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Influence of Anisotropic Permeability and Soret Effect on the Convective Heat and Mass Transfer through a Porous Cavity Saturated by a Non-Newtonian Fluid

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Abstract

In this work, an analytical study is carried out on double-diffusive natural convection through a horizontal anisotropic porous layer saturated with a non-Newtonian fluid by using the Darcy model with the Boussinesq approximations. The horizontal walls of the system are subject to vertical uniform fluxes of heat and mass, whereas the vertical walls are assumed to be adiabatic and impermeable. The Soret effect is taken into consideration. Based on parallel flow approximation theory, the problem is solved in the limit of a thin layer and documented the effects of the physical parameters describing this investigation.

Keywords

Soret Number, Heat and Mass Transfer, Non-Newtonian Fluid, Isotropy, Anisotropy, Supercritical Rayleigh Number

1. Introduction

The study of natural convection in porous media has been the subject of much research in the past. For an exhaustive review on this subject, see for example the book of Nield and Bejan [1]. The fluids studied in these research works are in general Newtonian fluids. However, there are many situations in practice where the considered fluids are non-Newtonian (oil drilling, design of chemical reactors, storage of radioactive materials, soil pollution problems, separation processes, geothermal systems, etc.). An exhaustive review of work on the convection of non-Newtonian fluids in porous media is presented as follows.

1.1. Convection of Non-Newtonian Fluids in Isotropic Porous Media

Chen *et al.* [2] were the first to consider the natural convection of non-Newtonian fluids along an impermeable horizontal plate in an isotropic porous medium, subjected to non-uniform heat fluxes. The power-law model, originally proposed by Christopher and Middleman [3] and later modified by Dharmadhikari and Kale [4], was used to model non-Newtonian fluids. The study was based on the boundary layer approximation valid for high Rayleigh numbers. The effects of non-uniform heat flux distribution on heat transfer characteristics were discussed. Pascal ([5] [6]) proposed a modified Darcy equation deduced from experimental measurements of the flow of non-Newtonian fluids through a simple capillary tube filled with an isotropic porous medium. Unlike the model defined by Dharmadhikari and Kale [4], Pascal's model takes into account the power law dependence on temperature. Indeed, some complex substances, such as oil sands, are sensitive to ambient temperature. For relatively high temperatures, their behavior tends towards that of a newtonian fluid. However, for intermediate temperatures, the behavior becomes non-Newtonian. Chen and Chen [7] studied the free convection of a non-Newtonian fluid along a vertical plate embedded in a porous medium. The authors used the boundary layer approximation and the modified Darcy model [Pascal ([5] [6])] to predict the resulting flow. The behavior of the fluid was modeled with the power law model. The results showed the influence of the behavior index (n) on the thicknesses of the dynamic and thermal boundary layers. For $n < 1$, the thickness of the thermal layer was greater than that of the dynamic boundary layer. The opposite happened for $n > 1$. This phenomenon is explained by the fact that the increase in the behavior index leads to an increase in the viscosity of the fluid. Pascal and Pascal [8] studied the rheological effects of non-Newtonian fluids on the mechanism of natural convection in a porous medium. These effects are predicted for the case of a power law fluid with flow stress when this is temperature dependent. The case of a heated vertical cylinder and inserted in a porous medium has been studied. The surface of the cylinder was kept at a constant temperature or heated by a constant flow of heat. Approximate self similar solutions in finite form and numerical solutions were obtained. The results show a significant difference between the velocity and temperature profiles of the Newtonian and non-Newtonian cases. It turned out that the threshold of natural convection was a function of the shear induced in the fluid. Amari *et al.* [9] obtained numerical results for the case of the natural convection within a horizontal porous cavity, saturated by a non-Newtonian fluid and subjected to horizontal and vertical heat flows. The modified Darcy model and the power fluid law were used. Results have been achieved for Rayleigh numbers varying from 30 to 1000 and for behavior indices n varying between 0.6 and 1.4. These results showed that the increase in the behavior index ($n > 1$, *i.e.* fluid for a dilatant) causes a strong reduction in heat transfer. Indeed, the increase in the apparent viscosity in this case considerably slows down the flow of the fluid. The opposite occurs for a behavior index $n < 1$ (*i.e.* fluid for a pseudo-elastic). In addition, an analytical solution was considered on the basis of an approximation of a flow parallel in the case of a cavity with a very large extension. The analytical and numerical results were in excellent agreement. Getachew *et al.* [10] considered the natural convection in steady state, within a porous cavity of square shape, saturated by a non-Newtonian fluid. A numerical method and a dimensional analysis have been developed by these authors. The modified Darcy model and the power law were used to model the problem. Correlations were made between the Nusselt number, the consistency index and the Rayleigh number. Four different modes of heat transfer have been identified: a pure conduction regime, a strong convection regime (with sufficiently large Rayleigh numbers), a convective regime with a horizontal boundary layer (crushed cavity) and a convective regime with a boundary layer vertical (slender cavity). The results showed that the Nusselt number depends on the Rayleigh number and the behavior index. Ching-Yang Cheng [11] studied the free convection heat transfer over a truncated cone embedded in a porous medium saturated by a non-Newtonian power-law nanofluid with constant wall temperature and constant wall nanoparticle volume fraction. The effects of Brownian motion and thermophoresis are incorporated into the model for nanofluids. A coordinate transformation is performed, and the obtained nonsimilar equations are solved by the cubic spline collocation method. The effects of the power-law index, Brownian motion parameter, thermophoresis parameter and buoyancy ratio on the temperature, nanoparticle volume fraction and velocity profiles are discussed. The reduced Nusselt numbers are plotted as functions of the power-law index, thermophoresis parameter, Brownian parameter, Lewis number, and buoyancy ratio. Results show that increasing the thermophoresis parameter or the Brownian parameter tends to decrease the reduced Nusselt number. Moreover, the reduced Nusselt number increases as the power-law index is increased.

1.2. Double-Diffusive Convection of Non-Newtonian Fluids in Isotropic Porous Media

Double-diffusive or thermosolutal natural convection is a fluid motion due to simultaneous variations of temperature and concentration in the gravity field. The work available on the phenomena of heat and mass transfer by natural convection in porous media is recorded in the books of Bejan [12], Platten and Legros [13] and Nield and Bejan [1]. According to the literature review of Redha *et al.*, the problem of thermosolutal natural convection in enclosures filled with saturated porous media had been the subject of numerous recent and past studies. The interest rose from the occurrence of the phenomenon in many engineering applications such as geothermal energy, diffusion of moisture in fibrous insulations, food processing, drying processes, spread of pollutants in soil, solar ponds, crystal growth in fluids, and metal casting [14] [15] [16]. The enormous interest in the double diffusive convection in the recent years has led researchers to an extensive study on this topic. The various aspects related to the heat and mass transfer have also been addressed in the extensive literature [17]-[28]. These researchers considered that the porous medium is saturated by a Newtonian fluid. Little research takes into account a non-Newtonian fluid. Rastogi and Poulikakos [29] studied doubly diffusive convection on a vertical surface embedded in a porous medium saturated with a non-Newtonian fluid. Cases where the vertical surface is heated and salted according to a constant temperature and concentration distribution or by heat and mass fluxes have been considered. A scaling analysis identified several flow regimes related to the volume force ratio N and the Lewis number Le. A numerical solution made it possible to highlight the dependence of the current, temperature and concentration function fields on the behavior index of the non-Newtonian fluid. Getachew *et al.* [30] considered the double diffusion within a rectangular porous cavity, saturated by a non-Newtonian fluid, and subjected to horizontal temperature and concentration gradients. A scaling analysis made it possible to highlight the variation of the Nusselt number and the Sherwood number with the control parameters, namely the behavior index n, the Rayleigh number R, the Lewis number Le and the ratio of volume forces N. The influence of the behavior index on the current function fields, temperature and concentration was also discussed. The numerical solution of the governing equations is in good agreement with the analytical model. It highlights the impact of the control parameters (n, R, Le and N) on the mean Nusselt and Sherwood numbers, as well as the influence of the fluid behavior index on the heat transfer rates and mass. Jumah and Mujumdar [31] considered free, doubly diffusive convection in the case of non-Newtonian viscoplastic fluids above a vertical plate embedded in a porous medium. Power law and the modified Darcy model were used. The influence of the control parameters (n, Le and N) on velocity, temperature and concentration profiles were discussed. The results showed that the threshold of free convection is a function of the shear of the fluid and consequently of the law which governs the behavior of the latter. Darcy model with the Boussinesq approximations is used by K. Benhadji and P. Vasseur [32], to study double-diffusive convection in a shallow porous cavity saturated with a non-Newtonian fluid. A power-law model is used to characterize the non-Newtonian fluid behaviour. The problem is solved analytically, in the limit of a thin layer, using a parallel flow approximation. Solutions for the flow fields, Nusselt and Sherwood numbers are obtained explicitly in terms of the governing parameters of the problem. A good agreement is obtained between the analytical prediction and a numerical solution of the full governing equations.

1.3. Double-Diffusive Convection of Newtonian Fluids in Isotropic Porous Media

According to the literature review of Redha *et al.* [33], Changhao and Payne [34] presented a mathematical study on the thermosolutal convection in a porous medium where the Darcy model was employed. The authors established a continuous dependence of the flow solution on the Soret effect. Theoretical and numerical analysis of Soret-driven convection in a horizontal porous layer saturated by an n-component mixture was investigated by Mutshler and Mojtabi [35]. In the first part, an analytical and numerical study of the onset of Soret driven convection was presented. The study was based on the classical Darcy-Boussinesq equations, which admitted a mechanical solution associated with the pure double-diffusive regime. In the second part, the analytical solution for the unicellular flow was obtained, and the separation was expressed in terms of the Lewis number, the separation ratio, the cross-diffusion coefficient and the Rayleigh number. Benano-Molly *et al.* [36] investigated the effect of Soret coefficient within a rectangular porous medium saturated by a binary fluid mixture when the thermal and solutal buoyancy forces were opposing each other. It was shown that, when the solutal buoyancy force ratio was negligible, the theory represented well the solute behavior. Mansour *et al.* [37] studied the Soret effect on double diffusive convection and on heat and mass transfer rates in a square cavity. The heat transfer rate was found to be significantly affected by the Soret effect. Furthermore, Joly *et al.* [38] presented an analytical and numerical study of the influence of the Soret effect on the onset of convection in a vertical porous cavity saturated with a binary mixture. The vertical walls were subjected to uniform heat fluxes. The Brinkman extended Darcy model was used to solve the governing equations. The results indicated that the critical Rayleigh number depended strongly upon of the control parameters such as the aspect ratio of the cavity, the Darcy and the Lewis numbers. Gaikwad *et al.* [39] made an analysis of thermosolutal convection in a horizontal anisotropic saturated porous layer with Soret effect. The heat and mass transfer rates increased with the anisotropy parameters and the Lewis number; in addition, the heat transfer increased with the negative Soret parameter while it decreased with the positive one. A reverse trend was found for the mass transfer rate. Malashetty *et al.* [40] presented a numerical investigation of thermosolutal convection in a porous layer saturated by a couple-stress fluid with Soret effect. Linear and weak nonlinear stability analyses were performed. The heat and mass transfer rates decreased with increasing the Taylor number and the couple-stress parameter, while both increased with increasing the solute Rayleigh number. The heat transfer rate decreased with increasing the Lewis number while the mass transfer rate increased significantly.

1.4. Double-Diffusive Convection of Newtonian Fluids in an Anisotropic Porous Media

There is little research done on double-diffusive convection through an anisotropic porous media saturated by a Newtonian fluid. Thermohaline convection with cross-diffusion in an anisotropic porous medium was studied in 1989 by PRABHAMANI and PARVATHY [41], using normal mode technique. Their results shown that 1) values of the anisotropy parameter are important in deciding the mode of convection in a doubly diffusive fluid saturating a porous medium, 2) depending on the values of the Soret and Dufour parameters, an increase in anisotropy parameter either promotes or inhibits instability, 3) cross-diffusion induces instability even in a potentially stable set-up and 4) for certain values of the Dufour and Soret parameters there is a discontinuity in the critical thermal Rayleigh number, which disappears if the porous medium has horizontal isotropy. Malashetty *et al.* [42] conducted a study on the onset of double diffusive convection in a binary viscoelastic fluid saturated anisotropic porous layer They used The modified Darcy law for the viscoelastic fluid of the Oldroyd type to model the momentum equation. The effect of anisotropy parameters, Darcy-Prandtl number, relaxation, and retardation parameters on the stability of the system is investigated. The nonlinear theory based on the truncated representation of Fourier series method is used to find the transient heat and mass transfer. The effect of various parameters on heat and mass transfer is also brought out. In 2011, Malashetty *et al.* [43] studied the onset of double diffusive convection in a binary viscoelastic fluid-saturated anisotropic rotating porous layer using a linear and a weakly non-linear stability analyses. The modified Darcy law for the viscoelastic fluid of the Oldroyd type is used to model the momentum equation. The onset criterion for stationary and oscillatory convection is derived analytically. The effect of anisotropy parameters, Vadasz number, relaxation and retardation parameters on the stability of the system is investigated. It is found that contrary to their usual influence on the onset of convection in the absence of rotation, the thermal anisotropy parameter and Vadasz number show contrasting effect on the onset criterion. Abdelraheem and Mitsuteru [44] studied double-diffusive natural convection with cross-diffusion effects in an anisotropic porous enclosure using Incompressibe Smoothed Particle Hydrodynamics (ISPH) method. Their results show that an increase of the permeability ratio parameter leads to decrease in the both of heat conduction and flow regime. As the Soret number increases with decreasing the value of Dufour number, the average Nusselt number increases. While, the average Sherwood number decreases as the Soret number increases with decreasing the Dufour number. Ajay and Kanchan [45], conduced, in 2018, a study on double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer with internal heating and soret effect. The problem has been solved analytically, performing linear and nonlinear analyses. The linear analysis is done using normal mode technique. Results show that the Taylor number T_a, Couple stress fluid C, solute Rayleigh number Ra_S and thermal anisotropic parameter has a stabilizing effect on both stationary and oscillatory modes of convection. A numerical study of double-diffusive convection in the anisotropic porous layer under rotational modulation with internal heat generation was conducted by Samah *et al.* [46] using the normal mode technique. The effects of time varying rotation, internal heat generation, anisotropy parameters, concentration Rayleigh, Vadasz, and Lewis numbers on the heat and mass transfer are shown graphically. Modulation amplitude and internal heating have been found to enhance the rate of heat mass transfer hence advancing the onset of thermal convection in the system. Gangadharaiah *et al.* [47] conducted a study on Darcy-Brinkman Double Diffusive Convection in an Anisotropic Porous Layer with Gravity Fluctuation and Throughflow. The critical Rayleigh numbers for the onset of stationary and oscillatory modes have been found via linear instability analysis. The impact of various gravitational functions in the presence of throughflow on stability is studied. The analysis has been carried out for decreasing and increasing gravity fluctuations. The results show that the mechanical anisotropy parameter and Lewis number have a destabilizing effect, while the thermal anisotropy parameter, Darcy number, solutal Rayleigh number, throughflow parameter, and gravity parameter have a stabilizing effect on stationary and oscillatory convection.

1.5. Double-Diffusive Convection of Non-Newtonian Fluids in an Anisotropic Porous Media

Works that have addressed the study of double-diffusive convection through anisotropic porous media saturated with non-Newtonian fluid are very rare. Yovogan J. *et al.* [48] conducted an analytical study on double-diffusive natural convection in a shallow porous cavity saturated with a non-Newtonian fluid by using the Darcy model with the Boussinesq approximations. Based on parallel flow approximation theory, the problem is solved analytically, in the limit of a thin layer and documented the effects of the physical parameters describing this investigation. Solutions for the flow fields, Nusselt and Sherwood numbers are obtained explicitly in terms of the governing parameters of the problem. The results obtained show that The Sherwood number for mass transfer is an increasing function of the Rayleigh number. The heat transfer increases (or decreases) when the permeability in the vertical direction is smaller (or higher) than the permeability in the horizontal direction. The characteristic parameter of the mass transfer (Sh) is minimal (or maximal) when the main axis having the most elevated permeability of the porous layer is perpendicular (or parallel) to the gravity.

1.6. Present Work

Taking into account the literature review which had just been carried out, we can note that no study has yet been done on double-diffusive convection through a horizontal anisotropic porous medium saturated by a non-Newtonian fluid with the contribution of the Soret effect.

From a physical perspective, convective motions in a porous layer have two main effects. First, they tend to homogenize the entire volume of the fluid in which they arise. Second, they produce a non-uniform in situ temperature distribution characterized by hot zones and cold zones. Double-diffusive convection in aquifers must be taken into account in the following real situations:

- The contribution of "homogenizing effects" of these convective flows to the diffusion of a contaminant. Indeed, from a local source of pollution in the aquifer, the effects of dispersion due to the average flow velocity and convection due to the geothermal gradient tend to disperse the polluting agent (nanoparticle of used oils, chemicals, harmful waste…, infiltrated into the water table) through the entire porous layer. In this case, the fluid consisting of water and nanoparticles can be considered as a non-Newtonian fluid;
- The complex dykes made in the fissured zones of the volcanic formations can generate thermal sources for the heating of the water table of the aquifer medium. When the dyke is adjacent to an almost horizontal rocky drop serving as a channel for the flow of the water table, the physical problem in this situation is comparable to forced or mixed convection on a horizontal plate in a saturated porous medium.

Our objective is to study how the double-diffusive convection in the aquifer is affected by the Soret effect, the behavior index and the anisotropy parameters, and also to know their effect on heat and mass transfers.

2. Mathematical Formulation and Solution

Figure 1 shows the problem under consideration. It consists of a two-dimensional horizontal porous layer of height H and width L. The generated out-flow is laminaire. The transfer of heat by radiance is negligible. The fluid is binary, non-newtonian and incompressible. A Cartesian coordinate system is chosen with the x- and y-axes at the geometrical center of the cavity and the y'-axis vertically upward. The top and bottom horizontal boundaries are subject to constant heat (q) and mass (j) fluxes. The porous medium is anisotropic, the permeabilities along the two principal axes of the porous matrix are denoted by K_1 and K_2. The anisotropy of the porous layer is characterized by the permeability ratio $K^* = K_1/K_2$ and the orientation angle φ, defined as the angle between the

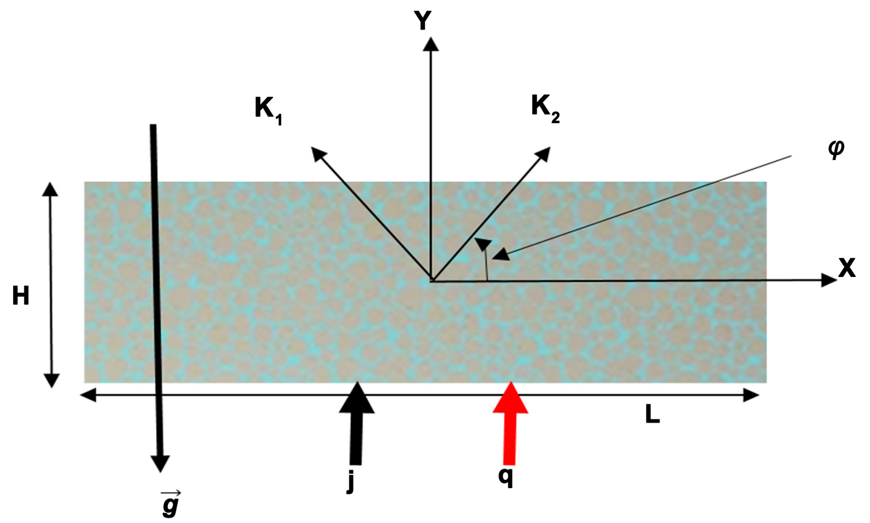

Figure 1. Physical model and coordinate system.

horizontal direction and the principal axis with the permeability K_2. The dimensionless equations describing conservation of momentum, energy and concentration are given respectively by:

$$a\frac{\mathrm{d}}{\mathrm{dy}}\left(\frac{\mathrm{d}\psi}{\mathrm{d}y}\right)^n = -Ra\frac{\partial}{\partial x}(T+NS), \quad (1)$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}y}\frac{\partial T}{\partial x} = \nabla^2 T, \quad (2)$$

$$Le\frac{\mathrm{d}\psi}{\mathrm{d}y}\frac{\partial S}{\partial x} = \nabla^2(S - SrT). \quad (3)$$

where $Ra = (\rho_0 g K_1 \Delta T H^3)/(\bar{\varepsilon}\alpha^n)$ is the modified thermal Rayleigh number, $N = (\beta_s \Delta S)/(\beta_T \Delta T)$ the buoyancy ratio, $Le = \alpha_T/\alpha_S$ the Lewis number $Sr = (D_{ST}\Delta T)/(\alpha_S \Delta S)$, the Soret number, D_{ST} the Soret diffusion coefficient, $a = K^* \cos^2\varphi + \sin^2\varphi$, $\mathrm{d}\psi/\mathrm{d}y$ the dimensionless horizontal velocity distribution, T the dimensionless temperature distribution, S the dimensionless concentration distribution and ψ the stream function.

The dimensionless boundary conditions for the Darcy model are given by:

$$x \pm \frac{A}{2} \quad \psi = 0, \frac{\partial S}{\partial x} = 0, \frac{\partial T}{\partial x} = 0, \quad (4)$$

$$y = \pm\frac{1}{2} \quad \psi = 0, \frac{\partial S}{\partial y} = Sr - 1, \frac{\partial T}{\partial y} = -1. \quad (5)$$

where $A = L/H$. The system is then governed by the following parameters: the Soret number (*Sr*), the modified thermal Rayleigh number (*Ra*), the buoyancy ratio (*N*), the Lewis number (*Le*), the anisotropic permeability ratio (K^*), the orientation angle (φ) and the power-law index (*n*).

In large aspect ratios ($A \gg 1$), the present problem can be significantly simplified by the approximation of the parallel flow in which $v = 0$ and $u(x, y) = u(y)$, in the central part of the enclosure. Such an approximation follows from the fact that, for a shallow cavity, the flow in the core of the enclosure is approximately parallel to the horizontal boundaries. The temperature and the concentration field, in the central part, can be divided into the sum of a linear dependence on x and an unknown function of y. Thus, it is assumed that

$$\psi(x, y) = \psi(y) \quad (6)$$

$$T(x, y) = C_T \cdot x + \theta_T(y) \quad (7)$$

$$S(x, y) = C_S \cdot x + \theta_S(y) \quad (8)$$

The solutions of Equations (1)-(3) satisfying the boundary conditions, Equations (4) and (5), are given by:

$$u_n(y) = -(\psi_0)^{\frac{1}{n}} y^{\frac{1}{n}} \quad (9)$$

$$T_n(x, y) = C_T \cdot x - \frac{C_T(\psi_0)^{\frac{1}{n}}}{1+\frac{1}{n}}\left(\frac{n}{2n+1} y^{2+\frac{1}{n}} - \frac{y}{2^{1+\frac{1}{n}}}\right) - y \quad (10)$$

$$S_n(x,y)=C_S\cdot x-\frac{(LeC_S-SrC_T)(\psi_0)^{\frac{1}{n}}}{1+\frac{1}{n}}\left(\frac{n}{2n+1}y^{2+\frac{1}{n}}-\frac{y}{2^{1+\frac{1}{n}}}\right)+(Sr-1)y \quad (11)$$

In Equations (9)-(11) the expression of ψ_0 is given by:

$$\psi_0=\frac{Ra(C_T+NC_S)}{\sin^2(\varphi)+K^*\cos^2(\varphi)} \quad (12)$$

The expressions of C_T and C_S can be deduced by integration of the following Equations (13) and (14), together with the boundary conditions (4) and (5), by considering the arbitrary control volume of **Figure 1** and connecting with the region of the parallel flow (Makayssi [49]). This yields:

$$\int_{-1/2}^{1/2}(U_nT_n)_{x=0}\,\mathrm{d}y=-C_T \quad (13)$$

$$Le\int_{-1/2}^{1/2}(U_nS_n)_{x=0}\,\mathrm{d}y=-(C_S+SrC_T) \quad (14)$$

Substituting the temperature, concentration and velocity profiles into Equations (13) and (14) and after performing the integration, it is readily found that the constant gradients of temperature and concentration along the *x*-direction, C_T and C_S, are respectively expressed by:

$$C_T=\frac{a_2(\psi_0)^{\frac{1}{n}}+a_1a_2Le^2(\psi_0)^{\frac{3}{n}}}{1+a_1(1+Le^2)(\psi_0)^{\frac{2}{n}}-a_1^2Le^2(\psi_0)^{\frac{4}{n}}} \quad (15)$$

$$C_S=\frac{b_0(\psi_0)^{\frac{1}{n}}+b_1(\psi_0)^{\frac{3}{n}}+b_2(\psi_0)^{\frac{4}{n}}-b_3(\psi_0)^{\frac{5}{n}}+b_4(\psi_0)^{\frac{6}{n}}}{1+b_5(\psi_0)^{\frac{2}{n}}+b_6(\psi_0)^{\frac{4}{n}}-a_2a_1^2Le^3(\psi_0)^{\frac{6}{n}}} \quad (16)$$

Substituting the expressions of C_T and C_S, into the expression of ψ_0, Equation (12), the following polynomial equation is obtained:

$$\begin{aligned}&-ab_{11}(\psi_0)^{\frac{10}{n}}+ab_{10}(\psi_0)^{\frac{8}{n}}-ab_9(\psi_0)^{\frac{6}{n}}-ab_8(\psi_0)^{\frac{4}{n}}-ab_7(\psi_0)^{\frac{2}{n}}\\&-Ra\omega_5(\psi_0)^{\frac{10-n}{n}}+Ra\omega_4(\psi_0)^{\frac{9-n}{n}}+RaNb_{20}(\psi_0)^{\frac{8-n}{n}}+Ra\omega_3(\psi_0)^{\frac{7-n}{n}}\\&+RaNb_{18}(\psi_0)^{\frac{6-n}{n}}+Ra\omega_2(\psi_0)^{\frac{5-n}{n}}+RaNb_2(\psi_0)^{\frac{4-n}{n}}\\&+Ra\omega_1(\psi_0)^{\frac{3-n}{n}}+Ra\omega_0(\psi_0)^{\frac{1-n}{n}}-a=f_n(\psi_0)=0\end{aligned} \quad (17)$$

The constants $a_{i(i=1,2)}$, $b_{i(i=1,\cdots,20)}$ and $\omega_{i(i=0,\cdots,5)}$ which depend on *Sr*, *Ra*, K^*, φ, *n* and *Le* are given by the following expressions:

$$\begin{cases}a_1=\dfrac{n}{n+1}\left(\dfrac{n^2}{(2n+1)(3n+2)}\left(\dfrac{1}{2}\right)^{2+\frac{2}{n}}-\dfrac{n}{2n+1}\left(\dfrac{1}{2}\right)^{2+\frac{2}{n}}\right)\\[2ex] a_2=\dfrac{n}{2n+1}\left(\dfrac{1}{2}\right)^2\left[\left(\dfrac{1}{2}\right)^{\frac{1}{n}}-\left(-\dfrac{1}{2}\right)^{\frac{1}{n}}\right]\end{cases} \quad (18)$$

$$\begin{cases} b_0 = Lea_2(1-Sr) - a_2Sr, \\ b_1 = Lea_1a_2(1-Sr)(1+Le^2) - a_1a_2SrLe^2, \\ b_2 = Lea_1a_2Sr, \\ b_3 = a_2a_1^2Le^3(1-Sr), \\ b_4 = Sra_2a_1^2Le^3, \\ b_5 = a_2Le^2 + a_1(1+Le^2), \\ b_6 = a_1a_2Le^2(1+Le^2) - a_1^2Le^2, \\ b_7 = b_5 + a_1(1+Le^2), \\ b_8 = b_6 + b_5a_1(1+Le^2) - a_1^2Le^2, \\ b_9 = b_6a_1(1+Le^2) - a_2a_1^2Le^3 - b_5a_1^2Le^2, \\ b_{10} = b_6a_1^2Le^2 + a_2a_1^3Le^3(1+Le^2), \\ b_{11} = a_2a_1^4Le^5 \\ b_{12} = a_2a_1Le^2 + b_5a_2, \\ b_{13} = b_5a_1a_2Le^2 + b_6a_2, \\ b_{14} = b_6a_1a_2Le^2 - a_1^2a_2^2Le^3, \\ b_{15} = a_1^3a_2^2Le^5, \\ b_{16} = b_1 + b_0a_1(1+Le^2), \\ b_{17} = b_1a_1(1+Le^2) - b_3 - b_0a_1Le^2, \\ b_{18} = b_4 + b_2a_1(1+Le^2), \\ b_{19} = b_3a_1(1+Le^2) + b_1a_1Le^2, \\ b_{20} = b_4a_1(1+Le^2) - b_2a_1Le^2. \end{cases} \tag{19}$$

$$\begin{cases} \omega_0 = a_2 + Nb_0 \\ \omega_1 = b_{12} + Nb_{16} \\ \omega_2 = b_{13} + Nb_{17} \\ \omega_3 = b_{14} - Nb_{19} \\ \omega_4 = Nb_3a_1Le^2 - b_{15} \\ \omega_5 = Nb_4a_1Le^2 \end{cases} \tag{20}$$

3. Onset of Supercritical Convection When $n = 1$ (Newtonian Fluid)

Equation (17), for $n = 1$, can be written as follows:

$$\begin{aligned} &-ab_{11}(\psi_0)^{10} - Ra\omega_5(\psi_0)^9 + (ab_{10} + Ra\omega_4)(\psi_0)^8 + RaNb_{20}(\psi_0)^7 \\ &+ (Ra\omega_3 - ab_9)(\psi_0)^6 + RaNb_{18}(\psi_0)^5 + (Ra\omega_2 - ab_8)(\psi_0)^4 + RaNb_2(\psi_0)^3 \\ &+ (Ra\omega_1 - ab_7)(\psi_0)^2 + Ra\omega_0 - a = f_1(\psi_0) \end{aligned} \tag{21}$$

The onset of supercritical convection is obtained while taking $\psi_0 = 0$ in Equation (21). The supercritical Rayleigh number Ra_c^{sup} takes the following form:

$$Ra_c^{sup} = \frac{\overleftarrow{Ra_c}}{1 + LeN(1 - Sr) - NSr} \tag{22}$$

where $\overleftarrow{Ra_c} = 12a$.

4. Average Nusselt and Sherwood Numbers

The average Nusselt ($\overline{Nu}$) and Sherwood ($\overline{Sh}$) number can be obtained as follows:

$$\left.\begin{aligned} \overline{Nu} &= \frac{1}{A}\int_{-\frac{A}{2}}^{+\frac{A}{2}} \frac{\mathrm{d}x}{T(x,-0.5) - T(x,+0.5)} \\ \overline{Sh} &= \frac{1}{A}\int_{-\frac{A}{2}}^{+\frac{A}{2}} \frac{\mathrm{d}x}{\left[S(x,-0.5) - S(x,+0.5)\right] + Sr\left[T(x,-0.5) - T(x,+0.5)\right]} \end{aligned}\right\} \tag{23}$$

We obtain then:

$$\left.\begin{aligned} \overline{Nu} &= \frac{1}{1 + C_T\alpha(\psi_0)^{\frac{1}{n}}} \\ \overline{Sh} &= \frac{1}{1 + LeC_S\alpha(\psi_0)^{\frac{1}{n}}} \end{aligned}\right\} \tag{24}$$

where:

$$\alpha = \frac{n}{n+1}\left(\frac{n}{2n+1}\left[\left(\frac{1}{2}\right)^{2+\frac{1}{n}} - \left(-\frac{1}{2}\right)^{2+\frac{1}{n}}\right] - 2\left(\frac{1}{2}\right)^{2+\frac{1}{n}}\right) \tag{25}$$

5. Results and Discussion

5.1. Isotropic Porous Cavity Saturated by a Newtonian Fluid

The solutions, Equations (9)-(11), when $n = 1$ is given by:

$$\left.\begin{aligned} & u(y) = -\psi_0 y, \\ & T(x,y) = C_T \cdot x - \frac{C_T\psi_0}{2}\left(\frac{y^2}{3} - \frac{1}{4}\right)y - y, \\ & S(x,y) = C_S \cdot x - \frac{(LeC_S - SrC_T)\psi_0}{2}\left(\frac{y^2}{3} - \frac{1}{4}\right)y + (Sr - 1)y, \\ & \psi_0 = Ra(C_T + NC_S) \end{aligned}\right\} \tag{26}$$

which are in agreement with those reported by Amari *et al.* [9], Mamou *et al.* [50], Kalla *et al.* [51] and Yovogan *et al.* [48] for $Sr = 0$ and $K^* = 1$.

5.2. Comparison of ψ_0 and Supercritical Rayleigh Number with Other Results Reported in the Past

The supercritical Rayleigh number (Equation (22)), for an isotropic porous cavity saturated by a Newtonian fluid, is similar to results obtained by Attia *et al.* [55], Redha *et al.* [33] for $Ha = 0$, $Du = 0$ (Table 1 & Table 2).

Otherwise, when $N = 0$, the supercritical Rayleigh number (Equation (22) is agreement with the result ($Ra_c = 12a$) obtained in the past by Nield [56], Vasseur *et al.* [57] and Degan *et al.* [54].

5.3. Effect of Physical Parameters on the Onset of Convection When n = 1

The effects of the anisotropic permeability, K^*, on the critical Rayleigh number are presented in **Figure 2**, when $\varphi = 45^\circ$, $N = 0.5$, $Le = 2$, and $n = 1$. By considering cooperating convection ($N > 0$) and compared to the situation for which $K^* = 1$, results show that:

- For $Sr \le -0.9$, the supercritical Rayleigh number for the onset of convection decreases with an increase of Soret number when $K_1 > K_2$ ($K^* = 2$) and increases with an increase of Soret number when $K_2 > K_1$ ($K^* = 0.1$).
- For $Sr > -0.9$ the supercritical Rayleigh number for the onset of convection increases with an increase of Soret number and with an increase of the anisotropic permeability K^*.

Considering opposite convection, for which $N < 0$ ($N = -0.5$), the results presented in **Figure 3** (when $\varphi = 45^\circ$, $Le = 2$, and $n = 1$) indicate that the effect of

Table 1. Comparison of ψ_0 with previous studies.

$Ra = 100$, $n = 1$, $Le = 10$, $N = -0.24$, $Sr = 0$	Present study $K^* = 0.1$	Mamou *et al.* [52] $K^* = 1$	Present study $K^* = 1.2$
ψ_0	**3.4336**	3.685	3.9888

Table 2. Comparison of Ra_c^{sup} with previous studies.

$N = 0$	Present study $K^* = 0.1$	Alloui *et al.* [53], Degan *et al.* [54]. $K^* = 1$	Present study $K^* = 1.2$
Ra_c^{sup}	**1.2**	12	14.4

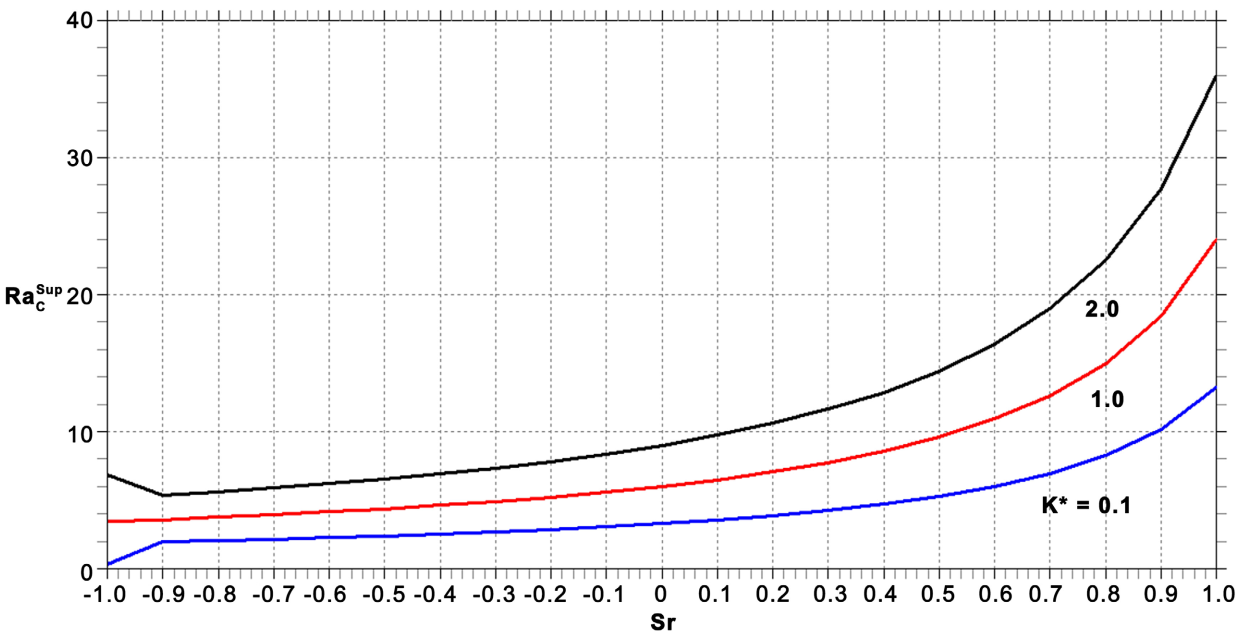

Figure 2. Effects of various values of K^*, on the supercritical Rayleigh number for the onset of the convection as functions of Soret when $\varphi = 45^\circ$, $N = 0.5$, $Le = 2$ and $n = 1$.

anisotropy in permeability is only noticeable for values of the Soret number such that $-0.1 < Sr < 0.1$. For this value interval and compared with the isotropic situation, the supercritical Rayleigh number (for negative values of Soret number) increases and is maximum when $Sr = 0$ and decreases with an increase of Soret number.

The influence of the anisotropic angle, φ, on the supercritical Rayleigh number are presented in Figure 4 ($N = 0.5$, cooperating convection) and Figure 5 ($N = -0.5$, opposite convection), when $K^* = 0.1$, $Le = 2$, and $n = 1$. It observed in

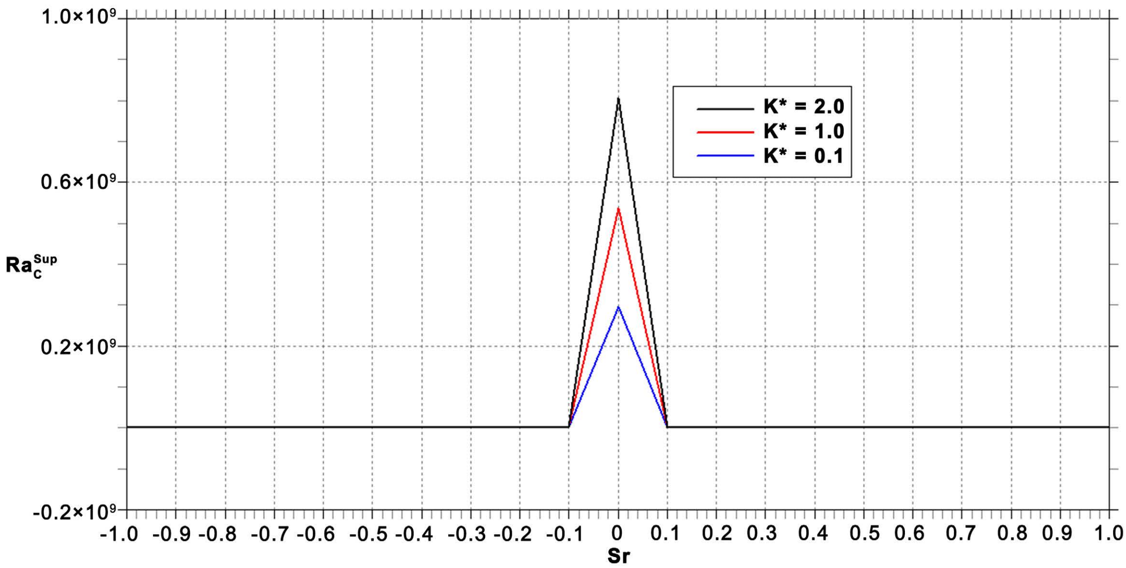

Figure 3. Effects of various values of Soret number, on the supercritical Rayleigh number for the onset of the convection when $\varphi = 45^\circ$, $N = -0.5$, $Le = 2$, $n = 1$ and various values of K^*.

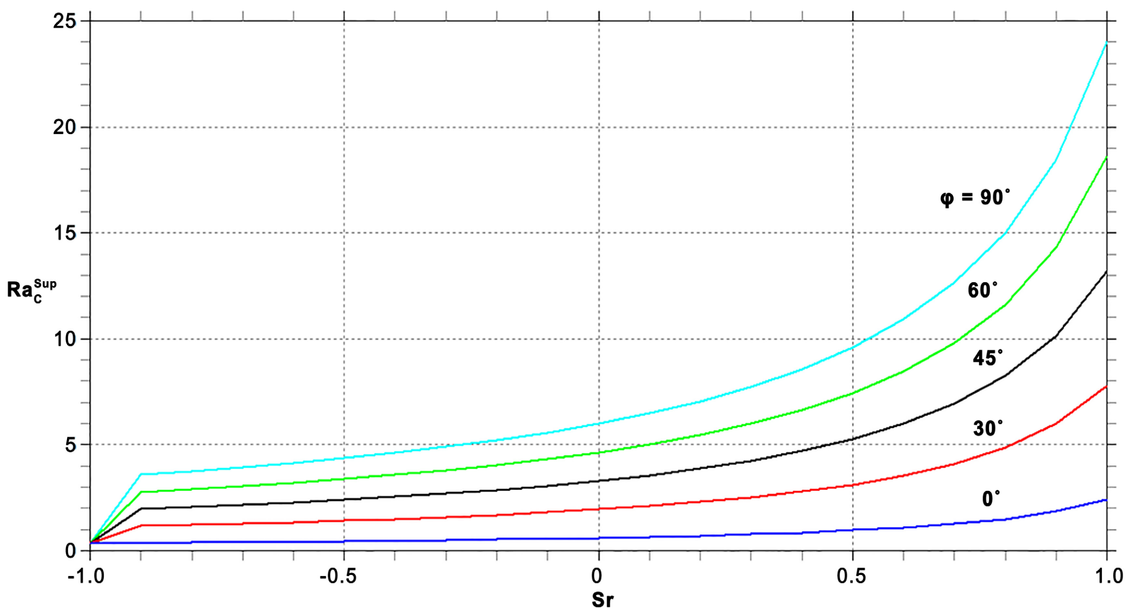

Figure 4. Effects of various values of φ, on the supercritical Rayleigh number for the onset of the convection as functions of Soret number when $K^* = 0.1$, $N = 0.5$, $Le = 2$ and $n = 1$.

Figure 4 that the supercritical Rayleigh number increases with an increase of Soret number and anisotropic angle. The results observed in Figure 5 are similar to those obtained in Figure 3.

The results obtained in Figure 6, show that for $Sr > 1$ the supercritical Rayleigh number takes negative values. However for $Sr < 1$, it takes positive values and is maximum (minimal) when $\varphi = 90^\circ$ ($\varphi = 0^\circ$).

The effects of Soret number on the profiles of the velocity, temperature and concentration distribution are presented respectively in Figures 7-9 when $\varphi = 0^\circ$,

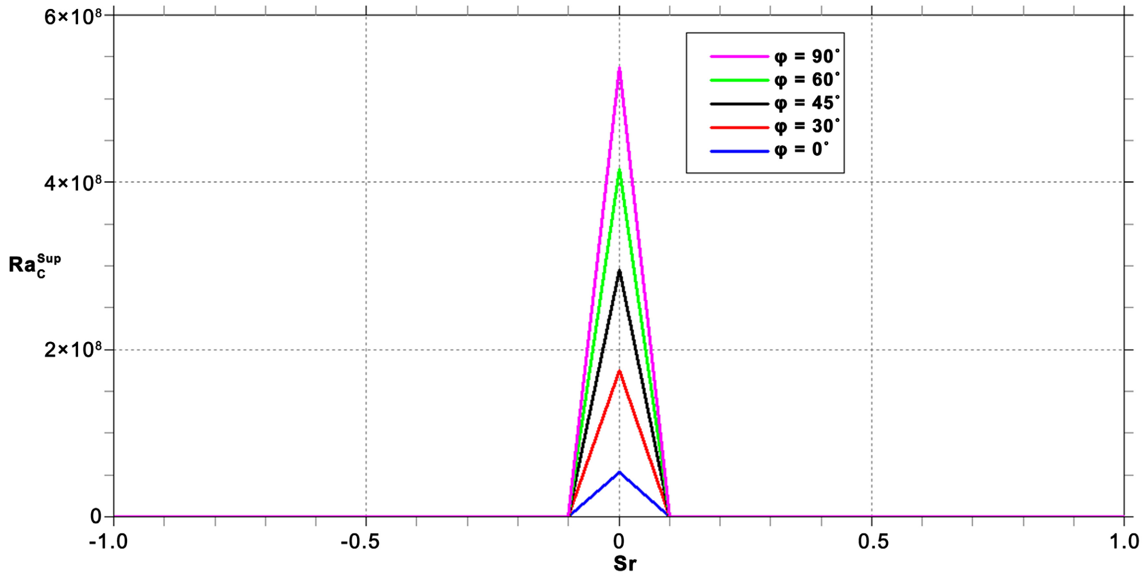

Figure 5. Effects of various values of φ, on the supercritical Rayleigh number for the onset of the convection as functions of Soret number when $K^* = 0.1$, $N = -0.5$, $Le = 2$ and $n = 1$.

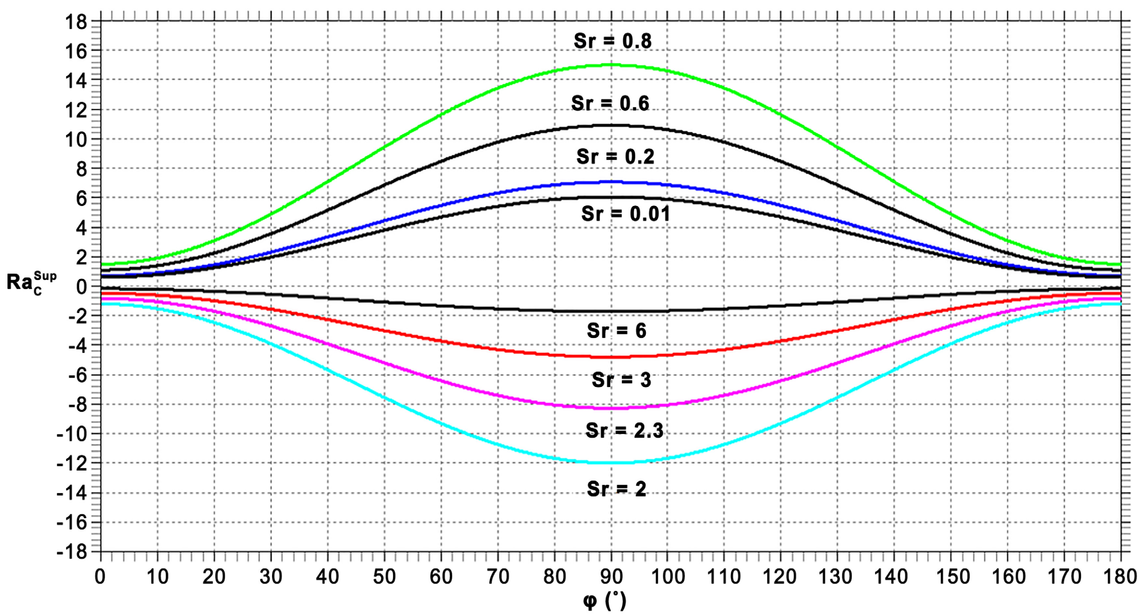

Figure 6. Effects of various values of Soret number, on the supercritical Rayleigh number for the onset of the convection as functions of φ, when $K^* = 0.1$, $N = 0.5$, $Le = 2$ and $n = 1$.

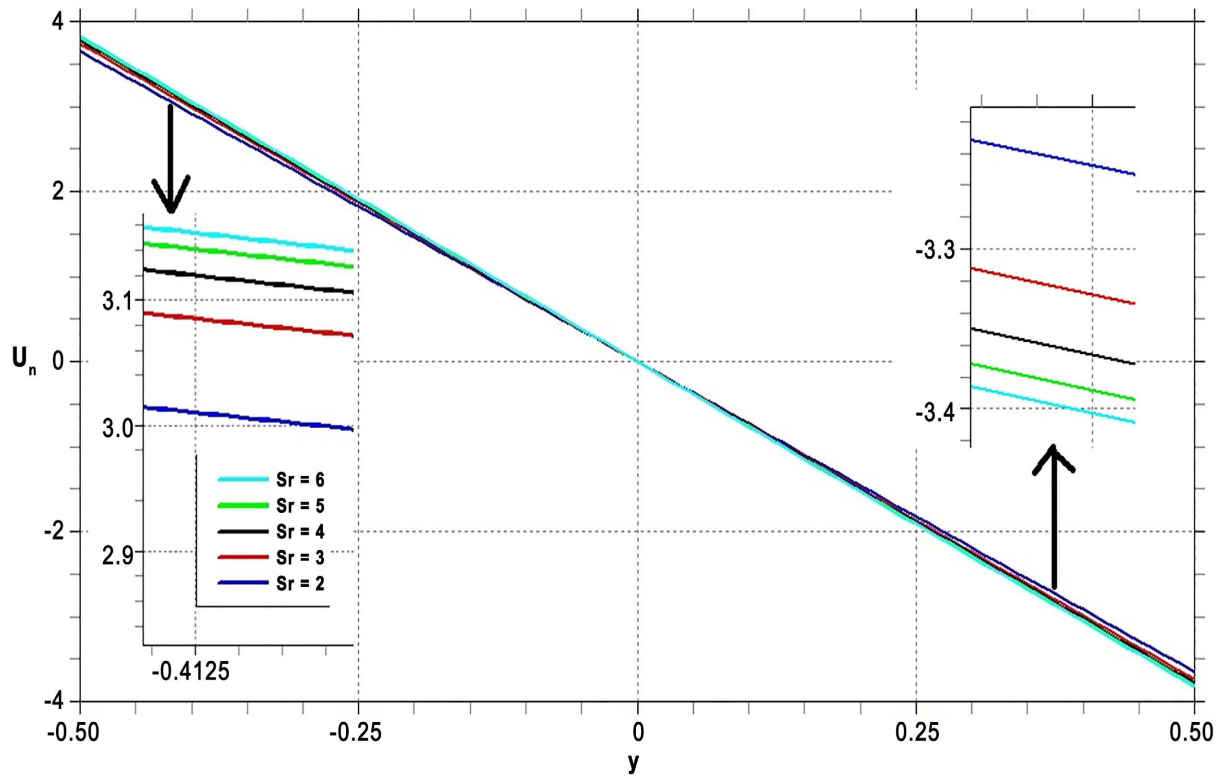

Figure 7. Effects of various values of Soret number, on the horizontal velocity distribution when $\varphi = 0^\circ$, $K^* = 0.1$, $N = 0.5$, $Le = 2$, $Ra = 150$ and $n = 1.5$.

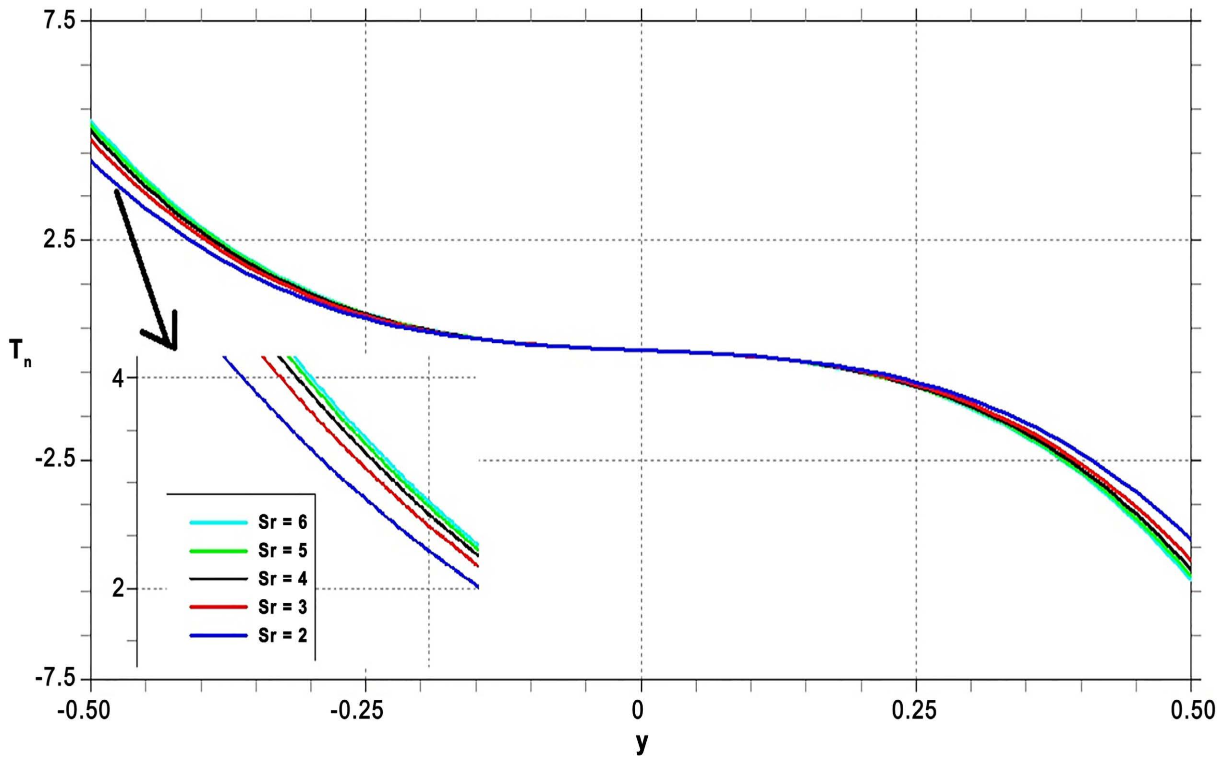

Figure 8. Effects of various values of Soret number, on the temperature distribution when $\varphi = 0^\circ$, $K^* = 0.1$, $N = 0.5$, $Le = 2$, $Ra = 150$ and $n = 1.5$.

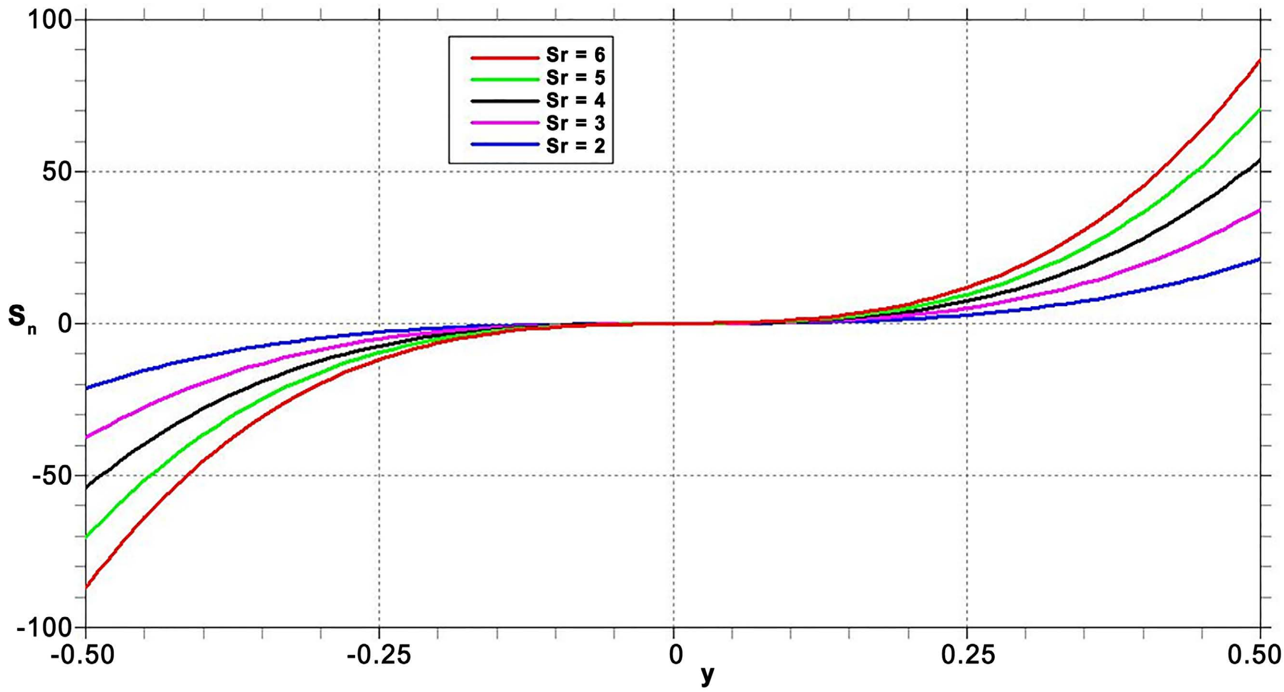

Figure 9. Effects of various values of Soret number, on the concentration distribution when $\varphi = 0^\circ$, $K^* = 0.1$, $N = 0.5$, $Le = 2$, $Ra = 150$ and $n = 1.5$.

$K^* = 0.1$, $Le = 2$, $Ra = 150$, $n = 1$ and $N = 0.5$ (cooperating convection). It is noted that whatever the value of $y \neq 0$, the velocity, temperature and concentration distribution increase with an increase of Soret number. Moreover the velocity and temperature distribution are decreasing functions of y while the concentration is an increasing function of y.

In **Figure 10** and **Figure 11**, the effects of the Rayleigh number on the velocity, temperature and concentration distribution are reported for the fixed values of the following physical parameters, $\varphi = 0^\circ$, $K^* = 0.1$, $Le = 2$, $Sr = 2$, $n = 1$ and $N = 0.5$ (cooperating convection). We notice that an increase in Rayleigh number tends to reduce the velocity and temperature of the convective flow, while the same increase in Rayleigh number tends to increase the concentration distribution whatever the value of y.

The resolution of the Equation (17) shows that no convection is possible when the porous cavity is saturated by a pseudo plastic fluid ($n < 1$). The onset of convection is only possible when $n \geq 1$ (Newtonians fluids and dilatants fluids).

Table 3 indicates that for the dilatants fluids, the flow intensity (ψ_0) increases with an increase of the behavior index (n) and for given values of the physical parameters. In **Table 4**, we can observe that the flow intensity (ψ_0) decreases with an increase of the Rayleigh number (Ra) and the effects are the same for both a Newtonian fluid and for the dilatants fluids. In **Table 5**, it should be noted that for the dilatants fluids and whatever the value of the Soret number, the average Nussekt (Sherwood) number decreases (increases) with the increase of the behavior index.

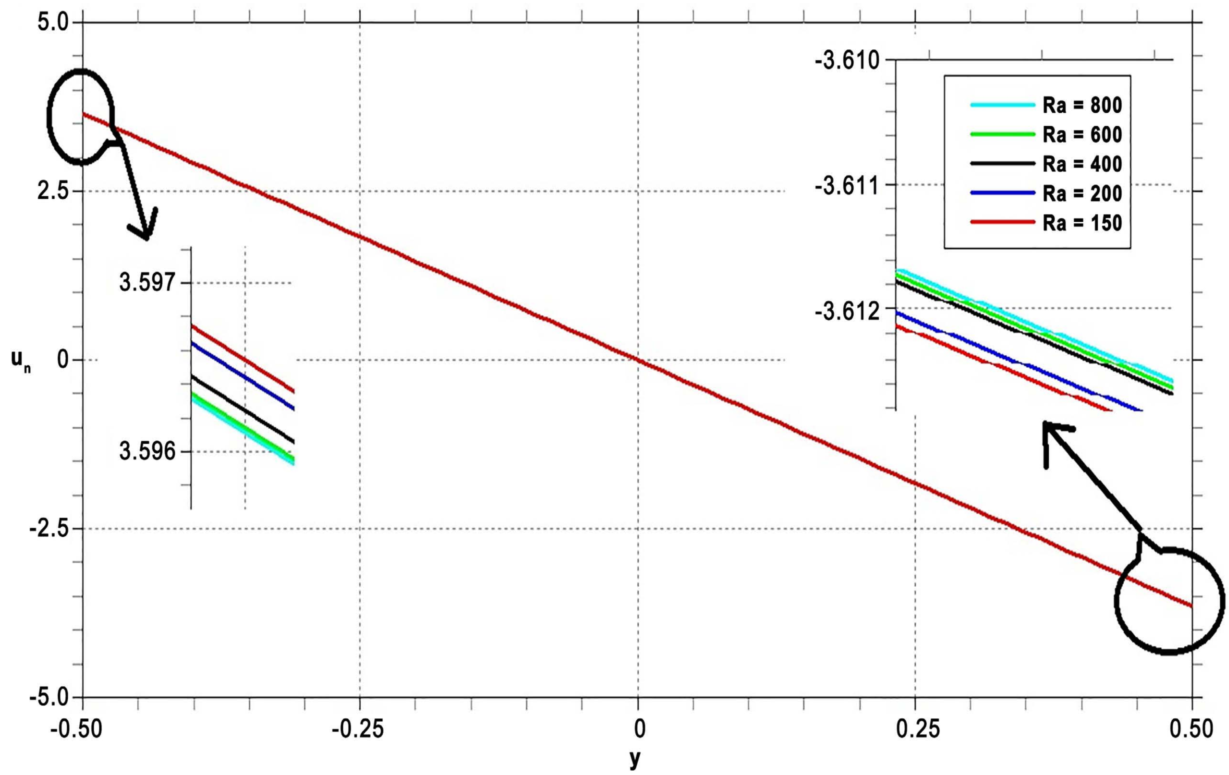

Figure 10. Effects of various values of Rayleigh number, on the horizontal velocity distribution when $\varphi = 0^\circ$, $K^* = 0.1$, $N = 0.5$, $Le = 2$, Sr = 2 and $n = 1.5$.

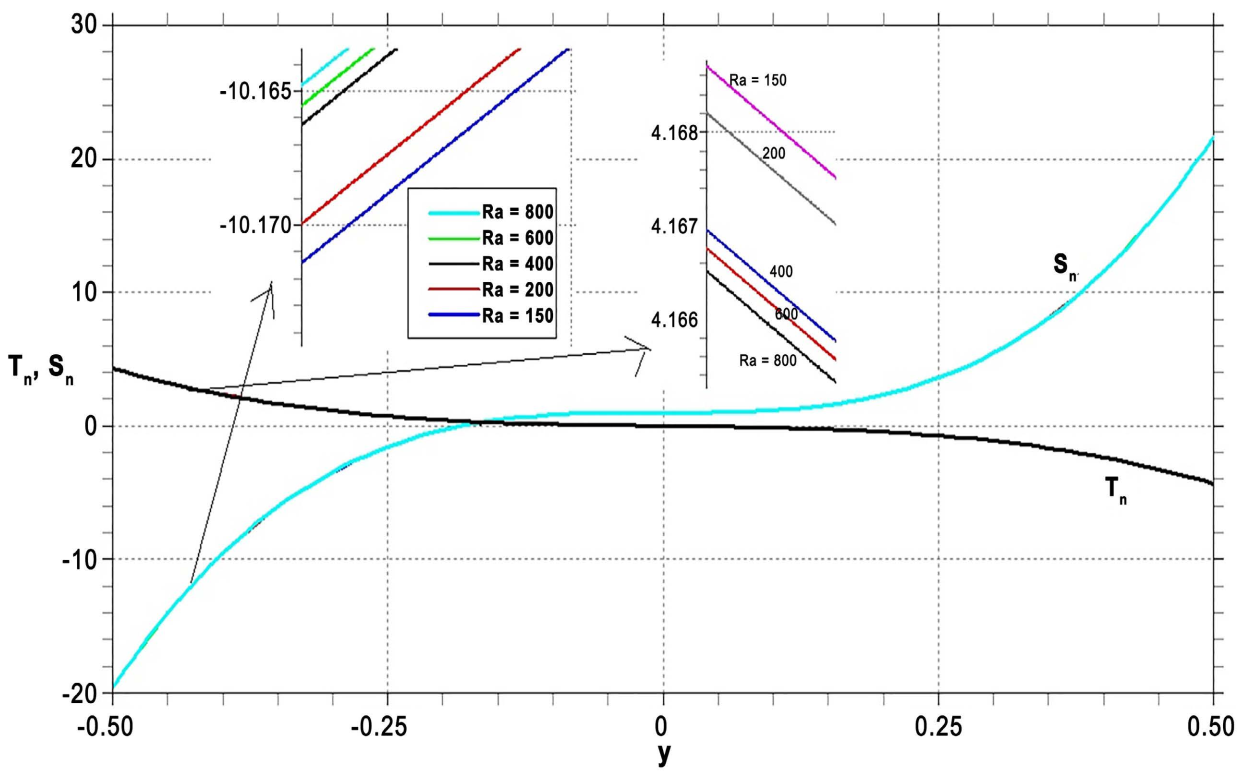

Figure 11. Effects of various values of Rayleigh number, on the temperature and concentration distribution when $\varphi = 0^\circ$, $K^* = 0.1$, $N = 0.5$, $Le = 2$, Sr = 2 and $n = 1.5$.

Table 3. Effect of behavior index on the stream function value at the center of the cavity.

Ra	*φ*	*K**	*Sr*	*N*	*Le*	*n*	Ψ_0	Fluid type
150	0˚	0.1	2.0	0.5	2	1.0	7.2982	Newtonian fluid
150	0˚	0.1	2.0	0.5	2	2	3.1931	Dilatant fluid
150	0˚	0.1	2.0	0.5	2	4	3.5504	
150	0˚	0.1	2.0	0.5	2	6	3.8765	
150	0˚	0.1	2.0	0.5	2	8	4.2113	
150	0˚	0.1	2.0	0.5	2	9	4.3853	
150	0˚	0.1	2.0	0.5	2	10	4.5647	
150	0˚	0.1	2.0	0.5	2	20	6.7353	
150	0˚	0.1	2.0	0.5	2	30	9.7801	
150	0˚	0.1	2.0	0.5	2	40	13.9283	
150	0˚	0.1	2.0	0.5	2	50	19.3218	

Table 4. Effect of Rayleigh number on the stream function value at the center of the cavity.

Ra	*φ*	*K**	*Sr*	*N*	*Le*	*n*	Ψ_0
			Newtonian fluid				
150	0˚	0.1	2.0	0.5	2	1.0	7.2982
200	0˚	0.1	2.0	0.5	2	1.0	7.2980
400	0˚	0.1	2.0	0.5	2	1.0	7.2975
600	0˚	0.1	2.0	0.5	2	1.0	7.2974
800	0˚	0.1	2.0	0.5	2	1.0	7.2973
			Dilatants fluid				
150	0˚	0.1	2.0	0.5	2	1.5	7.2982
200	0˚	0.1	2.0	0.5	2	1.5	7.2980
400	0˚	0.1	2.0	0.5	2	1.5	7.2975
600	0˚	0.1	2.0	0.5	2	1.5	7.2974
800	0˚	0.1	2.0	0.5	2	1.5	7.2973

Table 5. Effect of Soret number and the behavior index on the average Nusselt and Sherwood numbers.

Ra	*φ*	*K**	*Sr*	*N*	*Le*	*n*	$\overline{Sh}$	$\overline{Nu}$	Fluid type
150	0˚	0.1	1.0	0.5	2	2	0.9545	1.2910	Dilatant fluid
150	0˚	0.1	1.0	0.5	2	4	0.9647	1.1439	
150	0˚	0.1	1.0	0.5	2	6	0.9731	1.0899	
150	0˚	0.1	2.0	0.5	2	2	0.8884	1.1708	
150	0˚	0.1	2.0	0.5	2	4	0.9042	1.1043	
150	0˚	0.1	2.0	0.5	2	6	0.9214	1.0752	
150	0˚	0.1	5.0	0.5	2	2	0.5074	1.5214	
150	0˚	0.1	5.0	0.5	2	4	0.5530	1.2890	
150	0˚	0.1	5.0	0.5	2	6	0.5782	1.2297	

6. Conclusions

In this study, an analytical investigation is carried out on the effects of physical parameters (such as the Soret number (Sr), the anisotropy angle φ, the anisotropic permeability K^*, the Rayleigh number, the behavior index) on the thermal and mass flows through a rectangular porous cavity saturated by a non-Newtonian fluid. It emerges from this study that;

1) No onset of convection is observed when $n < 1$ (pseudo plastic fluid);

2) The supercritical Rayleigh number, for cooperating convection, increases with an increase of Soret number and anisotropic angle;

3) The velocity, temperature and concentration distribution increase with an increase of Soret number;

4) For the dilatants fluids and whatever the value of the Soret number, the average Nussekt (Sherwood) number decreases (increases) with the increase of the behavior index.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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