Creative Education, 2022, 13, 2264-2274
https://www.scirp.org/journal/ce
ISSN Online: 2151-4771
ISSN Print: 2151-4755

The Influence of Origami on Mathematics Study

Yutian Wong

Guangdong Country Garden School, Foshan, China
Email: harrywongcocoa@hotmail.com

How to cite this paper: Wong, Y. T. (2022). The Influence of Origami on Mathematics Study. *Creative Education, 13,* 2264-2274.
https://doi.org/10.4236/ce.2022.137143

Received: April 29, 2022
Accepted: July 17, 2022
Published: July 20, 2022

Abstract

This study aimed to investigate the influence of origami on mathematics study. Origami artists have been studying origami since 1893 to find the secret of math in it. In the past years, according to the studies that past sages have done for the world, origami helps mathematicians to solve several math puzzles. In addition, it may help students to understand geometry. As a high school student, the researcher was interested in origami and mathematics. Therefore, research related to these two themes is made, and connected with math education, to discover how origami helps learn math. In this article, origami was seen as something that may greatly assist mathematics and the education of mathematics. For further support, research is done for some examples that past people have done about origami and mathematics. Theories proposed by the pioneers were put into practice. Additionally, some of the current study experiences in origami with math knowledge points were added to find the benefits of students learning math by using origami. Related theories are taken into practice by the researcher, to further discover how mathematics and origami have combined. This article concludes with the examples of origami that have greatly helped to solve math problems and my discovery of how learning origami may help better comprehension of math. Spatial awareness, logical execution, non-cognitive skills, and finger dexterity has been greatly improved.

Keywords

Origami, Origami Geometry, Mathematics, Math Learning

1. Introduction

The researcher discovered this through origami and mathematics individually. During the process, the researcher found that origami was greatly connected to mathematics. Further on, origami techniques somehow could be explained by mathematics theories. The main problem that this study discusses is how origami affects the study of mathematics. Among the studies taken, how origami practice changes the way of thinking has become the thing that the researcher is trying to find.

Before getting notice about origami, we should first think about: what is mathematics? The definition of Mathematics, or in a shortened way math, in a dictionary is "an area of knowledge, which includes the study of such topics as numbers". Mathematics is also essential in many different fields, including natural sciences, finance, computer science, social science, and engineering. Some areas of mathematics, such as statistics and game theory, are developed in direct correlation with their applications. This type of mathematics is called applied mathematics. Other mathematical areas that are developed independently from any application, are therefore called pure mathematics. Mathematics could be said to be a part of human life, people used math to calculate every day, either by Purchasing things in the shopping mall, or the calculations for their phones to work, all of these cannot leave the existence of mathematics. For centuries, mathematics has always been one of the subjects in school. It is an essential part of a child's education. Beyond the obvious everyday applications, mathematics is seen as a needed element in developing student readiness for the workforce demands of the 21st century (Ferrini-Mundy, 2000). Thus, learning mathematics is very important, but now the question is: how to learn it?

Generally, people were taught mathematics by teachers in school. However, as modern origami appears in people's sight, some of its characteristics seemed to become a part of mathematics education. For example, geometry may be taught by combining origami with shapes to make students understand faster. Also, as a type of group activity, origami permits students to discuss and cooperate, which makes them notice how efficient group work is. That is one of the great things about origami, it is that even if you can do it on a one-by-one basis, it doesn't mean that it won't work well if you work with other people. These could be some of the advantages of combining origami with mathematics study. Therefore, the next question will be: what is origami?

Origami, which means "folding paper" in Japanese, the best-known origami model is the Japanese paper crane. It is a form of art that can be traced back to 905 CE in the Song Dynasty (Laing & Liu, 2004), being as a gift in the past. However, in Europe, it was used as a technique for folding napkins, which flourished during the 17th and 18th centuries. In the early 1900s, origami artists like Akira Yoshizawa, one of the founders of modern origami, began to create and record original origami artworks. Akira Yoshizawa in particular was responsible for several innovations, such as wet-folding and the Yoshizawa-Randlett diagramming system, and his work inspired a renaissance of the art form (Fox, 2005). And in the late 20th century, people started to understand origami in both aspects of artistic and scientific. Artists and scientists like Robert J. Lang and Erik Demaine combine it with mathematics and physics.

Origami has also applicated in sundry fields. Architects integrate elements in origami practice into architecture designs, scientists use origami principles to solve problems about astronomy, and also companies made bulletproof shields by applying origami structures to them. These facts show great prospects for origami in several industries.

The study of the mathematics behind origami starts in 1893, an Indian civil servant T. Sundara Rao published Geometric Exercises in Paper Folding which used paper folding to demonstrate proofs of geometrical constructions (Rao & Row, 1917). In 1980 was reported a construction that enabled an angle to be trisected, and trisections are impossible under Euclidean rules before it was proved by origami (Thomas, 2015). After that, loads of origami researchers published their discoveries on how origami helped prove mathematics theories for almost a century. However, the results are not affluence, there are still many problems waiting to be solved.

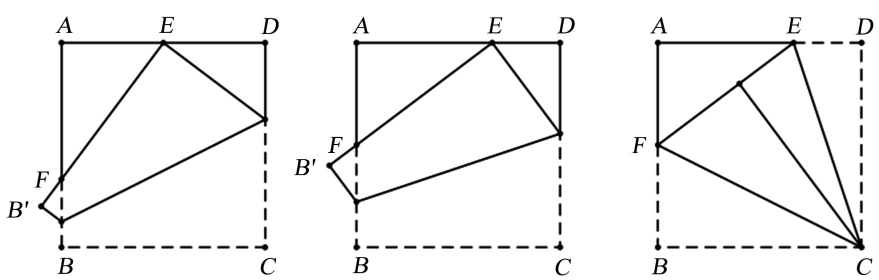
This thesis consists of three main parts, first serves some major examples of how origami helps study mathematics (a literature review), the second is about how origami influences my study, and the third part is about how can origami influence mathematics education. To find out how origami plays a guiding role in studying mathematics, research of works and literature that were published by past investigators was done, and next, two of them will be shown.

2. Examples of Related Math Theories

Among the existing literature that discusses this inquiry, one of them is about Haga's Theorems, written by Hiroshi Okumura, the thesis "A Note on Haga's Theorems in Paper Fold". As I have mentioned that origami helped mathematicians solve to divide a paper into three equal parts in the introduction part, Haga Kazuo was one of the pioneers of origami geometry. Haga's Theorems in the mathematics of square paper folding consist of three main parts.

First, let us assume that ABCD is a piece of square paper with a point E on the side AD. We fold the paper so that corner C coincides with E and the side BC is carried into BE, which intersects the side AB at a point F, see **Figure 1(a)**. And this is called Haga's fold of the first kind.

The first theorem and the proof: Haga discovered that, if E is the midpoint of AD, then F divides AB in the ratio of 2:1 internally.



(a) (b) (c)

Figure 1. Demonstration of Haga's fold.

Prove: As shown in Figure 2. Let the side length of the square be 1. Assuming DF = a, then FC = 1 − a. By the folding process FE = FC, so FE = 1 − a. Since E is a midpoint, DE = 1/2. Applying the Pythagorean theorem, $a^2 + b^2 = c^2$, we obtain a = 3/8. Therefore DF = 3/8 and FE = 1 − a = 5/8. In other words, by the above folding procedure, the right side of the square is divided in the ratio of 3:5. And further, the ratio of the three sides of the triangle EDF is FD: DE: EF = 3/8:1/2:5/8 = 3:4:5. Triangle EDF turns out to be a Pythagorean Triangle. Since vertex C of the square was folded onto point E and C is a right angle, then also ∠HEF is a right angle. Therefore, the angles adjacent to ∠HEF are complementary and triangle EAH and triangle FDE are similar. Therefore, triangle AEH is proportional to triangle EDF. The ratio between the two triangles will be DF/DE = AE/AH, then 3/8:1/2 = 1/2: AH. AH = 2/3, point H will be one of the trisections points on AB.

The second theorem and the proof: Let F be a point on the side AB such that the reflection of B in the line CF coincides with the reflection of D in the line CE, see Figure 1(c). This is called Haga's fold of the second kind, with the crease lines CE and CF. He discovered if F is the midpoint of AB, then E divides AD in the ratio of 2:1 internally.

Prove: As shown in Figure 3. Let the areas of the triangular flaps as R and S respectively, and the area of triangle AEG as T. Then the area of the whole square is $2R + 2S + T$. Assuming that the length of one side of the square is 1 and letting BG = x, then $R = 1/4$, $S = x/2$, $T = 1 - x/4$. Therefore, the expression of the square area will be $1/2 + x + 1 - x/4 = 1$. From this equation, we obtain x = 1/3.

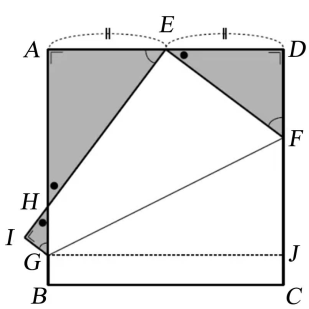


Figure 2. The proof of Haga's fold of the first kind theorem.

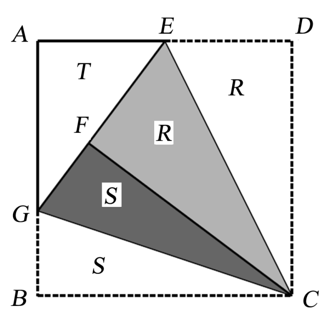


Figure 3. The proof of Haga's fold of the second kind theorem.

The third theorem and the proof: If F is the midpoint of AB, then E divides AD in the ratio of 2:1 internally, see Figure 1(b).

Prove: As shown in the Figure 4. Let the side length of the square be 1. Assuming HB = x and BG = y. By the paper folding procedure, CG = GH = 1 − y. By applying the Pythagorean theorem to triangle HBG, $x^2 + y^2 = (1-y)^2$, and therefore

$$y = (1-x^2)/2. \quad (1)$$

Since $\angle$EHG is right, then $\angle$AHE and $\angle$BHG are complementary. Therefore, triangle EAH and triangle HBG are similar, and AE: AH = HB: BG, which is $1/2 : 1-x = x : y$. This results in

$$y = 2x(1-x). \quad (2)$$

Put Formulas (1) and (2) together and we obtain $\frac{1-x^2}{4} = x - x^2$, which leads to $3x^2 - 4x + 1 = 0$ or $(3x-1)(x-1) = 0$. The roots are $x = 1$ and $x = 1/3$. Because the side length of the square is 1, so the value 1 is discarded, therefore $x = 1/3$ will be the solution.

The three theorems discovered by Haga are used to help people to be able to fold paper into equal proportions of separation (Haga, 2008).

Another one is about how to trisect an angle via folding, this method is by H. Abe (Okumura, 2014).

1) Let the angle you want to trisect originate from the lower-left corner. Call this angle A. (Note that here we assume that A is acute) Make two parallel, equidistant horizontal creases at the bottom, as shown in Figure 5(a).

2) Then fold p1 onto L1 and p2 onto L2, as shown in Figure 5(b).

3) With this folded, refold crease L1, now in its new position, and extend it up. This new crease, L3, is the crease we want. Unfold step 2 and extend crease L3 to the lower-left corner. The crease L3 will mark the angle (2/3) A, as shown in Figure 5(c).

4) After adding a few lines, as shown in the figure downward, then it shows that triangle AOB = triangle BOC = triangle COD, which proves that angles AOB, BOC, and COD are all equal, and thus must be A/3, as shown in Figure 5(d).

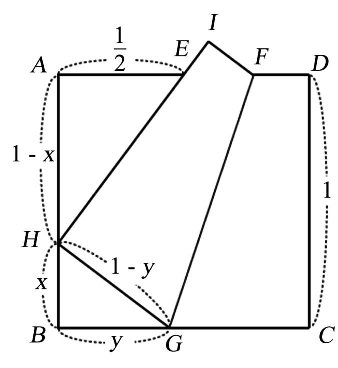


Figure 4. The proof of Haga's fold of the third kind theorem.

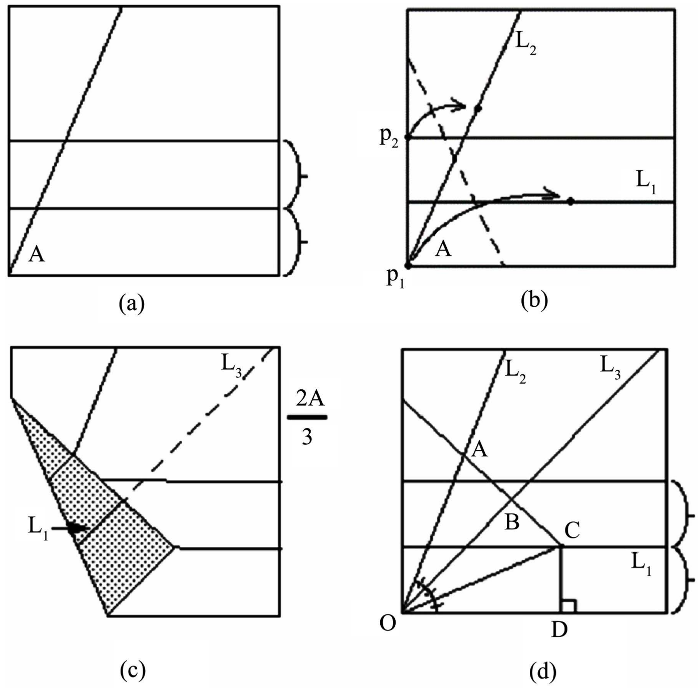


Figure 5. The process of H. Abe's method to trisect an angle.

What are the two major examples that were found in the works of literature written by past researchers; both the two examples helped solve mathematics problems greatly, especially the second one (Hull, 2012). It has been proved by people in the past that trisections are impossible under Euclidean rules. However, it is now proven that we may trisect by folding, which implies the importance of combining origami with mathematics. The two examples show that origami did help solve those mathematics problems.

3. Influence on the Current Study

Since started practicing origami and folding paper planes till now trying to fold a Diplodocus individually, I have gained a lot from this process. For instance, the ability of imagination, spatial awareness, and concentration. First, various improvements in the inner self, are what practicing origami brings.

Practicing origami enables people to think about the process of folding to make them fold the paper in different shapes that they want. This had exceedingly trained their ability to visualize things in their mind. Folding paper into complex three-dimensional objects is not as simple as it looks, you may fold hours for just a simple-looked maple. The main goal of this process is to fold the paper from a two-dimensional space into a three-dimensional, to describe it in another way, is to fold "space". To fold a square paper into objects without any cutting or gluing also requires a lot of concentration, if the paper is rotten, a new round of folding will be started. And this ensures people focus on the thing that was doing at any moment. This ensures the ability to study anywhere at any time. For example, whenever I was doing origami, I did it step by step carefully. I know that if I did something wrong, then I will have to fold it again from the start, which is a waste of time. To ensure my time management is efficient, whenever I was doing a task, I will treat it as if I am doing origami.

Secondly, it may also develop the way we think. Each time we fold on a piece of paper, we create a part of the entire structure. Understanding the links between each fold and how it relates to the overall structure is part of developing your critical thinking skills. Also, following the instructions, it teaches a way to enter into a state of thinking systematically. Origami isn't only an activity that children can do, some of them are very complicated, and even adults may not do it. People will try every step carefully until they finish it. It is just like you are trying to solve a mathematics problem, first think about the process and then give out the result. Therefore, we may improve our problem-solving skills during the process of doing origami.

Thirdly, origami practice helps us to get notices about math theorems and other contents at a more rapid speed. It is proven that math concepts could be shown by origami practice. After a long term of practicing, a frame of a system of imagination is being built, which makes people think in an orderly way that my mind follows a series of steps when thinking, which allows us to associate the content with origami in our mind. It makes math concepts into a visualized form; it may help us to have a more intuitive understanding of how those theorems work and think about how to prove them.

Next, how would origami assist the current study in the reality? I will write in two parts, my academic score, and how its culture influences my study. As mentioned, that origami helps to understand content more quickly in mathematics lessons, and it also helps a lot with my grade. Study in origami involves various subjects that we learn at school, including mathematics, and physics, and even requires an ability in organizing language. Mathematics and physics phenomena are shown as you fold papers into shapes and objects. The ability to organize language has two functions, the first is to ensure that you can read the instructions and accomplish them in reality; the second is to help you to sort out the main structure of the work more easily in your mind and the operating process. About how it improves current grade, the process of observing the instructions of origami enables people to remark the important information in a test paper, which improves accuracy. Another thing is, it also improves logical ability, as there is instruction for origami, it is folded in a specific sequence, but it is not easy to understand every process, it is because the crease for each step may be different when folding, you will have to think about the logic between steps. This helps to improve the ability of logic appreciably.

As an activity that has been developing for centuries, the development of origami in different countries was unlike. For instance, the origin of origami was different from its history in nonidentical regions. In western countries, the earliest evidence of paper folding is a picture of a small paper boat in the 1498 French edition of Johannes de Sacrobosco's Tractatus de Sphaera Mundi. However, whether origami in Europe originated by itself or not is still not known. In Japan, origami began sometime after Buddhist monks from China carried paper to Japan during the 6th century (Robinson, 2004), and the monks recorded their use of origami as early as 200 AD. Paper is expensive in that period in Japan due to there is difficulty in making it, so the first Japanese origami was used for religious ceremonial purposes only (Lang, 1988). In addition, a reference in a poem by Ihara Saikaku from 1680 describes the origami butterflies used during Shinto weddings to represent the bride and groom (Koshiro, 2006), which reveals how ingrained origami has become at the time in Japan. China, where people consider the origin of papermaking, is probably where the origin of origami. Origami has been developing in China for decades. As sino-foreign exchanges, it provides an opportunity for people who are interested in it. This also allows me to learn about how are those origami works designed by foreign artists around the world, enabling people to create origami work that was designed individually. During the process of questing the history of origami, I found out that it is something that I have never thought of before. I would never know what is the earliest origami if I did not make a thorough inquiry through it.

After knowing the history and the culture behind origami in different countries, a new awareness of this traditional art in China will be gained. And you may treat origami with more awe due to its bumpy history and minority to be known by the world. Origami is not just a form of art; it is also a procedure of inheriting culture. There is a spirit in origami, a spirit we called the "spirit of craftsman". The spirit is to let us meditate and work on something for hours without being distracted by others and things. Thus, studies may become more efficient than before since people will not be distracted by things. As a result, origami has helped us tremendously.

4. Influence on Mathematics Study

In this part, how origami may help to study mathematics will be mainly mentioned. Origami is divided into several types, one of them is modular origami. Modular origami is folding several papers into the same kind of component and assembling them into a fully done work. This is a way to make students work and study together in groups, something more is they may teach each other due to the speed of students accepting knowledge is different. Group study develops the thinking ability of each and the whole group and helps them to understand and solve problems quickly and efficiently. Group study allows students to gain a better extent of understanding math. Also, it permits students to discuss questions that they may not solve alone. This type of study will highly improve the efficiency of the class. They may participate more due to a team effect. When I was making the Diplodocus skeleton origami, it is made of hundreds of similar pieces, so I make it with my sister, by observing her reaction through the process, sentiments like feeling impatient and irritable have appeared. This indicates that, as I and my sister were of different ages, had unequal experiences, and had different student statuses, our productivity was inconsistent. Therefore, a group of students or friends who can discuss with each other may enhance efficiency and finally help each other solve the problems.

To those students who are deaf or dumb, origami could be a suitable tool for them to learn math concepts. It can be illustrated through origami. Origami activities enable students to build their experiential base relating to the development of certain mathematical concepts and to explore many geometric forms while problem-solving and constructing. The mathematical skills and concepts inherent in origami include spatial visualization, intersecting planes, areas, and volumes, mirroring, and more else. Origami can also be used for teaching symmetry. For example, with many folds, whatever is done to one side will be done to the other. Such activity makes math concepts visualize and allow deaf and dumb students to create and manipulate basic geometric shapes such as squares, rectangles, and triangles that might otherwise be taught through lessons that students may only understand the concepts by looking.

Also, mathematicians have already been investigating a wide range of questions relating to paper folding. Levenson (Chen, 2006) has found that origami has shown that, paper-folding, particularly in the elementary school years, is a unique and valuable addition to the math curriculum. After all, the process of transforming a piece of scrap paper into something not only makes children feel "a sense of achievement", it also links math and origami skills and helps children to understand spatial relationships of three-dimensional objects, investigate the symmetry, congruence, and angles of geometry, and develop their analytical and critical thinking skills.

A study was done by Norma J. Boakes, an associate professor of education at Richard Stockton College of New Jersey. Her study was against middle school students and college students, the result of the study shows that origami is an effective teaching tool capable of strengthening students' mathematical and spatial abilities, which is an important part of math study. This proves that the effect of origami works on students and improves their ability.

There are researchers been doing studies about the possibility of letting students from places all over the world obtain mathematics education with origami. However, a case in Indonesia shows that from 100% of incoming data obtained, only 12.5% of respondents used origami as a medium for learning mathematics (Toyib & Ishartono, 2018). This shows that the penetration rate of using origami as a tool to help mathematics study students still has space for improvement.

5. Conclusion

To conclude, as origami has now gradually become known by more people, it is assimilating into the students' mathematics study. Being a craft that has a hundred years of inheritance, origami has an important role in cultural study, practicing this may also ameliorate a student's self-behavior. The opportunity of learning origami gives us wider eyesight of the world and facilitates our academic study. Concerning the contributions of the origami activity, it was found that how potential origami is concerning embodying mathematical concepts. In the light of these results obtained with many aspects such as ensuring student participation, increasing motivation, stimulating curiosity, offering a playful environment, and ensuring hand-eye coordination, more focus should be laid on activities using origami.

About how origami may help math study, there are several examples below, it is mentioned that origami may be a tool for deaf and dumb students to study math. Second, it can improve students' mathematical and special abilities. Third, math concepts can be shown by origami. Forth, practicing origami may help students to participate more in class and ensure they focus on things that they are doing at the moment.

Although origami is not universal in mathematics study, only a few people are learning math through practicing origami. However, there are still many mathematical concepts waiting for exploration, which provides a tremendous opportunity for applying origami as a math learning medium in depth.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Chen, K. (2006). Math in Motion: Origami Math for Students Who Are Deaf and Hard of Hearing. *Journal of Deaf Studies and Deaf Education, 11,* 262-266. https://doi.org/10.1093/deafed/enj019

Ferrini-Mundy, J. (2000). Principles and Standards for School Mathematics: A Guide for Mathematicians. *Notices of the American Mathematical Society, 47,* 868-876.

Fox, M. (2005). *Akira Yoshizawa, 94, Modern Origami Master*. New York Times.

Haga, K. (2008). Origamics. *In* J. C. Fonacier, & M. Isoda (Eds.), *Mathematical Explorations through Paper Folding* (152 p.). World Scientific Publishing Company. https://doi.org/10.1142/7023

Hull, T. (2012). *Project Origami: Activities for Exploring Mathematics* (pp. 63-68). CRC Press. https://doi.org/10.1201/b14320

Koshiro, H. (2006). *History of Origami*. https://origami.ousaan.com/library/historye.html

Laing, E. J., & Liu, H. H.-L. (2004). *Up in Flames: The Ephemeral Art of Pasted-Paper Sculpture in Taiwan*. Stanford University Press. https://doi.org/10.1515/9781503617834

Lang, R. J. (1988). *The Complete Book of Origami: Step-by-Step Instructions in over 1000 Diagrams: 37 Original Models*. Courier Corporation.

Okumura, H. (2014). A Note on Haga's Theorems in Paper Folding. *Forum Geometricorum, 14,* 241-242.

Rao, T. S., & Row, T. S. (1917). *Geometric Exercises in Paper Folding*. Open Court Publishing Company.

Robinson, N. (2004). *The Origami Bible: A Practical Guide to the Art of Paper Folding*. Collins & Brown.

Thomas, H. (2015). Origami and Geometric Constructions: *A Comparison between Straight Edge and Compass Constructions and Origami*. http://origametry.net/omfiles/geoconst.html

Toyib, M., & Ishartono, N. (2018). An Analysis of the Possibility of Origami Implementation in Mathematics Learning Process in Indonesia. In *Proceedings of 2nd International Conference on Education Innovation* (*ICEI 2018*) (pp. 146-151). Atlantis Press. https://doi.org/10.2991/icei-18.2018.32