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# Shift, the Law of the Invention of Zero

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## Abstract

After posing the axiom of linear algebra, the author develops how this allows the calculation of arbitrary base powers, which provides an instantaneous calculation of powers in a particular base such as base ten; first of all by developing the any base calculation of these powers, then by calculating triangles following the example of the "arithmetical" triangle of Pascal and showing how the formula of the binomial of Newton is driving the construction. The author also develops the consequences of the axiom of linear algebra for the decimal writing of numbers and the result that this provides for the calculation of infinite sums of the inverse of integers to successive powers. Then the implications of these new forms of calculation on calculator technologies, with in particular the storage of triangles which calculate powers in any base and the use of a multiplication table in a very large canonical base are discussed.

## Keywords

Axiom, Axiom of Linear Algebra {ALA}, Any Base Calculation, ABC Theory, Number's Origin, Number Theory, Newton's Binomial Formula, Pascal's Triangle, Base Z, Canonical Bases, Calculator Revolution, Infinite Sums of Inverse of Integer to the Successive Powers, Information Completion Theory, Cipher, Factorizations That Are Numbers, Infinite Numbers That Are Infinite Sums

## 1. Introduction

For information completion theory, there is an information lack in the semantics of writing numbers in base ten and of course in other bases. The scene of the base is specified at the top of the page or by the context and we avoid mixing the numbers. But if we are interested in modeling a change of base, the numbers light up in a new light, and we dress them up with a note which specifies their base of writing. I formalized the question. I found that because it's easier to notate the base of a single symbol; we commonly count with the base of symbols, plus zero which implies this sneaky shift of the digit. The invention of zero did allow the instantaneous division and multiplication by the base of calculus that is, staying in an unic base, misexploited.

The comma is reserved for decimal writing, in accordance with the banking system even in the Anglo-Saxon world, so I use the point after the number to specify its writing base. We can also imagine to use the semicolon with prior advice in case of confusing situation. Anyway, we quickly realize that it is interesting to define large calculation bases, which have the virtue of increasing precision and of synthesizing the writing of very large numbers in an exponential way. These various virtues are all potentiated by the introduction of any base calculus and the axiom of linear algebra which allows it.

## 2. Linear Algebra Axiom and Any Base Calculus

We pose as a computational law the axiom "ir equals one, zero, base ir", ir is a circle with a point in its center, that is to say formally:

$$\odot = 10.\odot \quad \odot \in \mathbb{N}, \odot \geq 2$$

We thus free up the possibility of writing different formulas with any base, that is to say first of all for the powers of $\odot$:

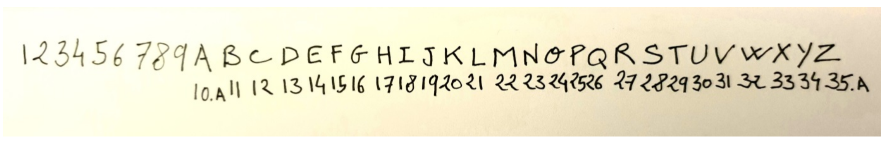
$$\odot^n = 1\overbrace{0..0}^{n}.\odot \quad \odot \in \mathbb{N}, \odot \geq 2, n \in \mathbb{N}$$

*The two successive dots meaning that the last symbol is repeated as many times as necessary.*

We also have:

$$(\odot^n - 1) = \overbrace{(\odot-1)..(\odot-1)}^{n}.\odot \quad \odot \in \mathbb{N}, \odot \geq 2, n \in \mathbb{N}$$

So, if we define the set of ordered digits followed by the set of ordered letters of the Latin alphabet, as the base thirty-five, we have *A* denoting ten.




So for $\odot = A$ we have:

$$A^n = 1\overbrace{0..0}^{n}.A \qquad n \in \mathbb{N}$$

And

$$A^n - 1 = \overbrace{9..9}^{n}.A \qquad n \in \mathbb{N}$$

By the way, we can also rewrite the axiom:

$$1 = 0,\odot.\odot \quad \odot \in \mathbb{N}, \odot \geq 2$$

"*One equals zero comma ir base ir*"

We then have the possibility to write

$$\frac{1}{3}=0,1.3 \quad \frac{2}{3}=0,2.3 \quad \frac{3}{3}=0,3.3 \quad \text{and} \quad \frac{4}{3}=1,1.3$$

So for all the elements of $\mathbb{Q}$.

We find results for $\left(\odot^{n}-1\right)^{p}$; $\left(\odot,n,p\right)\in\mathbb{N}^{3}$ and a condition on $\odot$ of the order of $\odot^{n}\geq q \quad q\in\mathbb{N}$ so that $\left(\odot^{n}-q\right)$ is defined as a positive integer or zero.

For example, with the laws $10.\odot^{n}=\odot^{n} \quad \odot\in\mathbb{N},\odot\geq 2,n\in\mathbb{N}$ and more generally

$$\alpha 0.\odot^{n}=\alpha*\odot^{n} \quad \mathit{for} \quad \alpha\in\mathbb{N}$$

We have:

$$\left(\odot^{n}-1\right)^{2}=\left(\odot^{n}-2\right)1.\odot^{n} \quad \mathit{for} \quad \odot^{n}\geq 2$$

where $1.\odot^{n}=\overbrace{0..01}^{n}.\odot$.

Indeed we have

$$\left(\odot^{n}-1\right)^{2}=\odot^{2n}-2\odot^{n}+1=\left(\odot^{n}-2\right)*\odot^{n}+1$$

$$=\left(\odot^{n}-2\right)1.\odot^{n}=\overbrace{(\odot-1)..(\odot-1)(\odot-2)}^{n}\overbrace{0..01}^{n}.\odot$$

For $\odot=A$ we find well with a normal calculator

$$\left(A^{n}-1\right)^{2}=\overbrace{9..98}^{n}\overbrace{0..01}^{n}.A$$

$n=1$ so $9^{2}=81.A$

$n=2$ so $(99.A)^{2}=9801.A$

$n=3$ so $(999.A)^{2}=998001.A$

and so on for any $n\in\mathbb{N}$.

In the same way

$$\left(\odot^{n}-1\right)^{3}=\left(\odot^{n}-3\right)2\left(\odot^{n}-1\right).\odot^{n}$$

$$=\overbrace{(\odot-1)..(\odot-1)(\odot-3)}^{n}\overbrace{0..02}^{n}\overbrace{(\odot-1)..(\odot-1)}^{n}.\odot$$

for $\odot^{n}\geq 3$.

With

$$\alpha 00.\odot^{n}=\alpha*\odot^{2n} \quad \mathit{for} \quad \alpha\in\mathbb{N}$$

moreover we will have more generally the lemma

$$\alpha\overbrace{0..0}^{p}.\odot^{n}=\alpha*\odot^{pn} \quad \left(\odot,\alpha,n,p\right)\in\mathbb{N}^{4}$$

Indeed we have

$$\left(\odot^{n}-1\right)^{3}=\left(\odot^{n}-1\right)*\left(\odot^{n}-1\right)^{2}$$

$$=\left(\odot^{n}-1\right)*\left(\odot^{2n}-2\odot^{n}+1\right)$$

$$=\odot^{3n}-3\odot^{2n}+2\odot^{n}+\odot^{n}-1$$

$$=\left(\odot^{n}-3\right)*\odot^{2n}+2\odot^{n}+\odot^{n}-1$$

So well $\left(\odot^n-3\right)2\left(\odot^n-1\right).\odot^n$ *for* $\odot^n \geq 3$

We can easily check with a current calculator.

For $\odot = A$ we find $\left(\odot^n-1\right)^3$ equals to

$$\left(A^n-1\right)^3=\left(A^n-3\right)2\left(A^n-1\right).A^n=\overbrace{9..97}^{n}\overbrace{0..02}^{n}\overbrace{9..9}^{n}.A$$

So for $n=1$ $9^3=729.A$

For $n=2$ $(99.A)^3=970299.A$

For $n=3$ $(999.A)^3=997002999.A$

And so on for any $n\in\mathbb{N}$.

Similarly

$$\begin{aligned}\left(\odot^n-1\right)^4&=\left(\odot^n-1\right)*\left(\odot^n-1\right)^3\\&=\left(\odot^n-1\right)*\left(\odot^{3n}-3\odot^{2n}+3\odot^n-1\right)\\&=\odot^{4n}-4\odot^{3n}+6\odot^{2n}-4\odot^n+1\\&=\left(\odot^n-4\right)*\odot^{3n}+5\odot^{2n}+\left(\odot^n-4\right)*\odot^n+1\\&=\left(\odot^n-4\right)5\left(\odot^n-4\right)1.\odot^n\end{aligned}$$

for $\odot^n \geq 5$

So for $\odot = A$

$$\left(A^n-1\right)^4=\left(A^n-4\right)5\left(A^n-4\right)1.A^n$$

$$\left(\overbrace{9..9}^{n}.A\right)^4=\overbrace{9..96}^{n}\overbrace{0..05}^{n}\overbrace{9..96}^{n}\overbrace{0..01}^{n}.A$$

We thus understand that we operate a factorization which is a number.

The calculator gives

For $n=1$ $9^4=6561.A$

$n=2$ $(99.A)^4=96059601.A$

$n=3$ $(999.A)^4=996005996001.A$

And so on for any $n\in\mathbb{N}$.

$$\begin{aligned}\left(\odot^n-1\right)^5&=\left(\odot^n-1\right)*\left(\odot^n-1\right)^4\\&=\left(\odot^n-1\right)*\left(\odot^{4n}-4\odot^{3n}+6\odot^{2n}-4\odot^n+1\right)\\&=\odot^{5n}-5\odot^{4n}+A\odot^{3n}-A\odot^{2n}+5\odot^n-1\\&=\left(\odot^n-5\right)\odot^{4n}+9\odot^{3n}+\left(\odot^n-A\right)\odot^{2n}+4\odot^n+\odot^n-1\\&=\left(\odot^n-5\right)9\left(\odot^n-A\right)4\left(\odot^n-1\right).\odot^n\end{aligned}$$

for $\odot^n \geq A$.

For $\odot = \mathrm{A}$

$$\left(\overbrace{9..9}^{n}.A\right)^5=\overbrace{9..95}^{n}\overbrace{0..09}^{n}\overbrace{9..90}^{n}\overbrace{0..04}^{n}\overbrace{9..9}^{n}.A$$

We also find with the calculator.

For $n=1$ $9^5 = 59049.A$
$n=2$ $(99.A)^5 = 9509900499.A$
$n=3$ $(999.A)^5 = 995009990004999.A$
And so on for any $n \in \mathbb{N}$.

$$\begin{aligned}(\odot^n - 1)^6 &= (\odot^n - 1) * (\odot^n - 1)^5 \\ &= (\odot^n - 1) * (\odot^{5n} - 5\odot^{4n} + A\odot^{3n} - A\odot^{2n} + 5\odot^n - 1) \\ &= \odot^{6n} - 6\odot^{5n} + F\odot^{4n} - K\odot^{3n} + F\odot^{2n} - 6\odot^n + 1 \\ &= (\odot^n - 6)\odot^{5n} + E\odot^{4n} + (\odot^n - K)\odot^{3n} + E\odot^{2n} + (\odot^n - 6)\odot^n + 1 \\ &= (\odot^n - 6)E(\odot^n - K)E(\odot^n - 6)1.\odot^n\end{aligned}$$

for $\odot^n \geq K$

$$\begin{aligned}(\odot^n - 1)^7 &= (\odot^n - 1) * (\odot^n - 1)^6 \\ &= (\odot^n - 1) * (\odot^{6n} - 6\odot^{5n} + F\odot^{4n} - K\odot^{3n} + F\odot^{2n} - 6\odot^n + 1) \\ &= \odot^{7n} - 7\odot^{6n} + L\odot^{5n} - Z\odot^{4n} + Z\odot^{3n} - L\odot^{2n} + 7\odot^n - 1 \\ &= (\odot^n - 7)\odot^{6n} + K\odot^{5n} + (\odot^n - Z)\odot^{4n} + Y\odot^{3n} \\ &\quad + (\odot^n - L)\odot^{2n} + 6\odot^n + \odot^n - 1 \\ &= (\odot^n - 7)K(\odot^n - Z)Y(\odot^n - L)6(\odot^n - 1).\odot^n\end{aligned}$$

for $\odot^n \geq Z$

## 3. Emmanuel's Triangle and Pascal's Triangle

We calculate the powers of $(\odot^n + 1)$

$$(\odot^n + 1) = 11.\odot^n \quad \odot^n \geq 2$$

$$(\odot^n + 1)^2 = \odot^{2n} + 2\odot^n + 1 = 121.\odot^n \quad \odot^n \geq 2$$

$$(\odot^n + 1)^3 = \odot^{3n} + 3\odot^{2n} + 3\odot^n + 1 = 1331.\odot^n \quad \odot^n \geq 3$$

$$(\odot^n + 1)^4 = \odot^{4n} + 4\odot^{3n} + 6\odot^{2n} + 4\odot^n + 1 = 14641.\odot^n \quad \odot^n \geq 6$$

$$(\odot^n + 1)^5 = \odot^{4n} + 5\odot^{4n} + A\odot^{3n} + A\odot^{2n} + 5\odot^n + 1 = 15AA51.\odot^n \quad \odot^n \geq A$$

For $\odot = A$

$n = 1$ $(A+1)^5 = (11.A)^5 = 15AA51.A$ which is indeed equal to $161051.A$ according to the linear algebra axiom.

$n = 2$ $(A^2+1)^5 = (101.A)^5 = 1050A0A0501.A$ which is indeed equal to $10510100501.A$ according to the linear algebra axiom.

$n = 3$ $(A^3+1)^5 = (1001.A)^5 = 100500A00A005001.A$ which is indeed equal to $1005010010005001.A$.

And so on for any $n \in \mathbb{N}$.

We also find

$$(\odot^n + 1)^6 = 16FKF61.\odot^n \quad \odot^n \geq K$$

And

$$\left(\odot^n+1\right)^7=17LZZL71.\odot^n \quad \odot^n\geq Z$$

These results are in accordance with Pascal's law of construction of the "arithmetical" triangle [1] which provides, outside the chevron of the 1 s, that a figure on the following line is obtained by adding the two figures overlying it to the left and to the right.

$$\begin{array}{c}
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad A \quad A \quad 5 \quad 1 \\
1 \quad 6 \quad F \quad K \quad F \quad 6 \quad 1 \\
1 \quad 7 \quad L \quad Z \quad Z \quad L \quad 7 \quad 1
\end{array}$$

$$\begin{array}{c}
\left(\odot^n-1\right) \\
\left(\odot^n-2\right) \quad 1 \\
\left(\odot^n-3\right) \quad 2 \quad \left(\odot^n-1\right) \\
\left(\odot^n-4\right) \quad 5 \quad \left(\odot^n-4\right) \quad 1 \\
\left(\odot^n-5\right) \quad 9 \quad \left(\odot^n-A\right) \quad 4 \quad \left(\odot^n-1\right) \\
\left(\odot^n-6\right) \quad E \quad \left(\odot^n-K\right) \quad E \quad \left(\odot^n-6\right) \quad 1 \\
\left(\odot^n-7\right) \quad K \quad \left(\odot^n-Z\right) \quad Y \quad \left(\odot^n-L\right) \quad 6 \quad \left(\odot^n-1\right)
\end{array}$$

Rows are numbers base $\odot^n$.

To count base ten($A$), there are eleven symbols. Is in Chinese, where the orthonormal cross is ten. So there are $\odot$ symbols plus zero to count base $\odot$.

We see that, for $\alpha\in\mathbb{N}$ small enough:

$$\left(\odot^n-\alpha\right)0.\odot^n=\overbrace{(\odot-1)..(\odot-1)(\odot-\alpha)}^{n}\overbrace{0..0}^{n}.\odot \quad \text{for} \quad \odot\geq\alpha$$

More generally $\left(\odot^n-\alpha\right)\overbrace{0..0}^{p}.\odot^n=\left(\odot^n-\alpha\right)*\odot^{pn}$ for $\odot^n\geq\alpha$

And $1\alpha0.\odot^n=1\overbrace{0..0}^{n}\alpha\overbrace{0..0}^{n}.\odot$ for $\odot^n\geq\alpha$

More generally $1\alpha\overbrace{0..0}^{p}.\odot^n=\odot^{(p+1)n}+\alpha\odot^{pn}$ for $\odot^n\geq\alpha$

## 4. Newton's Triangles

We know Newton's binomial formula [2] symmetric for all $x, y$ belonging to $\mathbb{R}$:

$$(x+y)^p=\sum_{k=0}^{p}\binom{p}{k}x^{p-k}y^k=\sum_{k=0}^{p}\binom{p}{k}x^k y^{p-k}$$

with $p$ and $k$ belonging to $\mathbb{N}$ zero inclusive and $\binom{p}{k} = \frac{p!}{k!(p-k)!}$ where $0! = 1$.

The binomial formula is giving all the coefficients to which we find $x^k y^{p-k}$ in the expanded polynomial, hence I call it the distribution law of Newton.

So we have, for $x = \odot^n$ and $y = 1$:

$$(\odot^n + 1)^p = \sum_{k=0}^{p} \frac{p!}{k!(p-k)!} \odot^{kn}$$

$p$ and $k$ belonging to $\mathbb{N}$ zero inclusive, $n \in \mathbb{N}$.

What describes well the triangle of Pascal as one notes:

$$(\odot^n + 1) = 11.\odot^n \quad \odot^n \geq 2$$

$$(\odot^n + 1)^2 = \odot^{2n} + 2\odot^n + 1 = 121.\odot^n \quad \odot^n \geq 2$$

$$(\odot^n + 1)^3 = \odot^{3n} + 3\odot^{2n} + 3\odot^n + 1 = 1331.\odot^n \quad \odot^n \geq 3$$

$$(\odot^n + 1)^4 = \odot^{4n} + 4\odot^{3n} + 6\odot^{2n} + 4\odot^n + 1 = 14641.\odot^n \quad \odot^n \geq 6$$

Likewise, if we take $x = \odot^n$ and $y = -1$, we have:

$$(\odot^n - 1)^p = \sum_{k=0}^{p} \frac{p!}{k!(p-k)!} (-1)^k \odot^{(p-k)n}$$

$p$ and $k$ belonging to $\mathbb{N}$ zero inclusive, $n \in \mathbb{N}$.

Which gives Pascal's triangle with alternating plus sign and minus sign in front of the digits starting from minus in front of the second digit of each line:

$$\begin{array}{ccccccccccccccc}
 & & & & & & 1 & & -1 & & & & & & \\
 & & & & & 1 & & -2 & & 1 & & & & & \\
 & & & & 1 & & -3 & & 3 & & -1 & & & & \\
 & & & 1 & & -4 & & 6 & & -4 & & 1 & & & \\
 & & 1 & & -5 & & A & & -A & & 5 & & -1 & & \\
 & 1 & & -6 & & F & & -K & & F & & -6 & & 1 & \\
1 & & -7 & & L & & -Z & & Z & & -L & & 7 & & -1
\end{array}$$

And what is solved by reducing each row number to a writing in its base $\odot^n$

$$\begin{array}{ccccccc}
(\odot^n - 1) & & & & & & \\
(\odot^n - 2) & 1 & & & & & \\
(\odot^n - 3) & 2 & (\odot^n - 1) & & & & \\
(\odot^n - 4) & 5 & (\odot^n - 4) & 1 & & & \\
(\odot^n - 5) & 9 & (\odot^n - A) & 4 & (\odot^n - 1) & & \\
(\odot^n - 6) & E & (\odot^n - K) & E & (\odot^n - 6) & 1 & \\
(\odot^n - 7) & K & (\odot^n - Z) & Y & (\odot^n - L) & 6 & (\odot^n - 1)
\end{array}$$

We re-apply Newton's distribution law with $x = \odot^n$ and $y = 2$:

$$\left(\odot^n + 2\right)^p = \sum_{k=0}^{p} \frac{p!}{k!(p-k)!} 2^k \odot^{(p-k)n}$$

for

$$\odot^n \geq \frac{p!}{k!(p-k)!} 2^k$$

*for all k for each p.*

$p$ and $k$ belonging to $\mathbb{N}$ zero inclusive, $n \in \mathbb{N}$.

We find the triangle of the numbers of the powers of $\left(\odot^n + 2\right)$ written in the base $\odot^n$ by taking Pascal's triangle and multiplying each of the coefficients by $2^k$, $k$ being zero for the first coefficient to the left of the line and increasing by one up to $2^p$ to the right of the $p$-th line.

$$\begin{array}{c}
1 \quad 2 \\
1 \quad 4 \quad 4 \\
1 \quad 6 \quad C \quad 8 \\
1 \quad 8 \quad O_{\text{ho}} \quad W \quad G \\
1 \quad A \quad 15.Z \quad 2A.Z \quad 2A.Z \quad W
\end{array}$$

*The middle of the fourth row is a "O" (ho!) base Z or 24.A We add a small tail at the top of the O in handwriting so as not to confuse it with zero.*

The elements appearing in lines in the triangles are numbers base $\odot^n$ that can be described in a base less than or equal to $Z$ = 35. $A$ to facilitate reading. We say they are machine-digits or ciphers. {A future machine will be able to be them as an arrangement of clear and opaque pixels to the square or the rectangle and the hover and the click of the mouse, on the machine-digit, will take place either the display of the value of the digit base ≤ $Z$, or the display of its decomposition into prime factors. One can even imagine colored machine digits with a color ordering convention the larger the digit.}

Always taking Newton's distribution law with $x = \odot^n$ and $y = -2$:

$$\left(\odot^n - 2\right)^p = \sum_{k=0}^{p} \frac{p!}{k!(p-k)!} (-2)^k \odot^{(p-k)n}$$

for

$$\odot^n \geq \frac{p!}{k!(p-k)!} 2^k$$

*for all k for each p.*

Which amounts to the same triangle with alternating the minus signs as described previously

$$\begin{array}{c}
1 \quad -2 \\
1 \quad -4 \quad 4 \\
1 \quad -6 \quad C \quad -8 \\
1 \quad -8 \quad O_{\text{ho}} \quad -W \quad G \\
1 \quad -A \quad 15.Z \quad -2A.Z \quad 2A.Z \quad -W
\end{array}$$

The expression of the powers of $(\odot^n - 2)$ base $\odot^n$ resolves as before

$$(\odot^n - 2)$$

$$(\odot^n - 4) \qquad 4$$

$$(\odot^n - 6) \qquad B \qquad (\odot^n - 8)$$

$$(\odot^n - 8) \qquad N \qquad (\odot^n - W) \qquad G$$

$$(\odot^n - A) \qquad 14.Z \qquad (\odot^n - 2A.Z) \qquad 29.Z \qquad (\odot^n - W)$$

So we check for $\odot = A$

$$(12.A)^2 = 144.A$$

For $n = 2$

$$(A^2 + 2)^2 = (102.A)^2 = 10404.A$$

$$(102.A)^3 = 1061208.A \quad (102.A)^4 = 108243216.A$$

$$(102.A)^5 = 11040808032.A$$

And

$$8^2 = 64.A \quad (98.A)^2 = 9604.A$$

$$(98.A)^3 = 941192.A \quad (98.A)^4 = 92236816.A$$

$$(98.A)^5 = 9039207968.A$$

*Likewise* with $x = \odot^n$ and $y = 3$:

$$(\odot^n + 3)^p = \sum_{k=0}^{p} \frac{p!}{k!(p-k)!} 3^k \odot^{(p-k)n}$$

for

$$\odot^n \geq \frac{p!}{k!(p-k)!} 3^k$$

*for all k for each p.*

$p$ and $k$ belonging to $\mathbb{N}$ zero inclusive, $n \in \mathbb{N}$.

We find the triangle of the numbers of the powers of $(\odot^n + 3)$ written in the base $\odot^n$ by taking Pascal's triangle and multiplying each of the coefficients by $3^k$, $k$ being zero for the first coefficient to the left of the line and increasing by one up to $3^p$ to the right of the $p$-th line.

$$1 \qquad 3$$

$$1 \qquad 6 \qquad 9$$

$$1 \qquad 9 \qquad R \qquad R$$

$$1 \qquad C \qquad 54.A \qquad 108.A \qquad 81.A$$

The conjugate triangle for the powers of $(\odot^n - 3)$ written in the base $\odot^n$

$$\left(\odot^{n}-3\right)$$

$$\left(\odot^{n}-6\right) \qquad 9$$

$$\left(\odot^{n}-9\right) \qquad Q \qquad \left(\odot^{n}-R\right)$$

$$\left(\odot^{n}-C\right) \qquad 53.A \qquad \left(\odot^{n}-108.A\right) \qquad 81.A$$

And so on. We therefore finally have, for any α belonging to $\mathbb{N}$:

$$\left(\odot^{n}+\alpha\right)^{p}=\sum_{k=0}^{p}\frac{p!}{k!(p-k)!}\alpha^{k}\odot^{(p-k)n}$$

for

$$\odot^{n}\geq\frac{p!}{k!(p-k)!}\alpha^{k}$$

*for all k for each p*

And

$$\left(\odot^{n}-\alpha\right)^{p}=\sum_{k=0}^{p}\frac{p!}{k!(p-k)!}(-1)^{k}\alpha^{k}\odot^{(p-k)n}$$

for

$$\odot^{n}\geq\frac{p!}{k!(p-k)!}\alpha^{k}$$

*for all k for each p*

We can check the results of each triangle with a base ten calculator, taking care to respect the condition for $A^{n}$. So taking for example the last triangle:

$$7^{2}=49.A \quad (97.A)^{2}=9409.A$$

$$(97.A)^{3}=912673.A$$

we have $100.A-9=91.A$ $Q=26.A$ and $100.A-R=73.A$.

In the same way we check

$$(997.A)^{4}=988053892081.A$$

If we want more base ten results, we need a base change strategy.

## 5. The Finitely Large and the Finitely Small

We see that if we try to calculate by the triangles any base a "small figure" to a power beyond the calculation condition on $\odot^{n}$, the Pascal's triangle then gives a false number whose coefficients are too large and come out of the standard calculation base (the "small figure" minus 1), the calculation of the polynomial with the coefficients multiplied by the respective powers of the base still gives a correct result once reduced to writing a correct number without numbers that exceed the base. The calculation of this kind of power by a machine will actually involve another calculation table, and even more calculation without going through the use of any base triangles.

The exponent denoted by lowercase e followed by a + or − sign followed by an arbitrarily base ten number precedes the low point meaning "base" and its machine digit.

For example:

$$(999999999.A)^9 \cong 9{,}99999991000000035999999916(e+80).A$$

with the precision of the Windows scientific calculator, the any base triangle giving it the finite and exact precision to which it corresponds. (Hence the advantage of using triangles with very large bases for very large and very small numbers).

Other results are found in any base for numbers corresponding to infinite sums.

First, let's specify some formulas along the lines of the axiom of algebra for rational numbers $1 = 0{,}\odot.\odot \quad \odot \in \mathbb{N}, \odot \geq 2$

So $\frac{1}{\odot} = 0{,}1.\odot$ and $\frac{1}{\odot^n} = \overbrace{0{,}0..0}^{n}1.\odot \quad n \in \mathbb{N}$

Also $0{,}1.\odot^n = \frac{1}{\odot^n}$ and $0{,}01.\odot^n = \frac{1}{\odot^{2n}} \quad n \in \mathbb{N}$

More generally $\overbrace{0{,}0..0}^{p}1.\odot^n = \frac{1}{\odot^{pn}} \quad p \in \mathbb{N}, n \in \mathbb{N}$

We therefore find for $n \geq 1$ and $\odot^n \geq 2$

$$\frac{1}{\odot^n - 1} = 0{,}1111111\cdots.\odot^n$$

*The three dots meaning that the last term is repeated in an infinite sum.*

Which amounts to

$$\frac{1}{\odot^n - 1} = 0{,}\overbrace{0..01}^{n}\overbrace{0..01}^{n}\overbrace{0..01}^{n}\overbrace{0..01}^{n}\cdots.\odot$$

*The three dots signifying that the pattern of the number repeats in an infinite sum.*

For example for $\odot = 2$ and $n = 1$:

$$0{,}1111111\cdots.2 = 1$$

Result that we already knew [3] in the form $\sum_{k=1}^{\infty} \frac{1}{2^k} = 1$

And for $\odot = 2$ and $n = 2$, we have:

$$\frac{1}{3} = 0{,}01010101010101\cdots.2$$

Equivalent to

$$\frac{1}{3} = 0{,}1111111\cdots.4$$

We also have for $\odot = 3$ and $n = 1$:

$$0{,}1111111\cdots.3 = \frac{1}{2}$$

Result that we already knew [3] in the form $\sum_{k=1}^{\infty}\frac{1}{3^k}=\frac{1}{2}$

So we can say we have

$$\sum_{k=1}^{\infty}\frac{1}{\odot^{nk}}=\frac{1}{\odot^n-1}\quad n\in\mathbb{N},\odot\in\mathbb{N},\odot\geq 2$$

In the same way for $n\geq 1$ and $\odot^n\geq 2$

$$\frac{1}{\odot^n+1}=0,0\left(\odot^n-1\right)0\left(\odot^n-1\right)0\left(\odot^n-1\right)0\left(\odot^n-1\right)\cdots.\odot^n$$

*The three dots signifying that the pattern of the number repeats in an infinite sum.*

Which amounts to

$$\frac{1}{\odot^n+1}=0,\overbrace{0..0}^{n}\left(\odot^n-1\right)\overbrace{0..0}^{n}\left(\odot^n-1\right)\overbrace{0..0}^{n}\left(\odot^n-1\right)\cdots.\odot$$

*The three dots signifying that the pattern of the number repeats in an infinite sum.*

For example for $\odot=2$ and $n=1$:

$$\frac{1}{3}=0,01010101010101\cdots.2$$

And for $\odot=3$ and $n=1$:

$$\frac{1}{4}=0,02020202020202\cdots.3$$

In all of these latter examples, the three dots mean that the pattern of the number repeats in an infinite sum.

We understand that a whole bunch of infinite sums can be drawn from these relationships.

Note that Bernoulli's formula written to calculate $\left(\odot^n-1\right)$ amounts to writing in Σ of $\left(\odot^n-1\right)$:

$$\odot^n-1=\left(\odot-1\right)*\sum_{k=0}^{n-1}\odot^k\quad n\in\mathbb{N},\odot\in\mathbb{N},\odot\geq 2$$

We can notice that

$$\sum_{n=1}^{\odot}n=\sum\odot=\frac{\odot}{2}\frac{\odot}{2}.\odot$$

for the canonical bases, meaning for the even bases.

## 6. Conclusions

The domain of exact calculus is the finite and the countable, and to push back the limits of this domain, in the finitely large and the finitely small, we will be interested in developing machines and calculating methods with large bases, even beyond base thirty-five (Z), and reserved exclusively for machines. In a first time, calculating base U {30.*A*} thirty should give a better result in the calculation of functions of a scientific calculator. The divisions with a factor of power of 3 at the denominator will be, this time, with a finite number of digits, within the algorithmic calculation of the functions. According to this fact, "canonical bases" will be multiplication of the first prime numbers.

The future will maybe allow the use of bigger bases, out of sight. {210.$A$ and much more}. The gain in space occupied by numbers is exponential. The numbers in the order of Gogol and its inverse will flow back into the exact calculation space in one line. The function power will gain to use the Newton's triangles, and the machine will also find a new assistance in calculations by a memory of the multiplication tables up to the largest base of the system. It will be used for operations and base changes.

Calculating any base affects the formation of the mind in the sense that the mind is trained in learning a base, it frees the mind from this body that has changed the meaning of the digit. I generalized the advantage of decimal writing to divide or multiply directly by the base.

The unreadable digits of a huge base of 2310. Ordered symbols can be evaluated by the use of a color scale according to the rainbow order. The digits at the beginning of the base will be shown red and the ones at the end will be shown violet, with all the colors in between according to the order of the digit. Nevertheless, the value of the digit can be shown base ten vertically over this digit by a simple command like hovering over the digit with the mouse or clicking on it. The color of the illegible figure gives an indication of its rank in the base. The scale of colors frames the screen as a permanently accessible reminder. That is to see where works the machine, but anyway there is a base ten display, even with many pages, one after the other.

## Drafts

$$\frac{1}{9^2} = 0,0123456790123456790123\cdots.A$$

$$\frac{1}{(11.A)^2} = 0,0082644628099173553719008264 4628\cdots.A$$

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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