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A Toolkit for Teachers and Learners, Parents, Carers and Support Staff: Improving Mathematical Safeguarding and Building Resilience to Increase Effectiveness of Teaching and Learning Mathematics

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Abstract

Mathematics anxiety is a longstanding, prevalent, global problem, and the scholarly literature demonstrates that it has significant consequences for both individuals and wider society. However, to date, there has been no simple, comprehensive, practical approach available to teachers, support staff, parents/carers and learners to address the issue. In this paper, previous work and existing approaches to addressing mathematics anxiety are reviewed; underlying concepts are examined, such as psychological safeguarding in the context of mathematics education and the distinction between the learner's psychological states of challenge and threat. Agency is important for the well-being of learners and those who support them; an argument is made for an original approach to improving mathematical safeguarding and building mathematical resilience that involves increasing agency. This new approach brings three recognisable tools into a toolkit designed to be accessible to teachers, learners, parents/carers and support staff. The three tools are all individually well-established in specialised practice; they are: the hand model of the brain; the relaxation response and the growth zone model. It is argued that these tools are particularly well suited to addressing mathematics anxiety, that they are complementary and that together they form an effective and accessible toolkit. Details from small-scale studies are shared and it is concluded that this approach warrants further research to properly establish efficacy.

Keywords

Mathematical Resilience, Mathematics Anxiety and Avoidance, Growth Zone Model, Mathematical Safeguarding, Resilience Education, Relaxation Response

1. Introduction

Mathematics anxiety (MA) inhibits mathematics learning and has ongoing deleterious effects on learner well-being and life opportunities, as well as limiting the contribution individuals can make to wider society (Dowker et al., 2016). However, despite the problem having been recognised and studied since the 1950s, there are currently few strategies to address MA and even fewer that can be implemented easily in the classroom (Dowker et al., 2016).

This article is a position paper, in which the authors review the significance of mathematics anxiety and construct a toolkit to address it. It proposes a way to give agency to, and empower, teachers, learners and their supporters, in a situation of increasing concern about MA (Carey et al., 2019), by offering a new way of thinking based on three tools. The approach is designed to enhance teachers' awareness of insights from psychology, coaching and neuroscience, such that they can use skills derived from these fields to become better-informed nurturers of mathematical resilience. In particular, effective psychological safeguarding designed to avoid learners developing anxiety associated with mathematics, is achievable through raised awareness of distress, on the part of teachers, support staff, parents/carers and learners, and the use of appropriate tools to communicate and address this distress. It is argued that increased understanding of the debilitating effects of MA, and building mathematics resilience, consequently avoiding psychological harm (Lyons & Beilock, 2014), is likely to result in saved time and money, and increase student attainment and self-efficacy.

Three tools are introduced into the mathematics context that can be used better to understand learner responses to mathematical stimuli and to address adverse emotional states associated with learning mathematics. This can be seen as a form of psychological safeguarding in the context of learning mathematics or *mathematical safeguarding* (see below). The aim is to provide educators, who may currently feel helpless or perplexed, with the tools and agency to support learners struggling with MA or choosing to avoid mathematics.

2. Background

2.1. Mathematics Anxiety (MA)

Mathematics anxiety has been recognised for over 60 years as disabling and, although estimates of its prevalence vary widely, "it is a very significant problem" (Dowker et al., 2016, p. 3; OECD, 2013). The problem is global. It is experienced on a spectrum, from mild tension to strong fear. In the PISA results of surveys of

15-year-olds across OECD countries (OECD, 2013), 59% reported worrying about mathematics, while around 30% reported helplessness and emotional stress when working on mathematics problems. Some very young learners have acquired mathematics anxiety (Carey et al., 2019), and its prevalence appears to increase with age during childhood (OECD, 2013). In most countries, females are more prone to MA than males (OECD, 2013). Such anxiety is not limited to school students: Johnston-Wilder et al. (2014) found that, amongst apprentices in the UK, MA scores indicated 30% would be visibly anxious during their mathematics studies and another 18% would experience significant impact on their work. Finlayson (2014) found that, amongst elementary school teacher trainees, "[n]early all indicated that they had suffered math anxiety at some point in their lives" (p. 104).

Not addressed, MA interferes with teaching and learning mathematics, resulting in avoidance (Ashcraft, 2002; Brown et al., 2008; Skemp, 1971), delayed progress and reduced competence (Ashcraft, 2002; Hopko et al., 2003; Richardson & Suinn, 1972). The importance of teachers' and learners' understanding MA has been long recognised (Dowker et al., 2016). However, MA has still not been addressed effectively (Dowker et al., 2016), despite associated waste of resources, and cost to individuals and economies (National Numeracy, n.d.).

In this paper, MA is seen as arising from emotional harm, actual or vicarious, associated with mathematics. Causes of MA include isolation, humiliation and being ignored (Cousins et al., 2019; Finlayson, 2014). The higher prevalence of MA in females (OECD, 2013) may be because they tend to have higher empathy (Mestre et al., 2009), and so be more sensitive to stories of others or acquire MA from older generations, be it teachers (Beilock et al., 2010) or carers (Maloney et al., 2015). A commonly-used definition of MA is "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in … ordinary life and academic situations" (Richardson & Suinn, 1972: p. 551). However, this definition does not convey the acuteness of the distress experienced by some learners. Lyons and Beilock (2012) found that when a mathematics-anxious individual thinks about doing mathematics, there is activity in areas of the brain that are involved in the sensation of pain. Thus, anxiety cannot be discounted as somehow less significant than physical pain.

In this paper, the term "mathematics anxiety" is taken to mean a subject-specific and disabling psychological injury that interferes with optimum learning of mathematics. It does not simply affect mathematical problem-solving when affected learners engage with mathematics but can have ongoing deleterious effects on learner well-being and life opportunities.

2.2. Safeguarding and Resilience in the Context of Mathematics Teaching

"Safeguarding" denotes actions taken to promote the welfare of learners and to protect them from harm. In the UK, safeguarding of children includes preventing impairment of health or development and taking action to enable all children to have the best outcomes (DfE, 2015); MA is one such impairment to be prevented.

Mathematical safeguarding is taken to be a fundamental requirement to support psychological safety (Newman et al., 2017) and to promote resilient self-efficacy in all learners (Bandura, 1997) in the context of mathematics.

Having mathematical resilience means having confidence, high satisfaction, and a sense of value, meaning or purpose in relation to mathematics, and being able to overcome threats to mathematical self-efficacy (based in Bandura, 1997). This involves teachers and other supporters having awareness of signs of elevated levels of arousal, and strategies to reduce arousal to levels consistent with productive learning. Without such awareness, the learner may be perceived as "not listening" when in fact they are panicking and may perceive themselves as "stupid". Learner experiences in such situations can be a factor in the development of chronic MA, with its associated ongoing effects.

In this paper, the notion of self-safeguarding is introduced in the context of learning mathematics, to describe the actions of learners to keep themselves safe from MA. This aspect of the toolkit for staying safe while doing maths parallels, for example, tools for staying safe online.

2.3. Challenge and Threat

The efficacy of the three tools is based upon recognition of two distinct emotional states which may occur in a learning context, namely challenge and threat. The two states and the distinction between them are described here. A situation is appraised as a challenge when the learner anticipates mastery or personal growth (Blascovich & Mendes, 2000) and believes that they have sufficient resources to cope. The situation is perceived as a threat when the learner believes that they lack sufficient resources to cope (Blascovich & Mendes, 2000).

Historically there is a lack of clarity about the distinction between these two states in discussion of mathematics anxiety. It is important to recognise that nervousness is a normal state in response to an unfamiliar situation, and anxiety is a normal state in response to perceived threat to well-being (Lazarus & Folkman, 1984). Similar states of physiological arousal are involved in challenge and threat, and their associated emotions. Cognitive appraisal determines which state is experienced (Lazarus & Folkman, 1984). However, this process can be short-circuited if the brain "flips" into the threat state (Siegel, 2010) before the cognitive appraisal is complete. Both challenge and threat cause the heart to beat faster and harder than during rest, but, after cognitive appraisal, challenge results in dilation of the arteries and more blood being pumped, enhancing both cognitive and physical performance, whereas threat results in constriction of the arteries and less blood pumped (Seery, 2013), reducing cognitive performance. Long-term exposure to perceived threat is inefficient and harmful to health and well-being (Black, 2003) and to mathematical performance.

Learners may experience physiological arousal as threat; however, teachers can help learners reappraise this arousal as challenge, which can facilitate improved performance (Jamieson et al., 2010). It will be shown below how the three tools can help with this reappraisal. Prior experience has a significant impact on a learner's appraisal of whether a situation represents a challenge or a threat (Lazarus & Folkman, 1984; Vygotsky, 1994). Different learners make different appraisals and have different transition points between states. Social experiences, such as previous exclusion, embarrassment, shame, humiliation or failure, can lead to perception of a similar experience as threat; this can trigger intense feelings, since survival often depends on social well-being and inclusion in a community (Simpson, 2005). Such prior experiences are not visible and can easily be underestimated unless understanding and communication about the affective domain is facilitated. Once a learner recognises that they are feeling threatened, they need to be allowed to take action to manage the associated emotions.

2.4. Existing Approaches to Addressing MA

Some teachers may not be aware that MA exists and can be addressed. Instead, they may complain that learners do not attend, do not engage, are argumentative, are unable to answer simple questions, lack motivation, have counterproductive attitudes (Alderton & Gifford, 2018) or avoid mathematics wherever possible. Such behaviours are frequently reported by mathematics teachers, who may be unaware that these are signs associated with anxiety.

Teachers may ascribe these behaviours to factors such as laziness, devaluing of school, and lack of parental support (Turner et al., 2002). Without understanding why learners present in this way, some teachers adopt a fixed mindset, maintaining that such learners are "not capable" or lack ability (Alderton & Gifford, 2018), in some cases voicing this directly to learners (Cousins et al., 2019), which then reinforces anxiety.

Dowker et al. (2016) mention possible approaches and strategies, highlighting the need to intervene early, to model positive attitudes to mathematics and to avoid expressing to children negative attitudes about mathematics. They discuss the complexity of treatment and mention: systematic desensitisation; behaviour therapy; cognitive reappraisal; writing about worries; specialised cognitive tutoring and transcranial electrical stimulation. Many of these strategies are beyond what can be implemented in a classroom, however; even modelling a positive attitude may be problematic if teachers, parents and support workers experience mathematics anxiety. Dowker et al. (2016) acknowledge that, after 60 years, research tells us little about "how mathematics anxiety may be treated, or, ideally, prevented" (p. 10), calling for further research on the effectiveness of different strategies. According to Lyons and Beilock (2012), the solution to this challenge lies in developing "[e]ducational interventions emphasizing control of negative emotional responses to math stimuli (rather than merely additional math training)" (p. 2102).

Discussion about MA is not yet high on the current teacher preparation agenda. Discussion about preparing teachers of mathematics (Copur-Gencturk et al., 2018) may mention the affective domain (Bloom et al., 1956) in terms of engagement; discussion of safeguarding of the kind that would address MA and those excluded by emotion from further study is rare (Nava et al., 2018). Educating teachers of mathematics about the affective domain is relatively new.

2.5. Contribution of the Proposed Approach

Addressing MA is often seen as specialised work beyond the remit of classroom teachers (Carey et al., 2019). However, an accessible way of exercising effective agency and attending to the affective domain, as proposed here, rather than feeling helpless, has significant implications. The consequences of MA are both personal and social; many writers have emphasised the way in which MA limits a person's contribution to society and the consequent wider effects. According to Lyons and Beilock (2012), the solution [to mathematics anxiety] will reveal "a population of mathematically competent individuals, who might otherwise go undiscovered" (p. 2102). As mathematics is increasingly required in the workplace (British Academy, 2012; Smith, 2017; UKCES, 2014), particularly in the context of "big data", solving this problem would have significance for potential employers who struggle to recruit enough staff with mathematics capability, and would enable more choice and opportunity for individuals. However, despite the implication that teaching mathematics without addressing anxiety is relatively unproductive, the practice continues. The challenge is to create a wider awareness and understanding of MA amongst practitioners (Finlayson, 2014) and to develop effective strategies to address it (Dowker et al., 2016). The three tools are found to be accessible to use in the classroom (Johnston-Wilder & Moreton, 2018).

Discussion here is framed in the context of mathematics; it is predicted that the tools proposed will also have efficacy in other subjects.

2.6. Originality of the Approach

The originality of the current paper is in engaging with the complex, global problem of mathematics anxiety, and showing how to combine and apply three existing tools to solve this problem, thereby creating a new educational intervention of the kind described in principle as needed by Lyons and Beilock (2012). It is not new to argue that teachers should learn about managing emotions in the classroom, or to argue that learners should be taught emotional regulation and self-safeguarding (OECD, 2010). For instance, in British Columbia, zones of regulation are taught (Kuypers, 2011). What is new here is applying self-regulation in an accessible way in the specific context of addressing mathematics anxiety, thus providing agency for individual teachers and learners and their supporters.

For instance, this paper takes the thinking behind well-established physical safeguarding and risk-taking in outdoor education (Rohnke, 1989) and applies it to psychological safeguarding of well-being in the mathematics classroom. Original to this paper is the holistic application of existing knowledge about concepts such as challenge, threat, safeguarding and self-safeguarding to the context of mathematics anxiety.

3. The Toolkit

3.1. Overview

It is proposed that an understanding of three tools forms a basis for mathematics teachers to support learners to manage their emotions, reduce the impact of anxiety in mathematics lessons and empower learners with agency to progress more effectively. The three tools are: the hand model of the brain (HMB; Siegel, 2010), the relaxation response (RR; Benson, 2000) and the growth zone model (GZM; Johnston-Wilder et al., 2013). The HMB helps learners to understand why they may feel "stupid" when faced with a perceived threat; the RR enables learners to recover from threat and return to a calmer state; and the GZM enables learners and teachers to develop an easily accessible shared language to raise awareness of and communicate emotions. These tools can help teacher educators, teachers, parents/carers and learners to understand better how teaching and learning approaches can be adjusted to reduce the impact of anxiety and to help learners build resilience. Teachers and learners who have piloted the toolkit find it manageable and effective. As stated above, these tools rely on the important distinction between challenge and threat. Each of the three tools will now be considered in turn, followed by a discussion bringing the tools together into an effective toolkit.

3.2. Hand Model of the Brain (Siegel, 2010)

The hand model of the brain (HMB), developed by Daniel Siegel (2010), forms the basis of several versions of emotional coaching that help people to understand why anxiety and panic occur, how to handle them, and effects on cognition. New here is the proposal to apply the HMB to the problem of mathematics anxiety, to be used both to explain to learners any feeling of "stupidity" they may experience, and to make it apparent that this experience can be temporary.

In **Figure 1(a)**, the closed fist represents the brain in a cognitively regulated

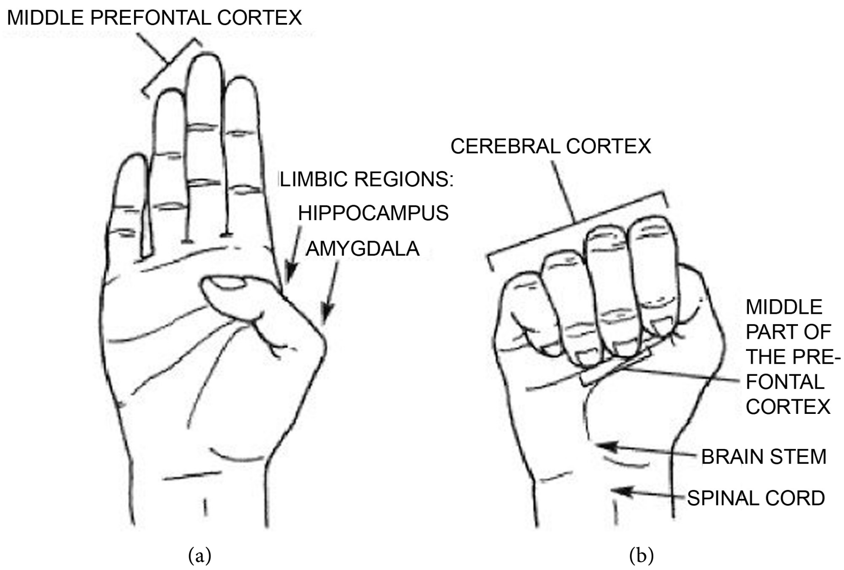

Figure 1. Hand model of the brain (Siegel, 2010) (copyright permission granted).

situation, able to learn. In **Figure 1(b)**, the hand is open to represent loss of regulation of the amygdala by the prefrontal cortex. This is known as the "fight or flight (or freeze)" response; see https://www.youtube.com/watch?v=gm9CIJ74Oxw. If a situation is perceived as similar to a previous situation experienced by the learner as harmful or humiliating, or which led to perceived failure, then it is more likely to be appraised by the amygdala as a threat. This triggers the fight, flight or freeze (Gallup, 1977) response, to "lower arousal and produce feelings of physical and emotional well-being by neutralizing a perceived threat or removing the person from the proximity of a perceived threat." (Pearce, 2011). When learners are in this state, their capacity to think and reason logically, and express themselves verbally, is [temporarily] reduced.

If a mathematics task triggers a threat response, then MA may lead to feelings or behaviours of distress, panic, or temporary "stupidity". Learners of mathematics can experience their mind going suddenly blank, or feelings of anger or fear, or an incremental increase in adrenalin that gradually interferes with their ability to think.

In response to anxiety, learners' behaviour may deteriorate and become more controlling, aggressive or destructive (fight), they may become agitated and seek to run or hide (flight), or responsiveness may decrease (freeze). Parents/carers with MA can experience similar emotions, and might, for example, suddenly burst into tears when asked to help a teenager with mathematics homework (Eggison, 2017). A common response is to join the growing ranks of the excluded, saying something like "I was never a maths person either; our family is better at English" (personal communication, student's mother).

Teaching the HMB can mitigate these responses in three ways. Firstly, it can help explain, in an accessible way, based in neuroscience, why such responses may occur when triggered: the learner is not "stupid" but perceiving threat based on prior experience. It also enables these responses to be reframed: they are simply behaviour directed at protecting the individual from perceived threat of harm. Recognising this enables the learner to re-evaluate their assumptions about their ability to learn mathematics. Siegel (2010) discusses the efficacy of naming feelings in order to tame them. Rather than name the state of MA as "I can't do maths", the HMB allows for a new name: "I've flipped; give me some time and I can rethink this". In a recent project (Baker, in progress), Devon (aged 11) reported that "the hand model of the brain … it sort of made more sense and you could like, understand why you get anxious, so then you could think about it and think, oh no, it's not because I'm dumb […] everyone is like it sometimes". In light of Dweck's (2006) work on growth and fixed mindset, it no longer seems appropriate to leave such learners believing that they are not capable of learning mathematics.

Secondly, the HMB can make visible a transition that might otherwise be hidden, and communicates the experience of the learner promptly and effectively if the learner is prepared to show what state they are experiencing using their hand to represent their brain state, either one-to-one or as part of a whole class. Many young people panic quite quickly in mathematics, developing avoidance strategies to safeguard themselves, such as moving to the back of the classroom or not attending (Cousins et al., 2019). For someone faced with a situation that causes psychological pain and disruption (Lyons & Beilock, 2014), who believes they are unable to do anything, whether practical or cognitive, to resolve the situation, there may seem little point in continuing learning. The problem with avoidance strategies is that, although they keep the learner psychologically safe in the short term, they reinforce anxiety, rather than addressing it (APA, n.d.). These strategies also fail to communicate the experience of the learner in a way that enables teachers and other learning supporters to be helpful. If the learner's transition into threat state becomes apparent, an opportunity is opened up for educators to help the learner to employ other protection strategies while continuing to engage.

Thirdly, some learners of mathematics behave in ways that are out of character, so as to be removed from or avoid engaging in the mathematics lesson. This may include:

- angry outbursts or behaving aggressively towards others;
- becoming withdrawn or appearing clingy or depressed [helpless];
- displaying a lack of confidence, wariness or caution;
- expressing a fear of making mistakes (NSPCC, 2017).

Use of the HMB can provide early warning that the risk of such behaviours is raised. In summary, it is suggested that educators need to find functional measures to identify the presence of MA when teaching mathematics, in order to optimise learning and provide effective safeguarding. The HMB is one way in which the experience of the learner may be communicated directly, hence enabling the learner to be supported effectively. The next tool can help provide both teacher and learner with appropriate responses to the threat state if it arises.

3.3. The Relaxation Response (Benson, 2000)

Having recognised that they have flipped into a threat state, if the learner comprehends that on this occasion there is no actual imminent danger, they need to be able to change state in order to re-engage. The "relaxation response" (RR; Benson, 2000), often known as "rest-and-digest", is the "opposite" of the fight or flight response, and offers a quick, effective way to self-soothe by engaging the parasympathetic nervous system; muscles and organs slow down and blood flow to the brain increases. Part of the originality of this paper is in proposing use of this tool in the mainstream mathematics classroom. It is argued that the RR has a particularly important role in the learning and teaching of mathematics when a learner's MA is triggered.

Within the mathematics or support classroom, and in exams, learners can be taught to trigger the RR and slow the heart rate by practising longer out-breaths, using 7 - 11 breathing (Griffin & Tyrell, 2004) and similar techniques. For a few learners, focus on breathing does not help. To promote relaxation, learners can alternatively: name three things they can hear; name three things they can see;

name three different textures they can feel. Such activities can help to "ground" the learner in the sensory reality around them rather than in their emotional upheaval. These activities are brief, simple and specific and do not require outside intervention to implement. They can help the distressed learner to redirect their attention away from the source of their distress, and hence help reduce the physiological manifestations of the threat response. In cases of more extreme distress, a learner may need to leave the room with permission, go for a walk to a safe, designated place, and then seek one-to-one support. In the most serious, rare cases, a learner may need formal therapeutic intervention (Bandura, 2007; Dowker et al., 2016).

The RR can be encouraged more broadly through mindfulness as a strategy for improving well-being and emotional regulation, and reducing anxiety, reactivity, worry and rumination (Gotink et al., 2016; Klingbeil et al., 2017). Mindfulness is the practice of focusing awareness, paying attention to the present moment with openness, curiosity and non-judgmental acceptance (Kabat-Zinn, 1994). Extensive mindfulness research in schools has demonstrated significant impact on attainment (see for example, Gouda et al., 2016). Helping learners find ways to calm adverse emotional responses to tasks and to pay attention to the moment may be achieved in any learning environment by any teacher. Use of the RR gives agency to the learner and the teacher to discuss emotional responses and different methods of focusing the mind on the moment. Brief interventions might be called "micro-mindfulness"; they can be applied in mathematics classrooms to help anxious or threatened learners.

The original proposition here is the introduction of "micro-mindfulness" within mainstream mathematics learning, giving an opportunity for learners to instantly use tools to self-manage anxiety—tools that a classroom teacher can introduce whether the school has introduced mindfulness or not. Micro-mindfulness can also be used by university lecturers to support learners who experience their mind going blank, for perhaps the first time. Important here is that once a learner recognises they can get out of the threat state, they become more willing to engage with challenge (Johnston-Wilder & Moreton, 2018; Lee, 2016).

3.4. The Growth Zone Model

In discussing the states of challenge and threat, implicit reference has been made to a third tool—the GZM. This brings together the preceding discussion in a simple, cohesive framework for becoming aware of, describing and communicating emotions. Similar tools are widely used in a range of contexts. Here it is proposed to include GZM as part of the toolkit to address mathematics anxiety.

Much of the work on MA has failed to distinguish across a range of anxious responses, from nervousness to panic. The GZM can be introduced to learners, teachers, parents/carers and support staff in the mathematical context, to help distinguish between productive nervousness and disabling panic, and as a way to help communicate these emotions without feeling "stupid".

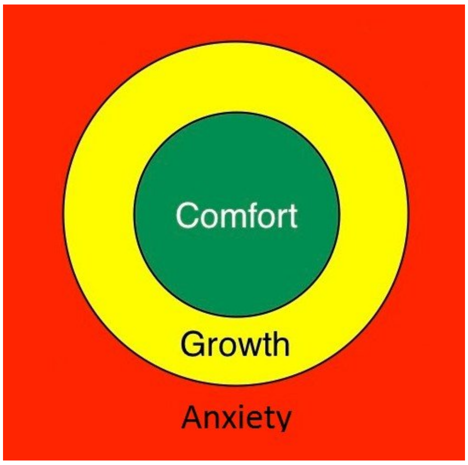

Figure 2. Growth zone model (Johnston-Wilder et al., 2013).

The model is represented (see **Figure 2**) by a green circle, representing the comfort zone where a learner feels safe, surrounded by an orange ring, representing the growth zone, where a learner experiences challenge. Outside the orange ring is a red background, representing the threat zone, where a learner experiences danger (consciously or subconsciously). The green and orange zones correspond to the closed hand in the hand model, and the red zone corresponds to the open hand.

This model extends the "flip" metaphor of the HMB and gives a framework for learners to distinguish between perceived challenge and threat, enabling them to name and communicate their current feelings. Learners can be introduced to the model in a brief lesson section, and encouraged to use their own words to describe the feelings they have experienced when faced with situations perceived as comfortable, challenging or threatening, whether related to mathematics or not (Para & Johnston-Wilder, submitted).

The comfort zone is where learners might feel comfortable, relaxed, safe, contented, and/or confident. Cruising in the comfort zone—that is, working on familiar tasks that a learner can do alone—builds self-confidence and provides opportunities for practice and automaticity.

The growth zone is where learners experience challenge. They might feel nervous, on edge, agitated, motivated, engaged, uneasy, trusting, questioning, productive, focused, uncertain, needing reassurance, excited, apprehensive, intrigued, proactive, and/or unskilled. New learning happens in the growth zone. Here, it should be explicitly safe to make mistakes, get stuck, require support, and find activities challenging and tiring. Learners can learn to reframe nervousness and need for help as indicators that they are in their growth zone rather than failing (Jamieson et al., 2010) and feeling "stupid". Learners can develop explicit strategies to deal with being stuck in the growth zone (Chisholm, 2017; Mason et al., 2010), such as underlining unfamiliar words, accessing resources online, talking with a peer, or preparing a question for the teacher. Teachers also need to allow time for mistakes, collaborative working and questions, in order to give agency to pupils experiencing growth, rather than "protecting" them from the emotions discussed.

The anxiety zone is where learners might feel threatened, incompetent, traumatised, scared, sick, overwhelmed, stressed, angry, out-of-control, hopeless, indecisive, afraid or "stupid". The anxiety zone is where what is being asked is not within the learner's perceived reach at the moment, even with support, and the learner does not have an effective self-safeguarding strategy. In this case, the learner experiences threat rather than challenge; stress increases and cognition decreases. Consequently, little or no useful learning takes place.

The idea of the GZM is well-established in the literature in other contexts in various guises which are exemplified as follows. Maslow (1962) said: "We can consider the process of healthy growth to be a never ending series of free choice situations … between the delights of safety and growth" (p. 49) but noted that there is a question of pace "if growth is not to look like an overwhelming danger instead of a delightful prospect" (p. 51). In a recent book on conflict resolution, Legge (2019) describes the zones as follows: "When doing the activities you might experience insights you would not have expected. Discovering your unacknowledged feelings or needs can't be predicted or controlled, so it could get scary or frustrating. You'll be asked to open to things that can be painful. Be ready to challenge yourself to the point of discomfort if you want to really grow. But it's not usually healthy to push past discomfort into full panic. Listen to yourself and discern when it may be time to take a break or sit out for an exercise" (p. 6). Rohnke (1989), an adventure educator, introduced a "comfort-stretch-panic" model. According to Rohnke, participants respond best to "the chance to try a potentially difficult and/or frightening challenge in an atmosphere of support and caring; the opportunity to 'back off' when performance pressures or self-doubt become too strong, knowing that an opportunity for a future attempt will always be available; the chance to try difficult tasks, recognizing that the attempt is more significant than performance results; respect for individual ideas and choices" (p. 14).

A model like the GZM, the Learning Zone Model, was developed by the German adventure pedagogue, Senninger (2000). Pearce (2011) implicitly described the GZM as connected to arousal:

> "When arousal is too high, a child's performance deteriorates, and they experience physical and emotional distress. Mastery experiences are less likely, and the child is vulnerable to repeated failure in their efforts to complete daily tasks. The result is that their self-confidence is undermined and their ability to cope with adversity is reduced. In contrast, if we can maintain a child's arousal in the middle part of the range, they are more likely to perform at their best, to have mastery experiences and to feel capable and competent when faced with adversity. So, in order to promote resilience in children we need to understand the relationship between arousal and performance, and to implement strategies to maintain optimum levels of

arousal".

Vygotsky's zone of proximal development tends to be considered a cognitive model, rather than an emotional one. Although some authors (Zaretskii, 2009) have sought to bring Vygotsky's insight about the role of emotions in learning to the fore, it can be helpful to use a three-zone model specifically of emotions. Small-scale work with teachers indicates the efficacy of this approach. One teacher reported:

> "The learning from this project has enabled children to make better choices of their appropriate level of challenge in all maths lessons and we have seen less [sic] children continuing with an activity which is easily within their comfort zone. Instead, they have found ways to make the task more difficult for themselves and to choose to move themselves into their growth zone. … Getting everything right is no longer seen as a positive achievement by the pupils. They are keen to tell us when they were in their growth zone and what they have learnt in lessons … There has been significant interest from other teachers in the school who have enjoyed hearing about the success of this project and we plan to roll out the Growth Zone Model to all classes throughout next year." (Lee, 2016: p.28)

A primary school teacher, Natasha, (Johnston-Wilder & Moreton, 2018) used a hard copy of the GZM and counters to enable year 5 pupils to show which zone they were currently in. Of one child, she reported:

> initially … it was always the red and I can't do it, I can't do it, I don't understand it, this is too much too hard … and slowly just from the little steps we were doing, so it might be using the resources, explaining the vocabulary … offering strategies to support each other so how can we help each other? … her counter slowly got into the [orange] ... she will always now ask for the challenge. (p. 8232)

Experienced teachers conducting action research such as Chisholm (2017) and King (2016) have reported that the GZM helped learners understand and communicate the emotions they felt when they were in their "growth zone". Prior to introducing the GZM, learners reported feeling hesitant to work in the growth zone as a consequence of fear of "getting stuck" and not knowing what to do. The GZM empowered them by giving them both a vocabulary for, and an understanding of, the emotions they were experiencing while learning, without judgment.

The GZM has been adapted for example by the Family Maths Charity into the toast model, which is found accessible by parents and young children, as illustrated here:

> My daughter was at the table, pen in hand, trying to figure out a maths activity. It was supposed to be fun—learning fun. But tension was rising. It wasn't plain sailing, or maybe she wasn't in the mood. She might burst out

> or give up at any moment, and I just don't want these small incidents to accumulate, as they undermine confidence. … I used to find it difficult to defuse the situation quickly enough. But no need this time. My daughter just turned around and incredibly calmly said: "Mama, where is that circle chart again?" I held my breath, took a sheet out from a pile of papers: "Do you mean this one, with the blue and green and red colours, The Growth Zone Model?" "Yes that one! See, Mama, this problem is here, in the red zone. I need something a bit easier first." … That was it. No tears, no sinking heart, no walking away. And on my side, no "Oh dear, what have I done again?"
>
> Since then "our" diagram is still on the fridge. My daughter coloured the challenge zone in blue, her favourite colour, on the grounds that "that's where we want to be when we learn". …
>
> Family Maths Charity (n.d.)

Thus, this learner does not feel panicked or threatened, but is aware when she has a problem she does not *yet* (Dweck, 2006) feel in a position to tackle, until she develops the necessary resources or cognitive ability.

3.5. Using the Tools Together

The three tools can help learners to recognise specific events of MA and address them rather than avoid them. They can then re-engage with the learning process. This is described vividly by Chris (aged 11), who took part in the same project as Devon:

> Because I was really stuck on what to do, and … I just had no idea and I was really confused, and so then I realised that I felt like I was about to go into brain freeze, and then I did the breathing … it really helped … when my brain freezes, I now know how to get it back, and be able to carry on with my work.

Devon described the process as "un-blank[ing] your brain and carry[ing] on doing [the mathematics], like understand[ing] that [the mathematics is] not that hard." Self-safeguarding in the mathematics context involves the learner knowing how to notice they are in their red zone and how to get out.

4. Discussion

In this discussion, the focus is the originality and potential impact of the paper and the challenges it seeks to address. In many places in the world, a competitive classroom culture is encouraged, and goals are articulated with a focus on performance relative to other students; this suits some learners. However, research has shown that other learners are less likely to seek help under these performance goal conditions than under mastery goal conditions, which are focused on improving individual performance (Butler & Neuman, 1995; Ryan, Gheen, & Midgley, 1998). A perceived emphasis on mastery goals in the classroom is positively correlated with lower avoidance (Turner et al., 2002). In schools where performance goal conditions predominate, "self-acceptance comes to depend on one's ability to achieve competitively" (Covington, 1992: p. 74). To protect self-worth, students who perceive themselves as lower down the achievement scale may develop strategies that deflect attention from their achievement in "the struggle to escape being labeled as stupid" (1992, p. 85). In particular, such students avoid seeking help (Ryan & Pintrich, 1997). Butler (1998) found that 10 to 12-year-olds with concerns about their competency were the least likely to request help when it was needed.

Thus, introducing the GZM offers the opportunity to discuss more helpful socio-mathematical norms focused on mastery goal conditions that support working in the orange zone, whilst reframing that a learner is not "stupid" in their individual red zone but feels threatened and temporarily unable to think clearly. The use of the GZM can reinforce the point that mastery is individual, while also encouraging the use of support of different kinds depending on which zone the learner is experiencing. Understanding of the GZM can be consolidated by learners giving their own words for the way in which they experience each zone (for example, Para and Johnston-Wilder, submitted), and adding these to a visual aid, taking care to distinguish between orange zone feelings of nervous anxiety associated with unfamiliarity and challenge, and red zone feelings of panic. The notion of stepping out of the comfort zone into the growth zone, also known as the "stretch zone" (Rohnke, 1989) or "brave zone" (Brown, 2006) is found widely in the literature. Over time as the GZM becomes internalised, learners increase their experience of being in their growth zone and also of noticing when they have moved into their threat zone.

The vocabulary of resilience and emotional zones is beginning to be introduced in primary schools in the context of learning (Kuypers, 2011). Young children instinctively keep on trying when learning a new skill, whether that is learning to write, learning to ride a bicycle or using technology. They are often good at recognising when support is needed or when a different strategy should be tried. Identifying familiar examples of learning like these as growth zone activities helps children differentiate between learning and panicking. Relating emotional zones to learning mathematics as early as possible has the potential to give learners a clear vocabulary even as they work with new teachers, which can help them feel confident in a new mathematics environment. This could encourage them to both ask for support in different forms, and to self-safeguard with a new professional who is not aware of the individual learner's attainment profile.

When introduced to the GZM, learners often say something like "my growth zone is too narrow in maths", and experience going straight from the green zone to the red zone. Faced with learners' mathematics anxiety, and side effects such as avoidance, learned helplessness or hostility, there is a tendency for teachers to "wrap the learners in cotton-wool", path-smooth (Wigley, 1992), remove challenge (Watson et al., 2003), give answers, and avoid giving any mathematics that would lead to struggle. Teachers often prefer to keep learners in their green, comfort zone, to avoid them panicking (Johnston-Wilder & Moreton, 2018). Thus, it is helpful to acknowledge the presence of the three zones, and to acknowledge that it is equally possible for learners to suffer a lack of challenge as to suffer from overwhelming challenge. It is a natural response to an anxious learner to be protective and supportive (Palethorpe & Wilson, 2011); this paper makes the case against over-protection and for developing in learners a willingness to be challenged safely. Once teachers know about and understand safeguarding in the context of learning mathematics, and once learners have learned to be aware of their emotions and communicate any perceived threat to a teacher or peers effectively, mathematics teachers can safely encourage risk-taking and development of new skills and resources to manage challenge (Johnston-Wilder & Moreton, 2018).

Resources to manage challenge and struggle in the growth zone include: a growth mindset (Dweck, 2006); self-efficacy (Bandura, 2007); confidence, persistence and perseverance (Williams, 2014); increased agency (Bandura, 2007); and a focus on approach goals (goals that can be achieved, with appropriate support), rather than on what should be avoided (Jones et al., 2009). Here, persistence is taken to mean not giving up easily but rather having another go, and perseverance to mean recruiting other strategies, resources, or support when persistence is not enough to meet a challenge. Therefore, techniques, strategies, and tactics that promote success when faced with challenge, that enable learners to deal with temporary failure and that can be employed when learners feel stuck, are highly valuable in a process of *safeguarded struggle*.

Learners who are given mathematics challenges can learn to find ways out of difficulty and around obstacles, identifying what resources they need to address the challenge presented, asking for resources, support and additional information where necessary, and reframing their need for help as an indicator of learning rather than of failure. Learners can learn to adopt a position of agency and responsibility for their mathematics learning and can become part of a supportive learning community (Lave & Wenger, 1991). Learners' perceived growth zones may expand over time, as they realise that with appropriate support there is room for learning, and thus there is an alternative to moving straight from the relaxed green zone to the panicked red zone (Johnston-Wilder & Moreton, 2018).

Work on MA has been documented since the 1950s; MA has been measured since the 1970s (Dowker et al., 2016). It seems that enough may now be known to begin to address the problem alongside further study. Tobias (1987) recognised mathematics avoidance to be a failure of nerve, not intellect; she proposed, long ago, that learners be encouraged to recognise when panic starts, then self-monitor using a split page to keep a record of thoughts and feelings experienced when not making progress with mathematics. This seems an additional, excellent strategy for moving from the red zone to the growth zone—one based on noticing and naming feelings, and consistent with the tools proposed in this paper. It enables learners to address being stuck by processing their own responses. One significant advantage of this process is that the teacher can see that time has been spent on the task and has data on which to base suggestions for the learner. However, some teachers find that the GZM is easier to implement initially (Johnston-Wilder & Moreton, 2018; Nyama, 2020).

As mentioned above (2.4), Dowker et al. (2016) acknowledge that, after 60 years, research tells us little about "how MA may be treated, or, ideally, prevented" (p. 10), which is why the three tools proposed may be highly significant. In comparison to solutions such as the specialised cognitive tutoring or transcranial electrical stimulation mentioned by Dowker's team, the three tools are inexpensive, unobtrusive, and appear effective; they can be put into the hands of learners, supporters and parents/carers, as well as teachers, giving feedback and agency to each.

Solutions to MA proposed by Finlayson (2014) for the learner include: relax; build self-confidence; practice; study and do homework; get help when needed. These solutions are consistent with implicit use of the GZM. However, some learners may value an approach in which the GZM is referenced explicitly. In that case, practice, study and homework would be comfort zone experiences, unless support is explicitly set up for more challenging homework. When learners experience getting help when needed with tasks that take them into their growth zone, they become more confident stepping out of their comfort zone (Johnston-Wilder & Moreton, 2018). Solutions proposed by Finlayson (2014) for the teacher include: encouraging risk taking; opportunities for practice; supporting diversity; slow pace; and engagement. The argument in this paper is that encouraging risk-taking requires provision of psychological safeguarding, for example by teaching the HMB and the RR in the context of learning mathematics. It is noted that without the GZM and explicit safeguarding, there is mention of the need for slow pace and hence there is still a possibility, with Finlayson's solutions, that the teacher might over-protect the learners and not give sufficient challenge to encourage learning (Johnston-Wilder & Moreton, 2018).

Schunk and Pajares (2005) recommended offering curiosity and tasks that provide moderate challenge in a warm, responsive environment. In a culture that has developed logical and empathetic skills separately (Findon & Johnston-Wilder, 2017), creation of such an environment may not be straightforward; some teachers may need to acquire skills and tools to do what Schunk and Pajares (2005) recommend. The GZM is a tool that facilitates responsiveness.

The three tools discussed in this paper involve much more than just adopting a positive approach. A transformation is sought of the prism through which the learner "becomes aware of, interprets, and emotionally relates" (Vygotsky, 1994: p. 341) to mathematical situations in order to overcome the impact of previous experiences. This transformation is helped to take place when immediate negative reactions can be managed, and this is made possible through tools that give learners an element of control (Bandura, 2007) and the ability to self-safeguard. Through such tools, teachers can help learners build self-efficacy and develop mathematical resilience, reappraising arousal as challenge, not threat, and thus improving performance (Jamieson et al., 2010). The implications and impact of use of the three tools in emergent practice so far have been far-reaching, with small-scale studies on three continents.

5. Conclusion

Teachers and learners can address MA with three accessible tools. The way in which these tools can help users to understand debilitating mathematics anxiety, and to reduce its impact and build resilience, has been illustrated. This can provide teachers in mathematical settings with agency to support their learners in an original way that is not yet part of official teacher expectations, but that makes explicit a process of safeguarding learners from psychological harm.

It would be helpful to test efficacy of these tools in large scale Random Controlled Trials. Based on experience to date, in small-scale studies illustrated above, learners have become able to address mathematics anxiety. It is argued that learners should acquire basic skills of self-safeguarding in mathematics. In claiming this, the paper builds upon and extends the pioneering work of thinkers such as Tobias (1987). The strong assertion of this paper is that teachers and learners of mathematics can benefit greatly by developing psychological safeguarding of learners in the mathematics context, thus improving progress in mathematics, and avoiding psychological harm for many (Lyons & Beilock, 2012). Teachers and learners can use tools that have been found to be: easily implemented; reliable; successful; and effective with learners of different ages. Teachers who have not previously understood MA have welcomed the GZM described in this paper. It is proposed that teachers and learners of mathematics be taught all three tools, with a view to reducing the harm and waste of time, money and potential that arises from teaching mathematics to learners with unaddressed MA (Lyons & Beilock, 2012).

The HMB helps learners to understand why they may feel "stupid" when faced with perceived threat; the RR enables learners to recover from anxiety, returning to a calmer state, perhaps still nervous but able to continue to learn; the GZM enables learners and teachers to develop an easily accessible shared language to communicate emotions. These interventions together help learners: develop agency to self-safeguard; understand the difference between challenge and threat; develop agency to recruit support when challenged; identify emotions associated with different zones of the GZM; and become aware of and address emotional exclusion from mathematics.

MA can be seen as a political issue (Tobias, 1978). So long as people view themselves as disabled with respect to mathematics, and excluded from participating in mathematics activities, and as long as they do not confront the social and pedagogical origins of their perceived disability, they do not yet have what Tobias (1978) called *mathematics mental health*: the willingness to learn mathematics when you need it. In an ever more technical society, having that willingness can make the difference between high and low self-esteem, and between failure and success in many spheres that use mathematics.

Learners need to become aware that it is possible for them to learn mathematics safely. They need to be empowered to self-safeguard against developing mathematics anxiety, and "preside as cognising agents over their own change" (Bandura, 2007: p. 95), rather than continuing to express their need for safety in terms of avoidance or non-cooperation. So, the task of addressing MA can be described simply: build learners' resources and experiences of meeting challenges successfully; strengthen their ability to manage their emotions and self-safeguard when threatened; and help them learn to re-appraise threats as challenges where appropriate. The three tools are specifically aimed at achieving these objectives and encouraging teachers to set challenging tasks that learners can rise to rather than avoid.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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