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Experimental Analysis of Hydraulic Conductivity for Saturated Granular Soils

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Abstract

Hydraulic conductivity is the ability of a porous media to transfer water through its pore matrix. That is a key parameter for the design and analysis of soil fluid associated structures and issues. This paper presents the test results of the vertical hydraulic conductivity k_v carried out on one poorly graded sand and three gap graded gravely sand. It was found that the vertical hydraulic conductivity of saturated soil depends on the grain size distribution curve, on the initial relative density of the soil. Compilation of these current test results and other test results published, shows that the common approaches predict well to some extent the vertical hydraulic conductivity k_v for the poorly graded sand materials and underestimate the k_v values for gap graded gravely sand materials. Therefore, new approaches are developed for the prediction of the vertical hydraulic conductivity in saturated poorly graded sand and gap graded gravely sand. The derived results from the new approaches lie in the range of the recommended values by (EAU 2012) and (NAVFAC DM 7 1974).

Keywords

Permeability, Hydraulic Conductivity, Tests, Saturated Granular Soils, Prediction Approaches

1. Introduction

Hydraulic conductivity is the ability of a porous media to transmit water through its pore matrix or voids. The hydraulic conductivity of soils is of great importance for the design and analysis of seepage related phenomenon and infrastructure.

In geotechnical engineering, knowledge of the hydraulic conductivity is important for many subsoil fluid-associated problems, seepage within and below earth dams and dykes, internal erosion in soil masses, settlement rates of consolidating clays, design of cut off structures or pressure-relief systems, and optimum design of oil or water well fields, (Chapuis and Gill 1989) [1]. The design of drainage systems for roads, airfields (Cedergren 1967) [2], or agricultural fields also requires a knowledge of hydraulic conductivity.

In the same way, groundwater seepage conditions are key parameters for drinking water supply, sustainable management of water resources, water contamination engineered facilities for waste storage, safety of waste repositories, basin-scale hydrogeological circulation, stability analyses, and many other problems on subsurface hydrology and geotechnical engineering, (Terzaghi and Peck 1964 [3], Chapuis 2012 [4]). Seepage is linked directly to hydraulic conductivity k_v through Darcy's law, (Darcy 1856) [5]. The k_v value of soils can be either measured or predicted. Most natural soils have spatially variable hydraulic properties. This implies that many hydraulic conductivity data are needed to adequately characterize the field k_v value. Most projects do not have the budget to carry out many fields and laboratory permeability tests, which are time consuming and more expensive than predictions. Alternatively, methods of estimating hydraulic conductivity are used to predict either the saturated hydraulic conductivity or the hydraulic conductivity dependent on the degree of saturation $k(S_r)$ at any degree of saturation S_r. Predictive methods use common soil properties such as porosity, grain size distribution curve, and consistency limits, which are routinely and economically determined for the most of construction projects.

In soil science, predictive methods consider the soil texture, its bulk density, clay content and organic matter content (Kunze *et al.* 1968 [6], Costa 2006 [7], Ghanbarian-Alavijeh *et al.* 2010 [8], Chapuis 2012 [4]). In this paper, the soil definition is that used for engineering or construction materials. It is not that used in soil science and agriculture, which corresponds to "top soil" in engineering. Therefore, the soils examined hereafter contain little or no organic matter and they have a single porosity (no fissures or secondary porosity that may be due to weathering effects or biological intrusions).

In theory, the hydraulic conductivity of saturated soil depends on the pore matrix, *i.e.* the pore sizes, and on how the pores are distributed and interconnected. Van De Genachte *et al.* (1996) [9] used principal component analysis in combination with multiple linear regression to model infiltration process and parameters, including the hydraulic conductivity of sandy soil. Another example of such development is the use of artificial neural networks (ANNs) for modelling soil hydraulic properties including the hydraulic conductivity as function of grain-size distribution and possibly other material parameters such as bulk density (Pachepsky *et al.* 1996 [10], Erzin *et al.* 2009 [11]).

Although a detailed description of the continuous complex void space is needed in theory to study seepage, this description is a scientific challenge (Silveira 1965 [12], Wittmann 1980 [13], Juang and Holtz 1986 [14], Witt 1986 [15], Xiong *et al.* 2016 [16], de Vries *et al.* 2017 [17]).

This explains why most methods of practice for predicting the hydraulic conductivity of saturated soil use the grain size distribution curve (GSDC) instead of the pore space such as the pore size distribution curve (PSDC). Evidently, the PSDC is linked to the GSDC (Ahlinhan and Adjovi 2019 [18]). Simplified descriptions of the pore space, such as bundles of straight tubes, have been used to predict the hydraulic conductivity of saturated soil k_{sat}. However, most predictive methods for k_{sat} use typical parameters such as the soil porosity n (or the void ratio e) and the grain size distribution curve, since the grain size distribution curve is easier to determine than the pore size distribution curve.

The paper presents the laboratory tests of the saturated vertical hydraulic conductivity for four (4) granular soils. Hereby, the initial relative density of each soil material is varied to study the influence of the initial relative density on the saturated hydraulic conductivity. Afterwards, data from excellent quality tests, performed on remoulded (homogenized) or intact soil specimens, which have been fully saturated using de-aired water and either a vacuum or back-pressure technique, and which are not prone to internal erosion, are used to assess the performance of predictive methods and to develop new prediction methods for poorly graded sand and for gap graded gravely sand.

2. Established Approaches

According to the state-of-the-art regarding the laminar and steady seepage in porous medium, the hydraulic conductivity can be predicted using empirical relationships, capillary models, hydraulic radius theories and statistical models (e.g. Artificial Neutral Networks ANN). The best models include at least three parameters to account for relationship between the flow rate and the pore matrix, for example the pore sizes, the pore size distribution, their tortuosity, their connectivity, etc.

The theory of laminar flow through homogeneous porous media is based the classical experiment performed by Darcy (1856) [5], who found the following empirical relationship by varying factors in the experiments:

$$\frac{Q}{A \cdot t} = \frac{q}{A} = k \cdot i \qquad \text{Equation (1)}$$

Or

$$v = k \cdot i \qquad \text{Equation (2)}$$

where Q is the total flux in m^3, q is the unit flux in m^3/s, A is the total cross-sectional area in m^2, t is the time in s, i is the hydraulic gradient, v is the flow velocity in m/s, and k is an experimentally determined constant called hydraulic conductivity in m/s. This constant k characterizes the permeability of a medium to a fluid, and it depends therefore on the properties of both porous media and fluid. For the porous medium it is the size the pore channels in the void network that is of key importance, and this depends on the representative or effective grain size d_e, the gradation factor S describing the shape of the grain size distribution curve, the shape factor η of the grains, and the porosity n (void ratio e or relative density D_r) of the medium. The controlling properties of the fluid are the unit weight γ and the coefficient of viscosity μ, which are dependent on the temperature T. Hence, the unit flux q can be expressed as a function the above-mentioned parameters as follows:

$$q = f\left(i, \gamma, \mu, T, d_e, S, \eta, n\right) \quad \text{Equation (3)}$$

From Equation (1) and Equation (3) it can be deduced that:

$$k = f\left(i, \gamma, \mu, T, d_e, S, \eta, n\right) \quad \text{Equation (4)}$$

2.1. Hazen Approach

One of the earliest predictive approaches for the hydraulic conductivity is the Hazen equation Hazen (1892) [19], which expresses the hydraulic conductivity (or coefficient of permeability) as function of the square of the 10th percentile grain-size diameter d_{10}.

Hazen (1892, 1911) [19] [20] developed the following empirical formula for the prediction of the hydraulic conductivity of saturated sand.

$$k = C_H \cdot d_{10}^2 \quad \text{Equation (5)}$$

where k is hydraulic conductivity or coefficient of permeability in cm/s, C_H is Hazen empirical coefficient, d_{10} is grain size in cm for which 10% of the soil material is finer.

The applicability of the Hazen formula is limited to grain size 0.01 cm < d_{10} < 0.3 cm, (Hazen 1892). Hazen developed this approach for the design of sand filter for water purification (loose, clean sand with a coefficient of uniformity C_u < 2 (Terzaghi and Peck 1964 [3])). However, this formula is used to assess the permeability of in situ soil. The value of C_H is generally assumed to be about 100 but varies in a large range from 1 to 1000 depending on the considered authors (Carrier 2003 [21]). Hazen (1892 [19]) found that the resistance to flow also varies with the temperature, being twice as great at the freezing point as at summer heat. That can be explained by the temperature dependent of the viscosity of the water. Therefore, Hazen (1892) [19] used a corrected Hazen coefficient equal to $C_H = 0.70 + 0.03T$, where T is the temperature in degrees Celsius. Thus, at 20˚C, the corrected coefficient is equal to $1.3C_H$, and the permeability would be 30% greater than at 10˚C.

Accounting for water temperature, Hazen equation becomes:

$$k = C_H \cdot \left(0.70 + 0.03T\right) \cdot d_{10}^2 \quad \text{Equation (6)}$$

2.2. Kozeny-Carman Approach

Another typical relation for the prediction of hydraulic conductivity was proposed by Kozeny (1927) [22] and later modified by Carman (1938, 1956) [23] [24]. The resulting equation is known as Kozeny-Carman (KC) equation. This equation was developed under the consideration of capillary tubes model for a porous material for which the flow equation of Navier Stokes can be used. The Kozeny-Carman (KC) depends on the void ratio, the specific surface area and can be expressed with the following relation:

$$k=\left(\frac{\gamma}{\mu}\right)\cdot\left(\frac{1}{C_{K-C}}\right)\cdot\left(\frac{1}{S_o^2}\right)\cdot\left(\frac{\mathrm{e}^3}{(1+e)}\right) \qquad \text{Equation (7)}$$

where γ is unit weight of fluid; μ is dynamic viscosity of fluid, C_{K-C} is Kozeny-Carman empirical coefficient, S_o specific surface area per unit volume of particles (1/cm), and e is void ratio. When the fluid is water at a temperature of 20˚C, $\gamma/\mu = 9.93\cdot10^4\cdot1/\text{cm}$. At 10˚C, $\gamma/\mu = 7.64\cdot10^4\cdot1/\text{cm}$,. Thus, the ratio of γ/μ at 20˚C and at 10˚C is 1.3, the same as Hazen used in his compound coefficient of permeability. For the effect of temperature on the viscosity of water reference is made to Lambe (1965) [25], Terzaghi *et al.* (1996) [26]. Carman (1956) [24] reported the value of C_{K-C} as being equal to 4.8 ± 0.3 for uniform spheres; C_{K-C} is usually taken to be equal to 5. Thus, Equation (7) becomes for water at 20˚C:

$$k=1.99\cdot10^4\cdot\left(\frac{1}{S_o^2}\right)\cdot\left(\frac{\mathrm{e}^3}{(1+e)}\right) \qquad \text{Equation (8)}$$

For example, if a soil material consists of uniform spheres with diameter d (cm),

$$S_o=\frac{\text{Area}}{\text{Volume}}=\frac{\Pi d^2}{\frac{\Pi d^3}{6}}=\frac{6}{d} \qquad \text{Equation (9)}$$

Thus, Equation (8) becomes

$$k=552.78\cdot d^2\cdot\left(\frac{\mathrm{e}^3}{(1+e)}\right) \qquad \text{Equation (10)}$$

Assuming the grain size distribution is log-linear between two consecutive sieve sizes, Carrier (2003) [21] transformed the Kozeny-Carman formula (Equation (8)) as follows:

$$k=1.99\cdot10^4\cdot\left[\frac{100\%}{\sum\left(\frac{f_i}{\left(d_{li}^{0.404}\cdot d_{si}^{0.595}\right)}\right)}\right]^2\cdot\left(\frac{1}{SF^2}\right)\cdot\left(\frac{\mathrm{e}^3}{(1+e)}\right) \qquad \text{Equation (11)}$$

where f_i is fraction of particles between two sieve sizes d_{li} (large diameter) and d_{si} d_{si} (small diameter), *SF* is shape factor. The shape factor *SF* accounts for the angularity, the roundness, and the sphericity of the grains through surface area concepts and is calculated as the surface area of a grain divided by its volume. Comparison chart regarding the roundness and sphericity of grain is reported by Santamarina and Cho (2004) [27], Cho *et al.* (2006) [28]. For the shape factor SF depending on the grain shape, reference is made to (Fair and Hatch 1933 [29], and Loudon 1952 [30]).

3. Test Materials

The hydraulic conductivity tests were performed with non-cohesive soil materials. The curve of the grain size distribution for the soils tested is shown in **Figure 1**. The soil materials consist of one poorly graded fine sand (mFSa) named A1, and three gap-graded soils: coarse sand (CSa) named E1, coarse sand (CSa) named E2 and fine gravel (FGr) named E5. For these three gap-graded soil materials, the gap is located by 15% finer by weight. The physical properties for the soils tested and the test programme are presented in **Table 1**.

4. Test Devices and Testing Procedure

Vertical permeameter tests have been carried out to analyse the hydraulic conductivity of saturated soil materials. For that a special test device have been designed and used for the vertical saturated conductivity in non-cohesive soils under vertical upward seepage (**Figure 2**, **Figure 3** and **Figure 5**) (Ahlinhan and Adjovi 2019) [18]. The clear internal size of the cylindrical pot is 28.5 cm for the diameter and 47 cm for the height for the vertical saturated hydraulic conductivity under upward seepage. Therefore, a soil sample with a diameter of 28.5 cm and a height of 30 cm is built in the cylindrical pot for the vertical saturated hydraulic conductivity under vertical upward seepage. The sample is supplied by de-aired water using an inflow pipe at the upstream side and an outflow pipe at the downstream side. The design details of the cylindrical pot are compatible with those of the vertical permeameter as recommended by ASTM D2434-68 standard (2006) [31]: equilibrium chambers, full-sections porous stones playing filter role, piezometer lateral outlets, constant-head tanks to supply de-aired water. The cylindrical pot has been equipped with lateral piezometers, as required by ASTM D2434-68 standard (2006) [31], which measure hydraulic head loss within the soil sample.

Chapuis *et al.* (1989) [32] reported possible common mistakes for rigid and flexible wall permeameter. Some preferential leakage can occur between the sample and the rigid permeameter wall. According to ASTM D2434-68 standard (2006) [31] the inner diameter of the permeameter must be at least 8 to 10 times the maximum particle size of the tested specimen to avoid preferential leakage. Since the maximum grain size of the used soil materials is 2 mm, the ratio of inner diameter of the permeameter to maximum particle size is 140, Therefore, the designed permeameter for the investigation of the saturated hydraulic conductivity fulfils the ASTM requirement regarding to the relation between the grain size of the material tested and the size of the permeameter. Hence, a preferential leakage between the sample and the rigid wall cannot be expected. Another possible reason for preferential leakage is the segregation of the solids or soil grains within the tested specimen, either during compaction or seepage (internal erosion), (Chapuis *et al.* 1989) [32]. The test sample has been built in a homogeneous way in the permeameter (see **Figure 4**). Furthermore, the inner surface of the designed permeameter showed a sufficient roughness, so that disturbing

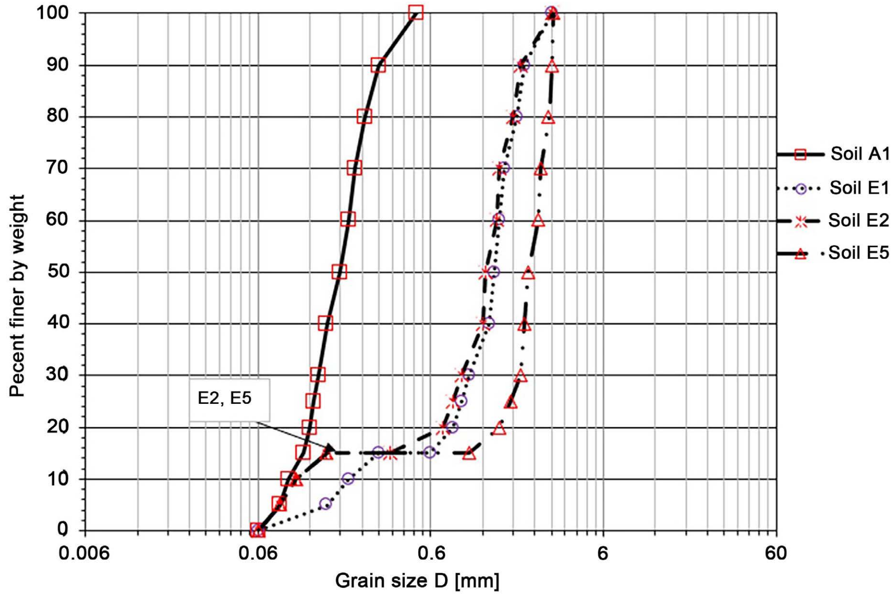

Figure 1. Grain size distribution curves of soils materials tested.

Table 1. Test programme and physical soil properties.

Parameters	Reference Soils			
	A1	E1	E2	E5
Minimum porosity $n_{\min}$	0.40	0.34	0.27	0.31
Maximum porosity $n_{\max}$	0.52	0.42	0.40	0.42
Coefficient of Curvature $C_C = (d_{30})^2/(d_{60}{*}d_{10})$	1.00	3.30	6.70	13.80
Coefficient of Uniformity $C_U = d_{60}/d_{10}$	2.10	6.80	13.50	23.40
	0.50			
Relative density $D_r = (n_{\max} - n)/(n_{\max} - n_{\min})$	0.75	0.50	0.40	0.75
	0.92	0.60	0.85	0.95

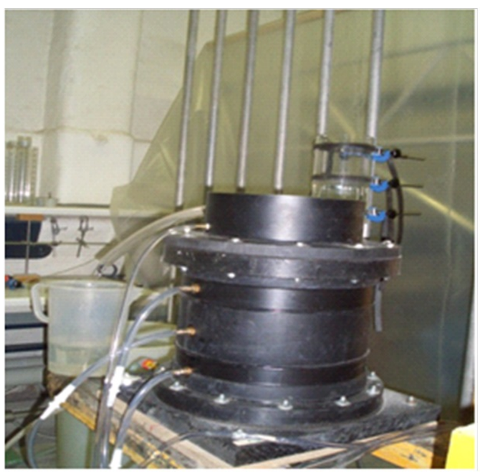

Figure 2. Photographic view of test device for vertical upward hydraulic conductivity.

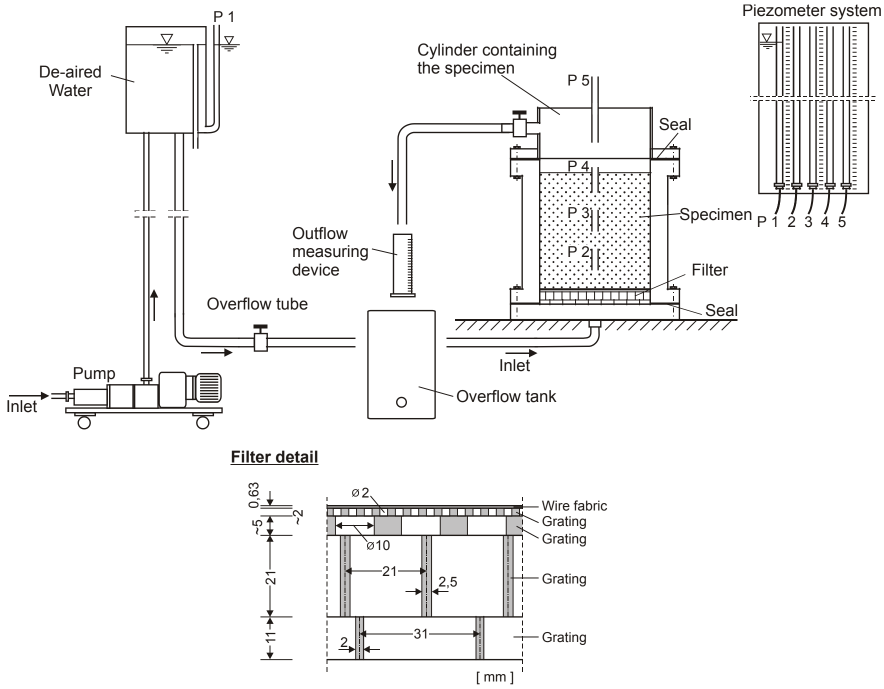

Figure 3. Schematic sketch of test device for vertical upward hydraulic conductivity.

Figure 4. Photographic top view of sample after pluviation of soil material A1 under de-aired water.

edge effects such as joint erosion or preferential leakage could be prevented. Therefore, preferential leakage due to solid segregation can be considered as negligible for the prepared samples in this study.

The potable water is primarily de-aired by boiling it up to a temperature of 120˚C and then cooling it at a laboratory temperature of 20˚C. The cohesionless soil is poured gently into the de-aired water-filled cylindrical box to ensure that almost no air voids remained, thus achieving a fully saturated soil specimen. The hydraulic head or gradient is set to a value and the seepage outflow is measured in each time interval of five minutes until the seepage outflow remained constant for three successive measurements. This indicates that a steady flow is reached, and the Darcy law is valid. The hydraulic head is then gradually increased, and the seepage outflow is measured again in each time interval of five minutes until the steady flow is reached again. This procedure was repeated as long as possible. It should be noted that from a certain hydraulic head some sand boils occur on the sample surface, which indicate the initiation of the internal erosion or suffusion (Ahlinhan and Adjovi 2019) [18]. This hydraulic gradient that leads to internal erosion or suffusion was not considered for the determination of the saturated hydraulic conductivity, due to significant increase of the hydraulic conductivity of the soil sample and migration of fine particles in the pore matrix that lead to sand boils on the sample surface. As result the grain size distribution curve of the initial sol material is modified.

The initial relative density of the soils is varied in the tests. The loose state is achieved by filling the soil material gently into the de-aired water, whereas the dense state is achieved by vibrating the designed permeameter by means of a plate vibrator while casting the soil specimen into the water. The performed compaction through light vibration is not expected to modify the grain size distribution curve of the soil materials tested. Chapuis *et al.* (1989) [32] stated that heavy compaction by means of compaction tampers can break solid soil grains thus modifying the grain size distribution curve.

The relative density is defined here with respect to the porosity of the soil as follows:

$$D_r = \frac{n_{\max} - n}{n_{\max} - n_{\min}} \qquad \text{Equation (12)}$$

Here $n_{\max}$ and $n_{\min}$ are the porosities at the loosest and the densest states, respectively, as determined in the corresponding laboratory tests. n is the actual porosity.

The filter velocity v_f is calculated as follows:

$$v_f = \frac{V}{A \cdot t} \qquad \text{Equation (13)}$$

Here v is the measured water volume through the soil sample during the time t, and A is the cross-section area of the sample.

The vertical hydraulic gradient i is calculated as the hydraulic head ΔH to the sample height h ($i = \Delta H/h$).

The sample is fully saturated due to of the sample preparation method *i.e.* pouring soil material under de-aired water. The role of the degree of saturation S_r and its influence on the hydraulic conductivity has been known for a long time (Hassler *et al.* 1936 [33], Houpeurt 1974 [34], Chapuis 2012 [4]). The role of trapped gas during permeability tests was studied by (Christiansen 1944 [35], Pillsbury and Appleman 1950 [36], Chapuis *et al.* 1991 [37], Chapuis and Aubertin 2003 [38]). Chapuis (2012) stated that most gas bubbles in the pore matrix of the testing specimens are too small to be visible. The specimen-built method used for the current tests (pluviation of soil material under de-aired water) prevented gas bubbles in the pore matrix. Furthermore, the inner roughness of the test containers prevented preferential leakage of the seepage around the soil sample.

5. Results and Discussions

Figure 5 shows the increase of the vertical filter velocity with the vertical hydraulic gradient. The vertical filter velocity increases linear proportionally with the vertical hydraulic gradient up to the critical gradient, that leads to the transport of the fine fraction of the soil material through the matrix of the coarse fraction. The coefficient of the proportionality for this segment of the curve $v_f(i_v)$ is defined as the vertical hydraulic conductivity k_v. Above the critical vertical hydraulic gradient $i_{crit,v}$, the vertical filter velocity increases significantly with the vertical hydraulic gradient. This can be explained by the transport of the soil material that leads to internal erosion.

Note that only the vertical hydraulic conductivity k_v determined for the first segment of the curve $v_f(i_v)$, *i.e.* before the initiation of the internal erosion, is considered and analysed in this paper. The vertical hydraulic conductivity for the soil material A1 (poorly graded fine sand) with a relative density $D_r = 0.50$ is read to 0.0004 m/s (**Figure 5**), while that of the soil material E1 (gap graded fine sandy coarse sand) with a relative density $D_r = 0.50$ is read to 0.0059 m/s (**Figure 6**). As expected, the vertical hydraulic conductivity of the saturated poorly graded fine sand A1 is significantly smaller than that of the gap graded fine sandy coarse sand E1 due to the larger pore matrix of the coarse sand in material E1 in comparison to the fine sand A1.

Figure 7 shows vertical hydraulic conductivity in function of the initial relative density in a semi-logarithmic diagram (abscissa axe in arithmetic scale, ordinate axe in logarithmic scale). Two similar trends can be seen from **Figure 7**. The vertical hydraulic conductivity decreases almost linearly with the initial relative density. However, this relationship between the vertical hydraulic conductivity is not the same for the poorly graded sand and the gap graded coarse sand or fine gravel. As expected, the gap graded soil materials (E1, E2 and E5) show larger vertical hydraulic conductivity than the poorly graded sand (A1).

Figure 8 presents a comparison of the vertical hydraulic conductivity of the current tests results and the approach by Hazen (1892, 1911) [19] [20]. The coefficient c_H for the Hazen approach depends not only on the soil material but also on its initial relative density. The larger initial relative density, the smaller is the value of the coefficient c_H, and the smaller is the vertical hydraulic conductivity. Good agreement can be achieved, when the coefficient c_H for the Hazen approach is varied to match to the tests results. Therefore, the coefficient c_H for the Hazen approach that depends on combined soil and fluid properties could be hardly predicted with acceptable accuracy without carrying out experimental tests.

Figure 9 presents a comparison of the vertical hydraulic conductivity of the current tests results and the approach of Kozeny and Carman (1956) [24] modified by Carrier (2003) [21]. The modified approach of Kozeny and Carman (1956) [24] by Carrier (2003) [21] predicts to some extent the vertical hydraulic conductivity for the poorly graded fine sand A1. But unlike, the modified

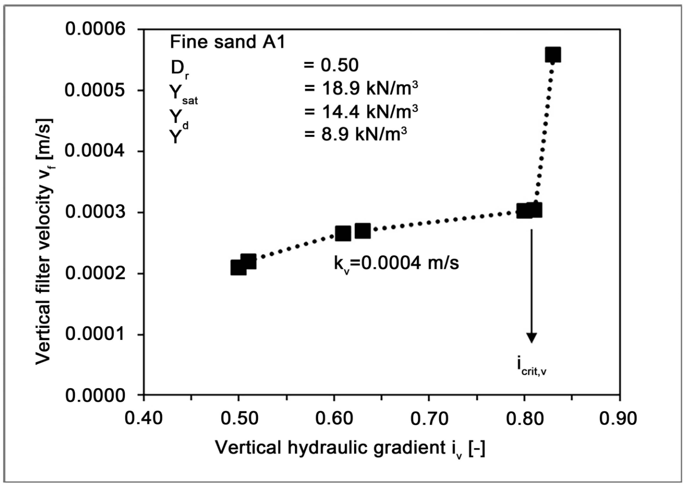

Figure 5. Filter velocity depending on vertical hydraulic gradient of soil material A1.

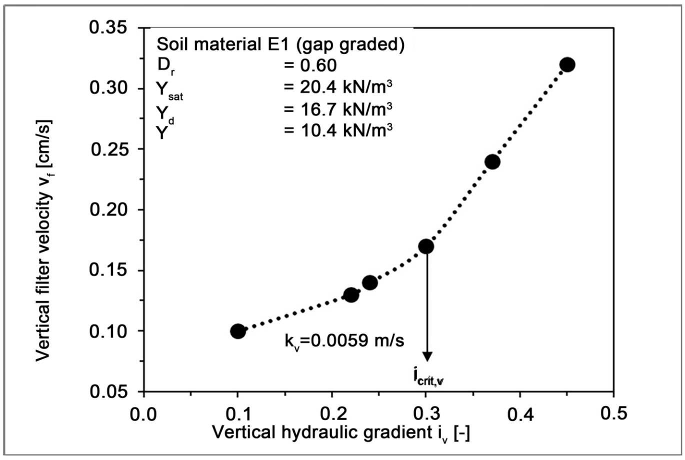

Figure 6. Filter velocity depending on vertical hydraulic gradient of soil material E1.

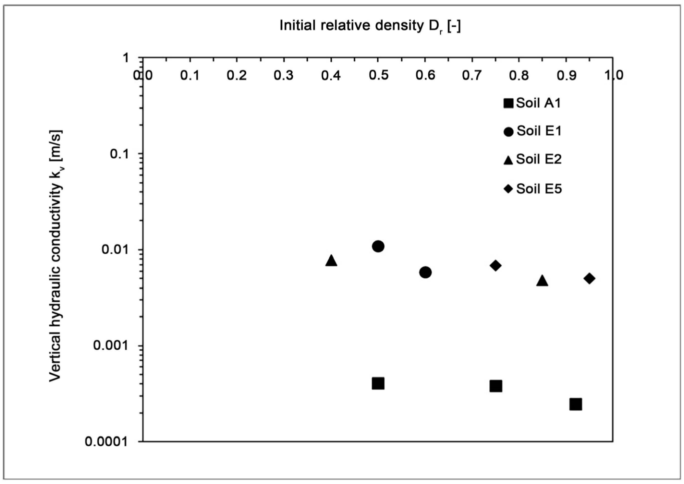

Figure 7. Vertical hydraulic conductivity of soil material tested depending on initial relative density.

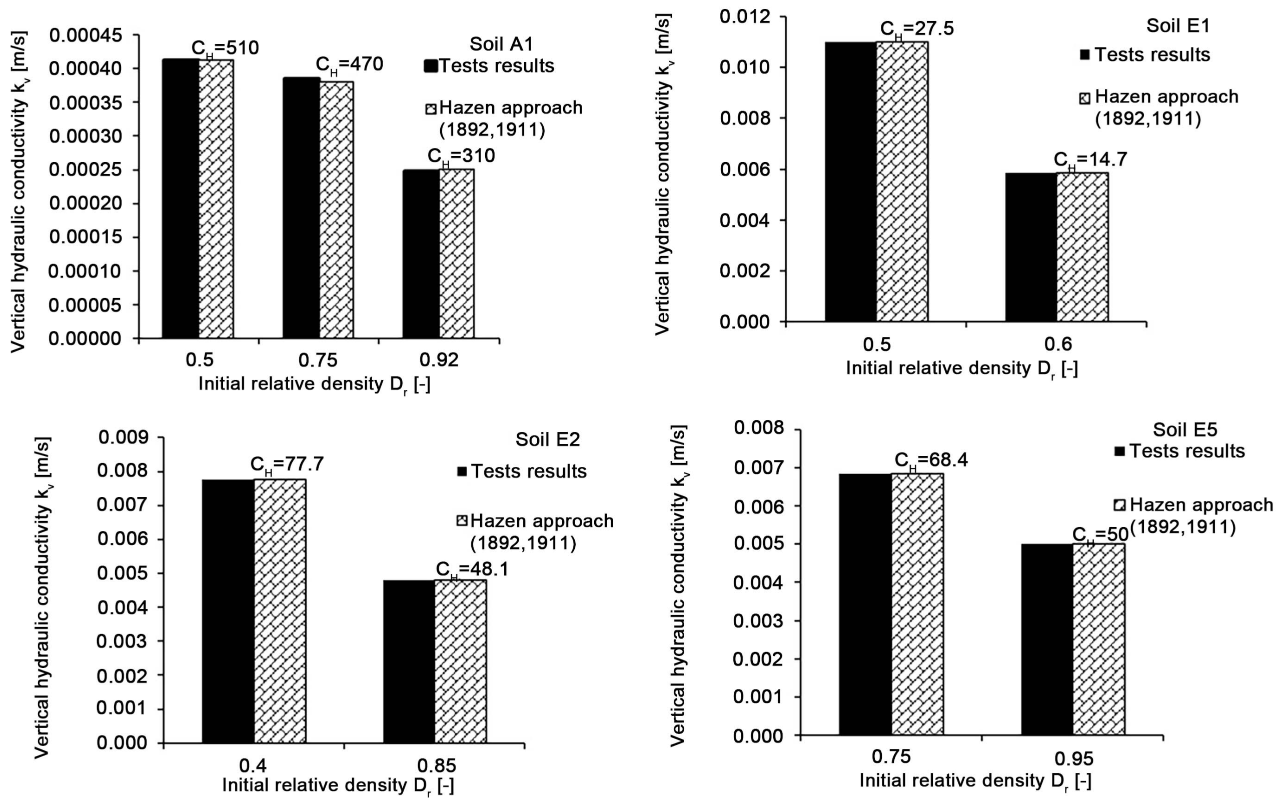

Figure 8. Comparison between tests results and whose of Hazen approach.

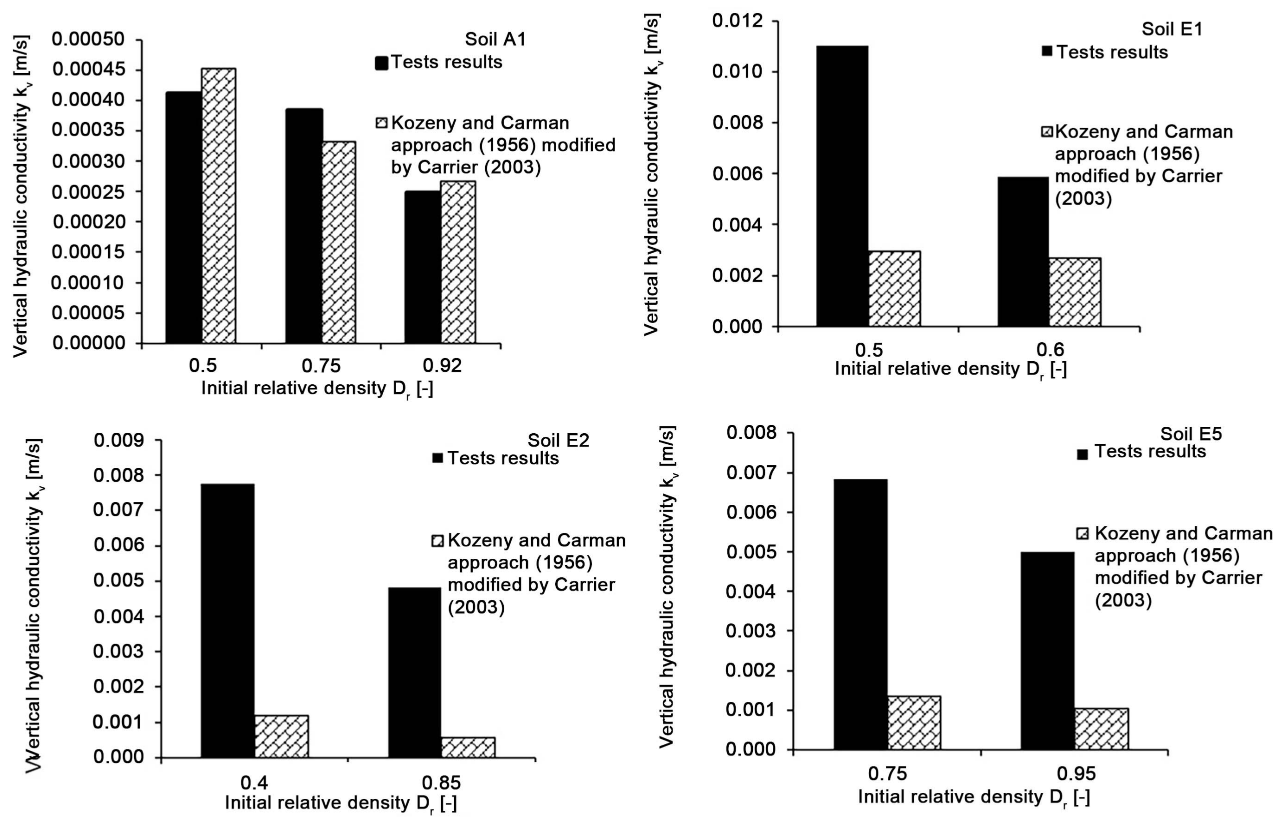

Figure 9. Comparison between tests results with Kozeny and Carman approach.

approach of Kozeny and Carman (1956) [24] by Carrier (2003) [21] significantly under-predicts the vertical hydraulic conductivity for the gap graded soil materials E1, E2, E5. However, such gap graded soil materials are often used for dyke construction and are subsoil or foundation soil for road and building construction for which a good prediction of the hydraulic conductivity is required for safe-to-safe analysis (den Adel *et al.* 1988 [39], Skempton and Brogan 1994 [40], Ahlinhan and Adjovi 2019 [18]).

Figure 10 and **Figure 11** present the compilation of the test results for the poorly graded sand and for gap graded gravely sand, respectively. The hydraulic conductivity decreases with the initial relative density. That can be explained with the decrease of the porosity with increasing initial relative density of soil material. The trend line is described by Equation (14) for poorly graded sand and by Equation (17) for gap graded gravely sand. Evidently, an estimation of the vertical hydraulic conductivity with a deviation of 15% is possible using the Equation (14) and Equation (17). Lines with ±15% deviation from the trend lines are also plotted in dash lines (**Figure 10** and **Figure 12**). This deviation can be explained with the shape, size and gradation effects and is within the expected margin of variation for laboratory permeability test results of round ±20% (Chapuis and Aubertin 2003 [38]).

$$k_v\left(m/s\right)=0.002\mathrm{e}^{-3.239D_r} \quad \text{for poorly graded sand} \qquad \text{Equation (14)}$$

Under the consideration of the grain size distribution curve, the shape factor and the initial relative density, Equation (14) can be rewritten in a form of the Kozeny-Carman equation as follows:

$$k=\left(\frac{\gamma}{\mu}\right)\cdot\left(\frac{1}{C_{K-C}}\right)\cdot\left(\frac{1}{S_o^2}\right)\cdot\left(-0.33\cdot D_r+0.55\right) \quad \text{for poorly graded sand} \qquad \text{Equation (15)}$$

For poorly graded sand and water at 20˚C Equation (15) becomes:

$$k=1.99\cdot10^4\cdot\left(\frac{1}{S_o^2}\right)\cdot\left(\frac{1}{SF^2}\right)\cdot\left(-0.33\cdot D_r+0.55\right) \qquad \text{Equation (16)}$$

$$k_v\left(m/s\right)=0.019\mathrm{e}^{-1.541D_r} \quad \text{for gap-graded gravely sand} \qquad \text{Equation (17)}$$

Under the consideration of the grain size distribution curve and the shape factor, Equation (17) can be rewritten in a form of the Kozeny-Carman equation as follows:

$$k=\left(\frac{\gamma}{\mu}\right)\cdot\left(\frac{1}{C_{K-C}}\right)\cdot\left(\frac{1}{S_o^2}\right)\cdot\left(-0.13\cdot D_r+0.18\right) \quad \text{for gap-graded gravely sand} \qquad \text{Equation (18)}$$

For gap-graded gravely sand and water at 20˚C:

$$k=1.99\cdot10^4\cdot\left(\frac{1}{S_o^2}\right)\cdot\left(\frac{1}{SF^2}\right)\cdot\left(-0.13\cdot D_r+0.18\right) \qquad \text{Equation (19)}$$

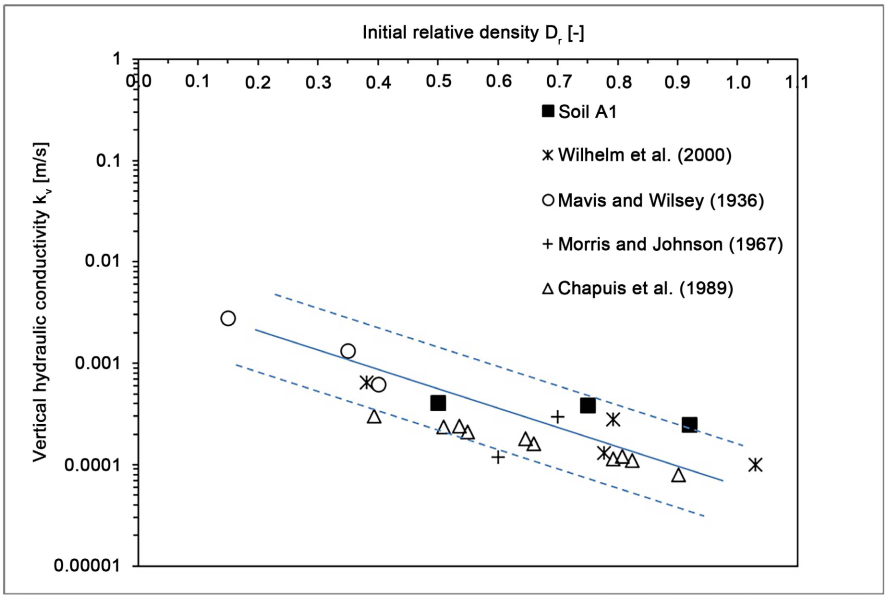

Figure 10. Compilation of test results for poorly graded sand.

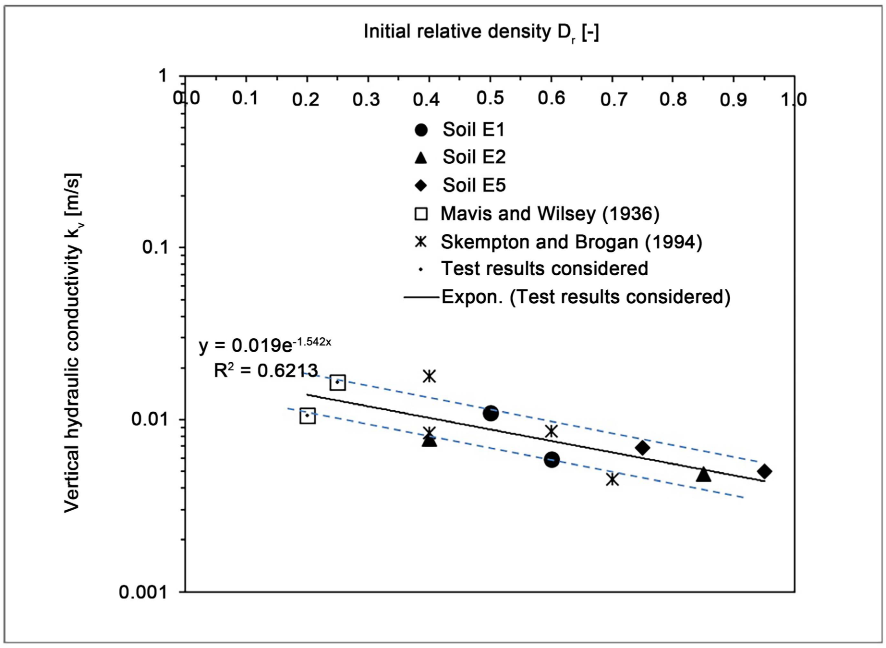

Figure 11. Compilation of test results for gap graded gravely sand.

Figure 12 presents a comparison between the proposed approach (Equation (15) or Equation (16)), the Navfac DM-7 (1974) [41] chart, the approach by Carrier (2003) [21], and the recommendations of the committee for waterfront structures harbours and waterways EAU (2012) [42]. For the calculation of the vertical hydraulic in accordance with Carrier (2003) [21] approach, a typical $1/s^2$ value of $6.1*10^{-4}$ cm^{-2} for sand is considered. It appears that the k_v values lie in general between the range of the lower bound and upper recommended by EAU (2012) [42].

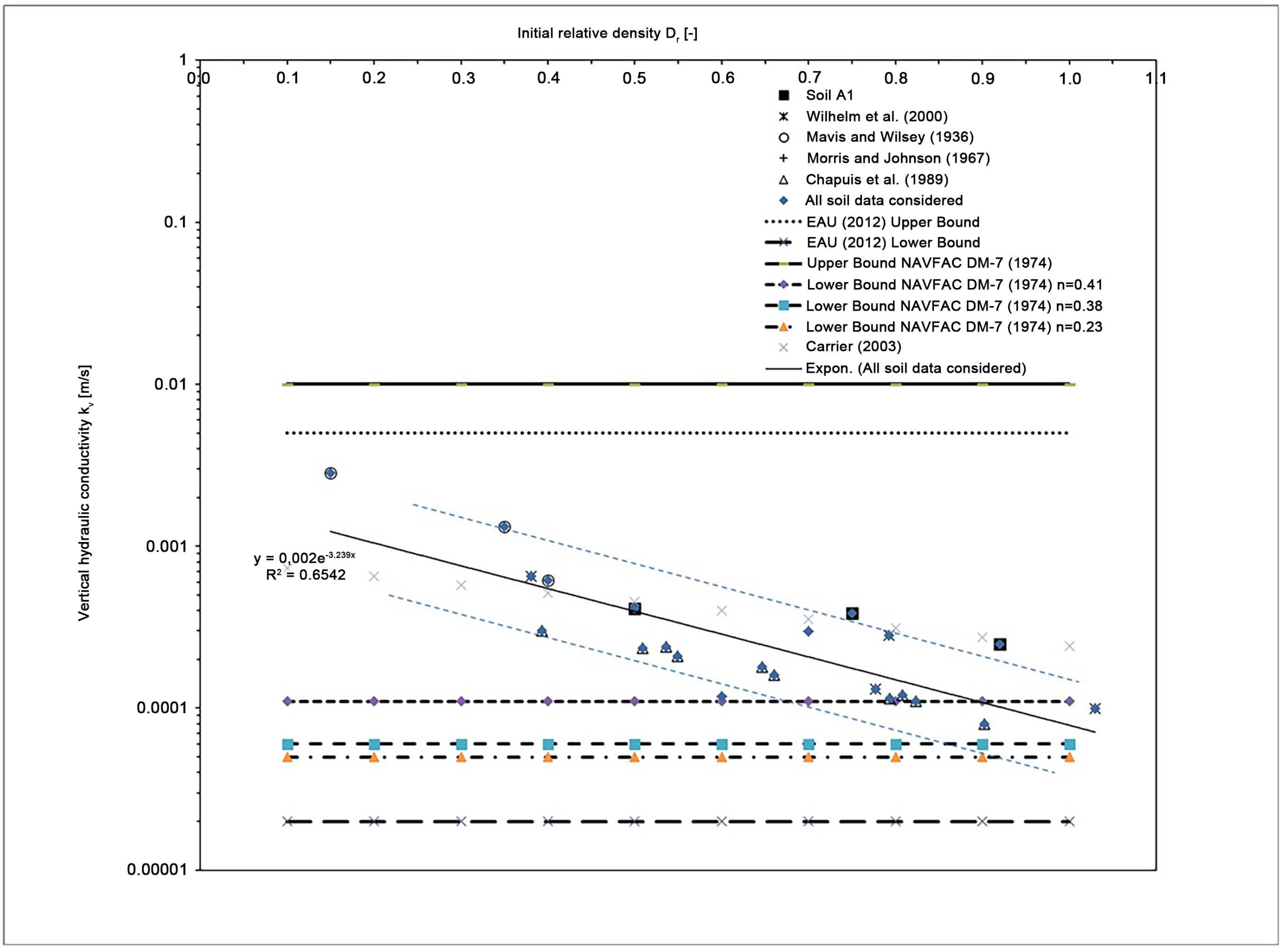

Figure 12. Comparison of test results with approaches for vertical hydraulic conductivity of poorly graded sand.

The results of the proposed approach are comparable with the results of approach by Carrier (2003) [21] for a range from 0.30 to 0.60 for the initial relative density of the soil material. For an initial relative density smaller than 0.3, the proposed approach shows larger vertical hydraulic conductivity in comparison to the approach by Carrier (2003) [21]. But for an initial relative density larger than 0.6, the proposed approach shows smaller vertical hydraulic conductivity in comparison to the approach by Carrier (2003) [21].

The transposition of laboratory results to field conditions must be done with fine engineering judgement and caution. If several precautions are not taken, there may have discrepancies between hydraulic conductivity values predicted by lab tests and field results. Possible discrepancies can be explained by test procedures, test conditions, scale effect, discrepancy in flow nets or flow rates *in-situ* and in laboratory, soil heterogeneity, anisotropy, etc. As result, it is usually assumed that the true k-value of a soil lie between 1/3 and 3 times the value given by a good laboratory test (Chapuis and Aubertin 2003 [38], Chapuis 2012 [4]). However according to standards for laboratory permeability tests ASTM (D2434, D5084, D5856) [43] [44] [45], the real precision of this testing seems unknown and therefore their bias cannot be determined. Therefore, more research in the field is needed.

6. Conclusions

The hydraulic conductivity k is a key parameter for the design and analysis of the seepage related phenomenon and infrastructure.

This paper presents laboratory results on the vertical hydraulic conductivity k_v of poorly graded sand and gap graded gravely sand. The k_v values were obtained with a design permeameter that fulfils the requirement of ASTM D 2434-68 standard (ASTM 1974) [31]. The specimens were prepared according to the typical method of pluviation under de-aired water. Therefore, full saturation of the sample was reached. Tests are performed at different relative densities to derive the curves k_v (D_r). As expected, the vertical hydraulic conductivity decreases with increasing initial relative density. These test results were compared with the published results, the approach by Hazen (1911) [20], the modified approach of Kozeny and Carman (1956) [24] by Carrier (2003) [21]. To some extent, the test results and those derived from both predictive approaches are well comparable. From an engineering viewpoint, a reasonably good fit is obtained between experimental k_v values and those derived from the predictive approach of Hazen (1911) [20] and the modified approach of Kozeny and Carman (1956) [24] by Carrier (2003) [21] for poorly graded sand. However, both approaches significantly underestimate the k_v values for the gap graded gravely sand.

Moreover, a comparison of the test results with the published results shows good agreement both for the poorly graded sand and the gap graded gravely sand. Based on this data compilation new approaches for the prediction of the vertical hydraulic conductivity account for soil and fluid properties are developed. The k_v values derived from the developed approaches lie in the range for k_v values in accordance with NAVFAC DM (1974) [41] and EAU (2012) [42].

The advantage of this approach resides in the use of common soil and fluid properties such as curve of grain size distribution, porosity or relative density, shape factor of soil grain, unit weight of fluid, dynamic viscosity of fluid, etc.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Highlights

- Permeameter tests were carried out on four granular soils.
- Test results were evaluated, analysed, and compared with the common predictive approaches for vertical hydraulic conductivity.
- Good agreement between results tests and common predictive approaches was shown for poorly graded sand.
- Poor agreement between results tests and common predictive approaches was found for gap graded gravely sand.
- Approaches for determination of vertical hydraulic conductivity were developed based the test results performed and published tests results.
- Results from these new approaches compared well to some common predictive approaches.

List of Symbols

C_U	Coefficient of uniformity, $C_U = d_{60}/d_{10}$
C_C	Coefficient of curvature, $C_C = (d_{30})^2/(d_{60}*d_{10})$
d	Grain size (mm)
d_x	Grain size (mm) such that x% of the solid mass is made of grains finer than d_x
D_r	Relative density, $D_r = (n_{max} - n)/(n_{max} - n_{min})$
e	Void ratio (m^3/m^3); $e = n/(1 - n)$
e_{max}, e_{min}	Maximum, minimum void ratio (m^3/m^3)
GSDC	Grain size distribution curve
H	Hydraulic head (m)
G_s	Specific gravity of solids, $G_s = \rho_s/\rho_w$
k_v	Vertical hydraulic conductivity (m/s)
n	Porosity (m^3/m^3); $n = e/(1 + e)$
n_{max}, n_{min}	Maximum, minimum porosity (m^3/m^3)
RF	Roundness factor (number)
S_r	Degree of saturation (% or m^3/m^3)
S_S	Specific surface (m^2/kg)
t	Time (s)
T	Temperature (degrees Celsius)
w	Water content (% or kg/kg)

Greek Letters

γ_s, γ_w	Specific gravity (kN/m^3) of solids, of water
μ_x	Water dynamic viscosity (Pa·s) at temperature x
μ_w	Water dynamic viscosity (Pa·s)
γ_d	Dry density (kN/m^3)
ρ_s, ρ_w	Density (kg/m^3) of solids, of water