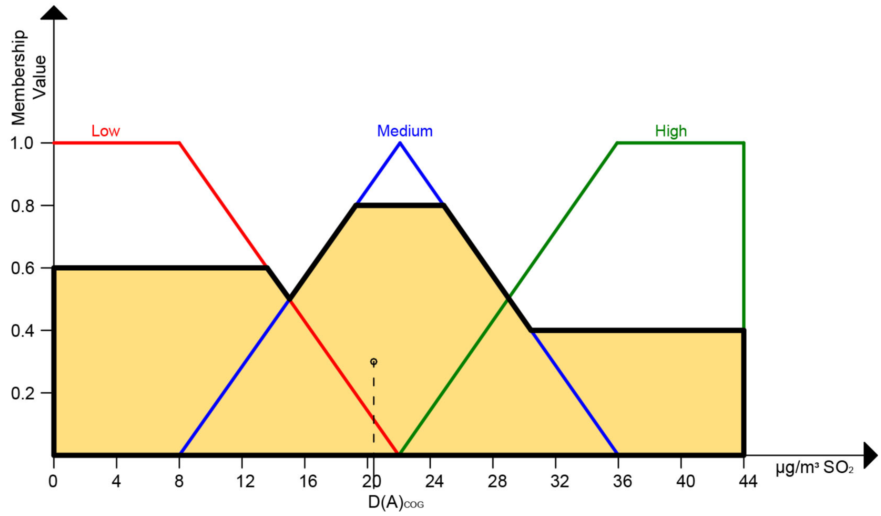


Figure 4. The COG operator (Pixel number 1).

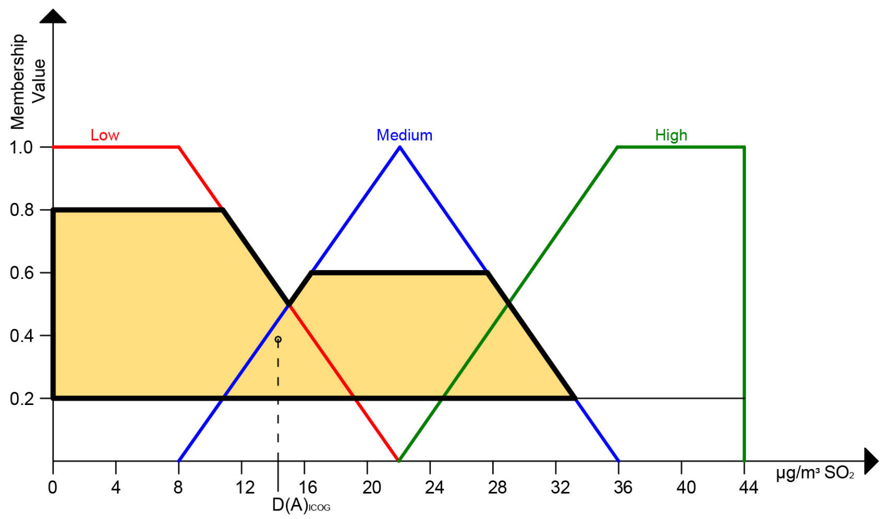


Figure 5. The ICOG operator (Pixel number 4).

result that all the membership values beneath this value are rendered equal to 0. The threshold value employed in the example is $a = 0.2$.

3.4. Mean of Maxima (MeOM)

The MeOM method displays a certain number of similarities to the COG method, since in fact it derives from the COG method. In particular, in the MeOM method, to calculate the defuzzified value D(A) the COG method is used. Here, however, the centre of gravity of the polygon is defined by the straight line that is defined by the highest membership value and by the corresponding branch of the membership function for the group in which the highest value appears (**Figure 6**).

4. Case Study

The aim of the fuzzy system developed in this application is to estimate the degree of pollution caused by Sulphur dioxide emissions. The factors employed in estimating the SO_2 pollution in this basic fuzzy system were the transportation network, sources of energy and industries. Geographic data, relating to the Athens area, Greece, was employed [22] (**Figure 7**).

To allow analysis by means of fuzzy logic, the three geographical layers were converted into raster format, with a pixel size of 5 m. The Euclidean distance for every layer was calculated. The layers of the Euclidean distance were overlaid to form one layer. The attributes of this layer were exported to the fuzzy logic software Data Engine 2.0., to perform an analysis of the data.

The first step involved in a fuzzy system is fuzzification, the definition of membership functions for every criterion. For every Euclidean distance criterion, we set a linguistic variable. Subclasses for every function were defined and a trapezoidal form was chosen as the appropriate form for rendering the function in question. The linguistic terms employed are "small-scale pollution",

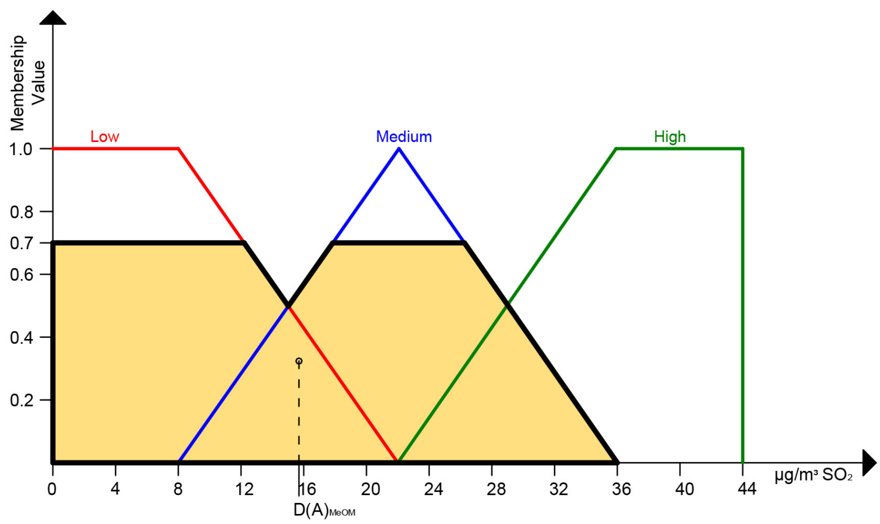


Figure 6. The MeOM opearator (Pixel number 3).

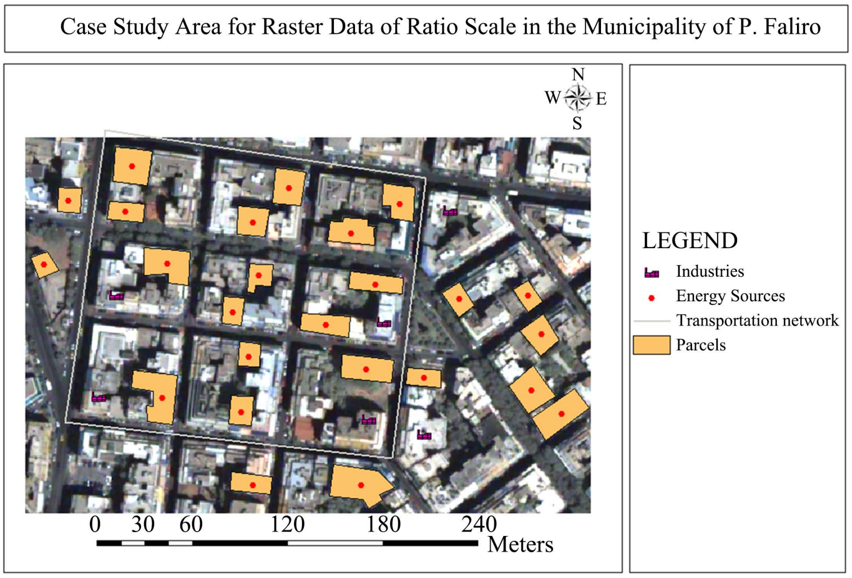


Figure 7. Case study area and criteria.

"medium-scale pollution" and "large-scale pollution". The membership functions for the three criteria are illustrated in **Table 2** and **Figure 8**.

The next step involved the definition of rules linking pollution to the three criteria. In total eight rules were created, of which three deal with small-scale pollution, two with medium-scale and three with large-scale pollution. These eight rules are given below.

Rules for small-scale pollution

If distance from transportation network is large and distance from energy sources is large and distance from industries is large, then pollution is small-scale to a degree of 100% certainty.

If distance from transportation network is large and distance from energy sources is large and distance from industries is small, then pollution is small-scale to a degree of 75% certainty.

If distance from transportation network is large and distance from energy sources is small and distance from industries is large, then pollution is small-scale to a degree of 75% certainty.

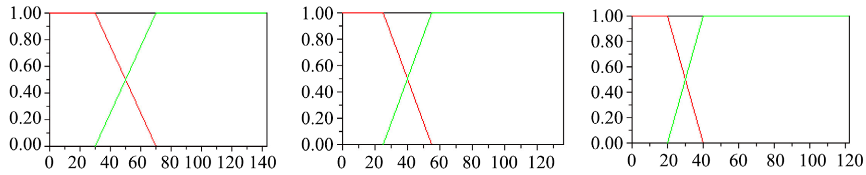
Rules for medium-scale pollution

If distance from transportation network is large and distance from energy sources is small and distance from industries is small, then pollution is medium-scale to a degree of 100% certainty.

If distance from transportation network is small and distance from energy sources is large and distance from industries is large, then pollution is medium-scale to a degree of 75% certainty.

Rules for large-scale pollution

If distance from transportation network is small and distance from energy sources is small and distance from industries is small, then pollution is large-scale to a degree of 100% certainty.



Membership function for the criterion "distance from transportation network" | Membership function for the criterion "distance from industries" | Membership function for the criterion "distance from energy sources"

Figure 8. Graphical representation of membership functions.

Table 2. Membership functions for the estimation of SO_2 pollution.

Criterion	Linguistic Variable	Membership Function
Transportation Network	Distance: Small (x) Large (x)	Large(x) = $\begin{cases} 0, & \text{if distance}(x) < 30 \\ (\text{distance}(x) - 30)/40, & \text{if } 30 \le \text{distance}(x) \le 70 \\ 1, & \text{if distance}(x) > 70 \end{cases}$
Industries	Distance: Small (x) Large (x)	Large(x) = $\begin{cases} 0, & \text{if distance}(x) < 25 \\ (\text{distance}(x) - 25)/30, & \text{if } 25 \le \text{distance}(x) \le 55 \\ 1, & \text{if distance}(x) > 55 \end{cases}$
Energy Sources	Distance: Small (x) Large (x)	Large(x) = $\begin{cases} 0, & \text{if distance}(x) < 20 \\ (\text{distance}(x) - 20)/20, & \text{if } 20 \le \text{distance}(x) \le 40 \\ 1, & \text{if distance}(x) > 40 \end{cases}$

If distance from transportation network is small and distance from energy sources is small and distance from industries is large, then pollution is large-scale to a degree of 75% certainty.

If distance from transportation network is small and distance from energy sources is large and distance from industries is small, then pollution is large-scale to a degree of 75% certainty.

The next step, after the designation of the rules, deals with the procedure by which the results are drawn from the existing facts and available knowledge is exported. In order to export the conclusion arising from the fuzzy system of the application the following operators were employed.

Aggregation operator: Gamma (0.5)

Implication operator: Algebraic Product

Accumulation operator: Algebraic Sum

Table 3 gives the degree of participation for each of the three classes—small-scale, medium-scale and large-scale-regarding the results arising from the fuzzy system evaluation of SO_2 pollution.

Table 3. Results of the fuzzy system for the estimation of the pollution by SO_2 (partial).

Pixel ID	Pollution		
	Small-Scale	Medium-Scale	Large-Scale
1	0.6124	0.7906	0.0000
2	0.5000	0.8660	0.0000
3	0.3536	0.9354	0.0000
….	…	…	…
3546	1.0000	0.0000	0.0000
3547	1.0000	0.0000	0.0000
3548	1.0000	0.0000	0.0000

Figure 9 presents a depiction of the results arising from the fuzzy system evaluation of SO_2 pollution in the area studied.

It is necessary to employ a membership function for the conversion of the linguistic variables into crisp numbers. **Figure 10** shows the membership function defined to defuzzify the output of the fuzzy system.

The results arising from the application of the six defuzzification methods referred to in the previous subsection are given in **Table 4**.

In order to present the results in a more comprehensible form and for ease of interpretation, the equal interval method with five categories was employed. **Table 5** and **Figure 11** present this new statistical information grouped according to the five categories and this information in graph form respectively.

Figure 12 presents the spatial distribution of the defuzzified results grouped according to equal interval classes.

On upon examining **Figure 11** and the spatial distribution of the results in **Figure 12**, we observe the following:

As regards the maxima methods (FOM, LOM, MOM), it is obvious that for the first category (involving values for $SO_2 < 8$ μg/m^3), the MOM method is not to be observed at all, whilst in the fourth category (involving values for $SO_2 < 8$ μg/m^3), the LOM method is not to be observed, either. Moreover, the defuzzified values that arise present a considerable deviation. In the final category (involving values for $SO_2 > 32$ μg/m^3) the frequency of values for the FOM method is followed by the values for the MOM method. Lastly, the greatest values of all appear in the LOM method.

As regards the results of the COG, ICOG ($a = 0.1$) and ICOG ($a = 0.2$) methods, no significant differences are to be observed, other than the fact that there is no first category for the COG method. If we consider the results of the ICOG method for the two threshold values, it becomes clear that, in the case in which the value rises ($a = 0.2$), there is an increase in values only in the case of the first category (involving values of $SO_2 < 8$ μg/m^3) and the last (with values of $SO_2 > 32$ μg/m^3), whilst values in the remaining three categories correspondingly decrease.

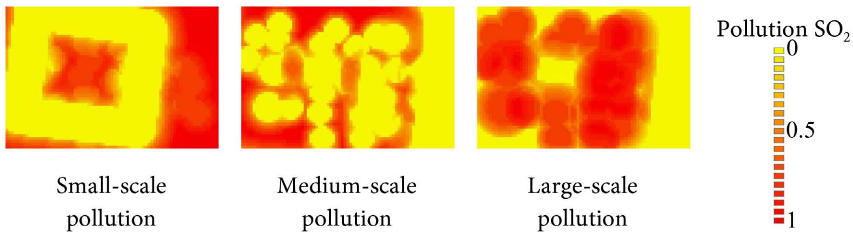


Figure 9. SO_2 Poluttion in the study area.

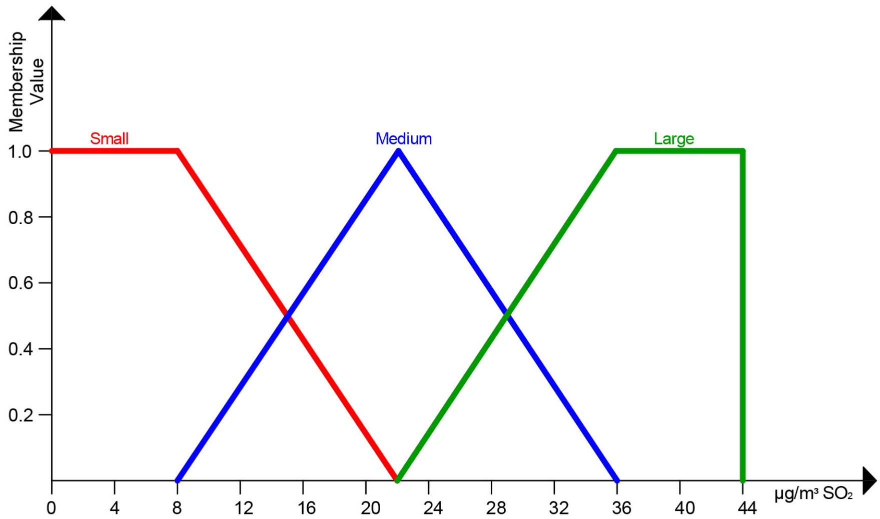


Figure 10. Defuzzification membership function of the variable "pollution—SO_2".

Table 4. Defuzzification results (partial).

Pixel ID	Pollution (SO_2 Values in μg/m³)						
	FOM	MOM	LOM	COG	ICOG (a = 0.1)	ICOG (a = 0.2)	MeOM
1	19.0680	24.9320	22.0000	16.3958	16.1380	15.9381	22.0000
2	20.1244	23.8756	22.0000	17.2604	17.1737	17.2334	22.0000
3	21.0958	22.9042	22.0000	18.5022	18.7148	19.2719	22.0000
…	…	…	…	…	…	…	…
3546	0.0000	8.0000	4.0000	8.0444	7.6126	7.1843	8.0444
3547	0.0000	8.0000	4.0000	8.0444	7.6126	7.1843	8.0444
3548	0.0000	8.0000	4.0000	8.0444	7.6126	7.1843	8.0444

Moreover, upon considering the results in Table 5, it will be seen that in the case of the LOM, MOM and COG methods, there are in fact four categories, although in anything pertaining to the MeOM method, there are only three categories to be observed. Only in the case of the FOM, ICOG (a = 0.1) and ICOG (a = 0.2) methods, results are observed for all of the four categories created for the

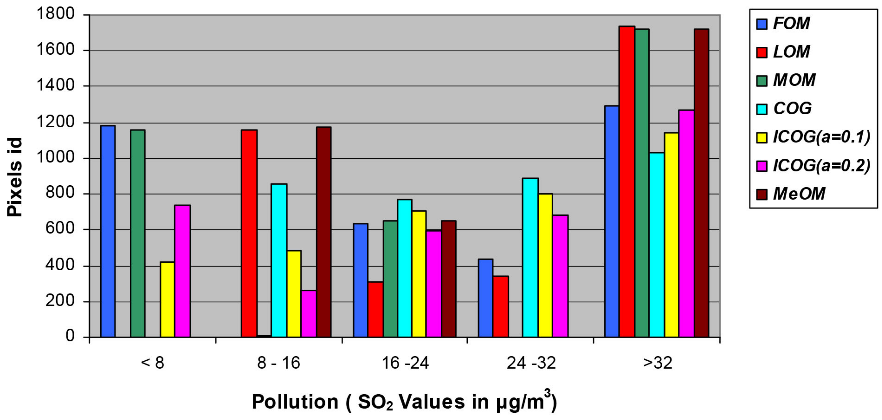


Figure 11. Chart of the defuzzified results grouped according to equal interval classes.

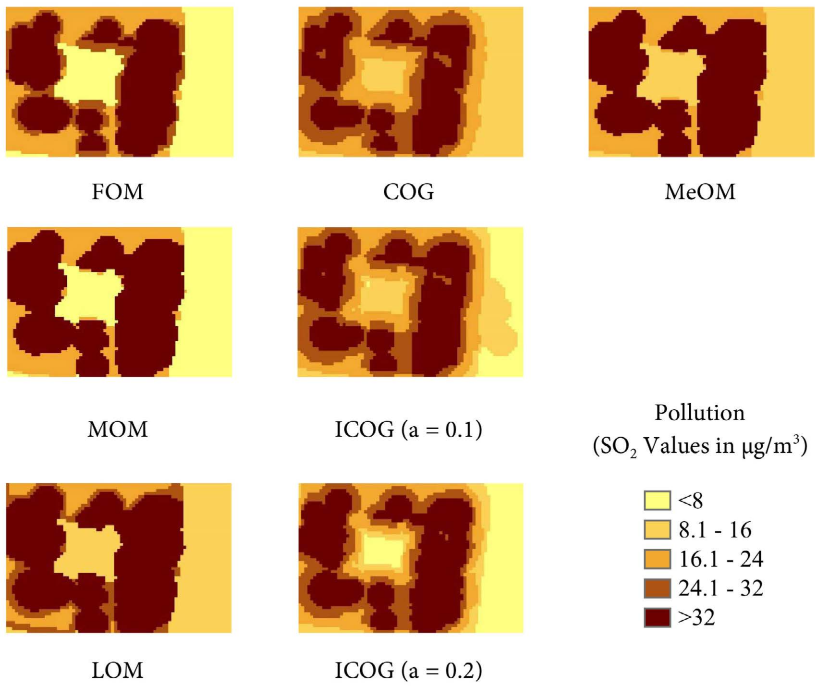


Figure 12. Depiction in visual form of the defuzzified results.

Table 5. Defuzzified results grouped according to equal interval classes.

Pollution (SO_2 Values in μg/m^3)	Number of Pixels						
	FOM	MOM	LOM	COG	ICOG (a = 0.1)	ICOG (a = 0.2)	MeOM
<8	1185	0	1159	0	419	740	0
8 - 16	3	1159	11	859	485	261	1170
16 - 24	636	306	654	769	704	598	654
24 - 32	433	344	0	885	801	680	0
>32	1289	1739	1724	1034	1139	1269	1724

needs arising from this particular application. Upon careful examination of the graphic, it is noticed that, in the first category (with values for $SO_2 < 8$ μg/m^3) only the FOM, MOM ICOG (a = 0.1) and ICOG (a = 0.2) methods appear. Moreover, in the case of the fourth category (with values for SO_2 24 - 32 μg/m^3), the FOM, LOM, COG, ICOG (a = 0.1) and IOG (a = 0.2) methods are to be observed, whilst all the methods appear in the remaining categories.

Observation of the spatial distribution of the layers of each method indicate that the GOG and IGOG methods give a gradual rendering of the differences between the pollution subgroups, in contrast with the Maxima and the MOM, MeOM methods, which depict these changes more sharply.

If the layers of defuzzified results deriving from the methods that yield them are subtracted, some interesting points emerge (Figure 13).

As for the couples derived from the Maxima methods, it is to be noticed that the difference between them is the highest of all, especially in the LOM-FOM couple. Moreover, the spatial pattern is the same in all the Maxima methods couples. The highest difference lies in the transitional zone between the subclasses.

The difference between MOM and MeOM is very low. Spatially, the difference is located in the highest values of the "large-scale pollution" subclass. The difference between GOG-IGOG (0.1) and GOG-IGOG (0.2) is also low. There is a

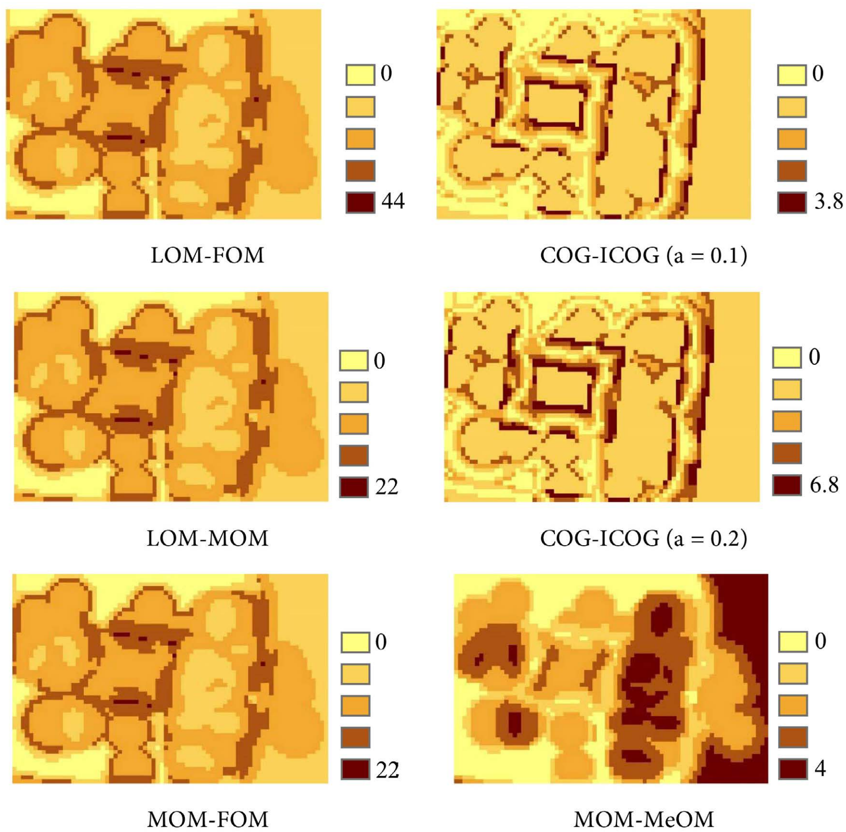


Figure 13. Visualization of the substraction of the associated defuzzification methods.

gradation in the transitional zone between the subclasses, in contrast to the results derived from the maxima methods.

5. Conclusions

In most geographic applications employing fuzzy sets, defuzzification is treated in far less detail than the other elements involved. The main reason for this is that the whole concept of defuzzification, namely the extension of crisp concepts and theories, is completely opposite to the main thrust of fuzzy set theory. The aim of this paper has been to review defuzzification operators that might be applied in geographic data of ratio scale. These operators are compared through an application of Spatial Planning.

The comparison of the methods employed may be summarized in the following terms.

- Maxima methods are simple and quick, but their results present a wide degree of internal deviation. We therefore suggest that they be employed in applications that are required to ensure large, medium, and correspondingly smallest possible values.
- Regarding the MOM and MeOM methods, it becomes clear that they follow almost the same pattern of distribution, except that in the case of the MeOM method the values are greater.
- Values deriving from the COG method display the pattern of normal distribution, their results show a gradation. In fact, it is this gradation that maximizes and maintains information, together with the ICOG method.
- The ICOG method can be regarded as specialized form, as it were, of the COG method. It indeed excludes, or rather, filters, so to speak, the classes that display a low membership value. It favors groups that show high membership values.
- Moreover, as regards any comparison among the results derived from the ICOG method, it becomes clear that for choice of threshold, the user should always perform a sensitivity analysis, if she/he is to arrive at the desired results. The analysis is of course always to be performed with the features of the problem being investigated in mind.

The application of defuzzification methods to the geographical problem selected shows that it is not possible to indicate a priori the most appropriate method of defuzzification in any GIS application. Trials always need to be made and a decision as to the most suitable method should be reached based on the requirements and peculiarities of each application. In fact, it is better to utilize all the results derived from all the methods available, to evaluate the complete extent of possible solutions.

Further research needs to be carried out on other defuzzification operators potentially capable of use on geographical data of ratio scale. The existence of an appropriate Graphical User Interface (GUI) for defuzzification purposes in a GIS environment would also be very helpful.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Zadeh, L.A. (1965) Fuzzy Sets. *Information and Control*, **8**, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Kosko, B. (1991) Neural Networks and Fuzzy Systems. Prentice Hall, Upper Saddle River.

[3] Bezdek, C.J. (1981) Pattern Recognition with Fuzzy Objective Function Algorithms. Plenum Press, New York. https://doi.org/10.1007/978-1-4757-0450-1

[4] Burrough, P. (1996) Natural Objects with Indeterminate Boundaries. In: Burrough, P. and Frank, A., Eds., *Geographic Objects with Indeterminate Boundaries*, CRC Press, London, 3-29. https://doi.org/10.1201/9781003062660-2

[5] Fischer, M.M. (1994) Expert Systems and Artificial Neural Networks for Spatial Analysis and Modelling: Essential Components for Knowledge Based Geographical Information Systems. *Geographical Systems*, **1**, 221-235.

[6] Openshaw, S. (1997) Artificial Intelligence in Geography. Wiley, London.

[7] Fischer, M.M. and Leung Y. (2001) Geocomputational Modelling: Techniques and Applications. Springer, Berlin. https://doi.org/10.1007/978-3-662-04637-1

[8] Leung, Y. (1989) Fuzzy Logic and Knowledge-Based GIS: A Prospectus. *Proceedings of IGARSS'*89 12*th Canadian Symposium on Remote Sensing*, Vancouver, 10-14 July 1989, 47-50.

[9] Robinson, B. (1988) Some Implications of Fuzzy Set Theory Applied to Geographic Databases. *Computers*, *Environment and Urban Systems*, **12**, 89-97. https://doi.org/10.1016/0198-9715(88)90012-9

[10] Robinson, B. and Strahler, H. (1984) Issues in Designing Geographic Information Systems under Conditions of Inexactness. *Proceedings of* 1984 *Machine Processing of Remote Sensing Data Symposium*, Lafayette, 12-14 June 1984, 198-204.

[11] Wang F. (1994) Towards a Natural Language User Interface: An Approach of Fuzzy Query. *International Journal of GIS*, **8**, 143-162. https://doi.org/10.1080/02693799408901991

[12] Kollia, V.J. and Voliotis, A. (1991) Fuzzy Reasoning in the Development of Geographical Information Systems FRSIS: A Prototype Soil Information System with Fuzzy Retrieval Capabilities. *International Journal of GIS*, **5**, 209-223. https://doi.org/10.1080/02693799108927844

[13] Klir, G. and Yuan, B. (1995) Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall, Upper Saddle River.

[14] Zadeh, L.A. and Kacprzyk, J. (1992) Fuzzy Logic for the Management of Uncertainty. Wiley, Hoboken.

[15] Van Leekwijck, W. and Kerre, E. (1999) Defuzzification: Criteria and Classification. *Fuzzy Sets and Systems*, **108**, 159-178. https://doi.org/10.1016/S0165-0114(97)00337-0

[16] Hatzichristos, T. and Potamias, J. (2004) Defuzzification Operators for Geographic Data of Nominal Scale. *Geoinformatics* 2004: *Proceedings of the* 12*th International Conference on Geoinformatics*: *Geospatial Information Research*: *Bridging the Pa-*

cific & Atlantic, 7-9 June 2004, Gavle.

[17] Hofmman, P. (2016) Defuzzification Strategies for Fuzzy Classifications of Remote Sensing Data. *Remote Sensing*, **8**, Article No. 467. https://doi.org/10.3390/rs8060467

[18] Driankov, D., Hellendorn, H. and Reinfrank, M. (1993) An Introduction to Fuzzy Control. Springer, Berlin. https://doi.org/10.1007/978-3-662-11131-4

[19] Runkler, T. and Glesner, M. (1993) Multidimensional Defuzzification—Fast Algorithms for the Determination of Crisp Characteristic Subsets. *Proceedings of the* 1995 *ACM Symposium on Applied Computing*, Nashville, 26-28 February 1995, 575-579

[20] Jager, R. and Filev, P. (1994) Essentials of Fuzzy Modelling and Control. Wiley, Hoboken.

[21] Užga-Rebrovs, O. and Kuļešova, G. (2017) Comparative Analysis of Fuzzy Set Defuzzification Methods in the Context of Ecological Risk Assessment. *Information Technology and Management*, **20**, 25-29. https://doi.org/10.1515/itms-2017-0004

[22] Warehouse of the Greek Infrastructure of Spatial Information. http://geodata.gov.gr