Engineering, 2023, 15, 793-797
https://www.scirp.org/journal/eng
ISSN Online: 1947-394X
ISSN Print: 1947-3931

The Meaning of Divergence

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How to cite this paper: Wu, W.B. and Yuan, X.J. (2023) The Meaning of Divergence. *Engineering*, **15**, 793-797.
https://doi.org/10.4236/eng.2023.1512055

Received: November 10, 2023
Accepted: December 1, 2023
Published: December 4, 2023

Abstract

The concept of the divergence is fundamental in electromagnetic field theory, yet they are especially difficult mathematical concepts. Understanding this concept requires strong spatial and abstract thinking. When teaching, through the graphical and quantitative methods presented herein, the significance of the divergence is displayed by a quantitative method for the first time. Through these methods, the concepts of the divergence can be grasped more easily. These explanations will be helpful for students to strengthen the understanding of this concept and has a certain reference significance.

Keywords

Divergence, Spatial Thinking, Quantitative Method

1. Introduction

The divergence is a quantity that describes the extent to which a gas converges from the surrounding area to or from a certain point. The divergence of three-dimensional space represents the change rate of the unit volume of any block of gas in a unit time. The volume expansion of a gas is called the divergence, and the volume contraction of a gas is called convergence. The difficulty for the lecturer lies in how to convey knowledge about these concepts vividly such that the students grasp this concept completely and accurately. This paper proposes methods to concretely convey the concept of the divergence. This method not only stimulates students' spatial imagination but also makes the abstract concept of the divergence simple to understand [1] [2] [3] [4] [5].

2. Definition of Divergence

A vector field $\boldsymbol{A}(x,y,z)$ is considered. A closed surface around the point M in the field is selected, and the area enclosed by Σ is V. When V shrinks to the point

M, if the limit $\lim\limits_{V \to M} \dfrac{\oiint_{\Sigma} \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{S}}{V}$ exists, the limit value is called the divergence at the point M, denoted as $\mathrm{div}\boldsymbol{A}$.

Where

$$\mathrm{div}\boldsymbol{A} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

The Hamilton operator is defined as follows:

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial x}\boldsymbol{j} + \frac{\partial}{\partial x}\boldsymbol{k}$$

thus:

$$\mathrm{div}\boldsymbol{A} = \nabla \cdot \boldsymbol{A}$$

The Gauss formula can be expressed as follows:

$$\int_V \nabla \cdot A \mathrm{d}v = \oint A \cdot \mathrm{d}s$$

From the Gauss flux theorem:

$$\oint_s \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{s} = \sum q$$

The following equation can be obtained:

$$\nabla \cdot \boldsymbol{D} = \rho \tag{1}$$

3. The Meaning of Divergence

According to the explanations of the divergence in general textbooks, divergence is a scalar, which can be understood as the flux passing through the closed surface of unit volume, that is, the change rate of the flux with respect to the volume at a given point in the field, as shown in **Figure 1** and **Figure 2**. The divergence represents the strength of a source. In a passive area, where a source does not exist, the divergence at each point should be equal to zero.

Scenario: A piece of glass is held in the reader's left hand, and a glass ball is held in the reader's right hand. The reader exits the light bulb at a constant speed, and the reader's eyes are focused on one light beam emitted by the bulb (**Figure 3**). The light in front of the reader passes through the piece of glass and the glass ball. The lines weaken or decrease proportionally with the distance from the bulb, and the meaning of the above three scenes is exactly the same. In this scenario, combined with **Figures 1-3**, the fact that the number of power lines changes with the distance is perfectly expressed in one-, two-, and three-dimensional space. The Gauss formula relates two-dimensional space with three-dimensional space:

$$\iiint_{\Omega} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) \mathrm{d}v = \oiint_{\Sigma} P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y$$

According to the results of the above analysis, the conclusion we get is that the rate of change of the electric flux density with volume is constant. We seek to

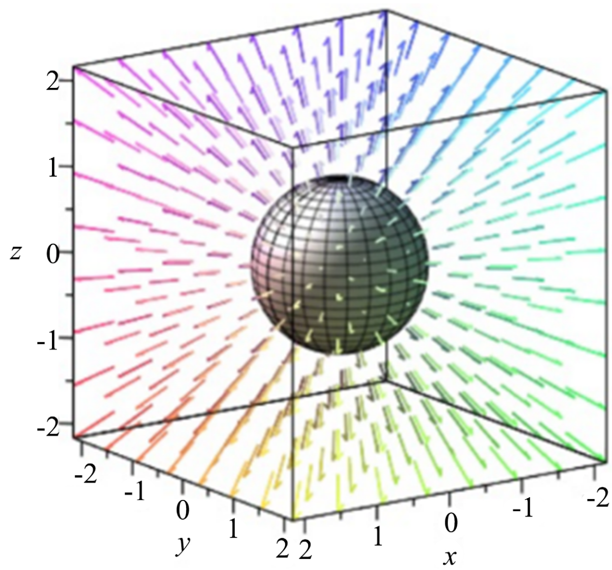

Figure 1. Charged sphere.

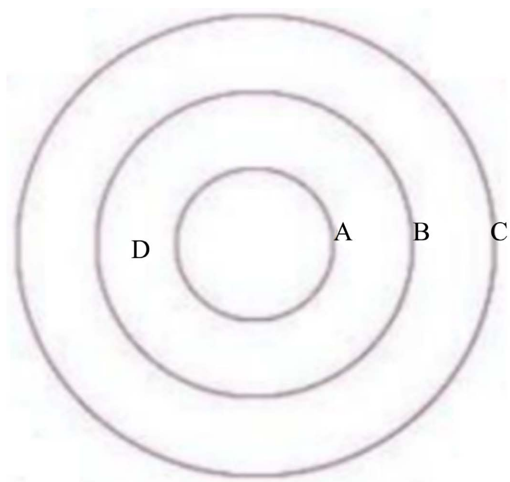

Figure 2. Divergence.

Figure 3. One-dimensional space interpretation of divergence.

determine whether this rate of change is the charge volume density ρ in equation [1]?

To confirm this speculation, it is assumed that the volume of the charged sphere in **Figure 1** is V. If the volume of the spherical surface surrounding the charge changes from V to V_0, since the electric quantity $Q = \rho V$ of the charged sphere is always constant, the larger the V_0, the fewer power lines pass through any unit volume element. These quantities are inversely proportional. At this time, the number of power lines in the unit volume element changes to Y:

$$Y = \rho\left(V/V_0\right) \tag{2}$$

Because V is a constant and V_0 is variable, (V/V_0) is variable. X is used to replace (V/V_0), and Equation (2) becomes the following:

$$Y = \rho X \tag{3}$$

Equation (2) shows the number of power lines passing through a unit volume that contains one selected point outside the charge sphere Q, and the number varies with the distance from the given point to the charged sphere Q (*i.e.*, the variable V_0).

According to the divergence, the rate of change of the flux with respect to the volume at one point in the electric field is expressed, while Equation (3) represents the relationship between the flux and the volume at one point in the electric field. Therefore, taking the derivative of Equation (3) with respect to X

$$\mathrm{d}Y/\mathrm{d}X = \rho \tag{4}$$

The derivative of Y with respect to X represents the rate of change of the flux to volume at a point in the electric field; that is, according to the Equation (1), the divergence $\nabla \cdot \boldsymbol{D}$ is just the slope of Equation (2)!

Through the above analysis, we can obtain the essential concept of the divergence symbol itself; that is, the divergence represents the rate of change of the number of power lines contained in one unit volume in the field, and this number will change with the distance of the unit volume from the charged sphere.

4. Conclusion

On the basis of the directional derivative, a cone being intercepted was visualized, and through the scene of a person climbing to the top of a mountain, the concept of the gradient was clearly explained [6] [7]. Through quantitative calculations, the concept of the divergence was deeply analyzed. First, visualizing a charged sphere, the change ratio of the flux with the change in volume was calculated. The results showed that this ratio is fixed at any point outside the charged sphere. Second, imagining the action of holding a piece of glass in one hand, the changes of the electric field intensity with the distance are perfectly expressed in one-, two-, and three-dimensional space. Finally, the conclusion that the divergence is the slope of a certain equation is presented. In short, this paper expounds the essence of the gradient and the divergence, so these concepts can be easily grasped for students.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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