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A Procedure for the Squaring of a Circle (of Any Radius)

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Abstract

This paper presents a graphical procedure for the squaring of a circle of any radius. This procedure, which is based on a novel application of the *involute profile*, when applied to a circle of arbitrary radius (using only an unmarked ruler and a compass), produced a square equal in area to the given circle, which is 50 cm^2. This result was a clear demonstration that not only is the construction valid for the squaring of a circle of any radius, but it is also capable of achieving absolute results (independent of the number pi (π), in a finite number of steps), when carried out with precision.

Keywords

Famous Problems in Mathematics, Archimedes, College Mathematics, Involute, Mean Proportional Principle, Squaring the Circle, Quadrature, Geometer's Sketch Pad, College Geometry

1. Introduction

The quadrature of a circle (*i.e.* squaring a circle or constructing a square that is equal in area to a given circle) was for more than two centuries, one of the three famous geometric problems that have intrigued mathematicians for centuries, dating back to the days of the ancient Greeks such as Hippocrates, Plato, and Archimedes (the creator of the number pi (π)). The other two famous problems are **the trisection of an angle and the doubling of a cube** [1] [2] [3].

Despite tireless efforts on the part of these mathematicians, this problem not only has remained unsolved for centuries, but proofs have been offered, notably by Ferdinand Von Lindermann (1882) [3] and others, who applied algebraic methods to geometry, to show that the problem cannot be **solved, using an unmarked straightedge and a compass alone**. The basis for this conclusion is the fact that the number pi (π), a function of the area of a circle, is not an algebraic number nor is it constructible by a straightedge and a compass.

Despite the formidability of the quadrature problem, the construction presented in this paper will demonstrate how a procedure, **using only an unmarked straightedge and a compass,** can be developed to produce, within a finite number of step a square that is equivalent to a circle of any radius (arbitrary). This procedure is a sequel to the article entitled, **"*A Method for the Squaring of a Circle*"**, published earlier [4], which represents one specific case of the quadrature problem, where the area of the given circle is known.

2. Theory

Like the referenced article [4], the procedure that is being presented in this paper is also based on the mechanics of the rolling wheel. However, in this case, where the area of the circle is not known, the mechanics involved rely on the application of *the involute profile* [5] [6] as shown in **Figure 1**.

The involute profile**,** as defined, is the path formed by a point on a string held taut as it is unwound from a cylinder. It may also be described as a point on a straight line as it rolls around a base circle Whereas this feature is normally applied in gear design and manufacture, in this procedure, its application is found to be useful for determining the ***exact*** measurement of the circumference of the wheel, or the distance that the wheel has travelled in one revolution. Determination of the circumference is key to the solution of the quadrature problem.

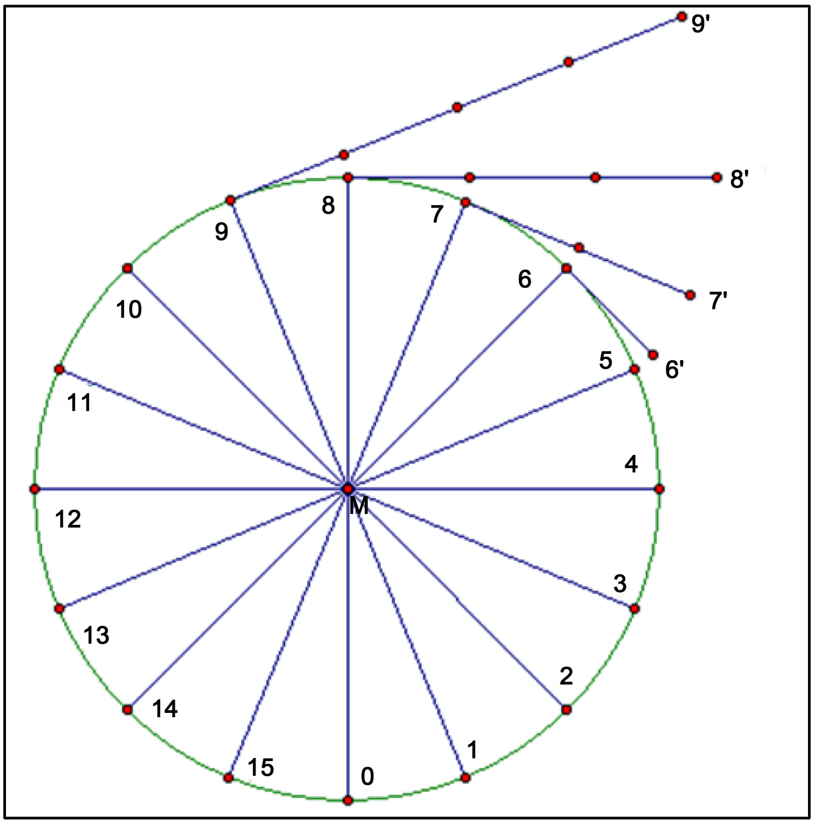

Figure 1. Involute profile.

In constructing the **involute profile,** Figure 1 shows the unwound portions of the string in four stages. At each stage, **tangents** (namely **6-6’, 7-7’, 8-8’, and 9-9’**) equal in length to **arcs** (respectively, **5⌒6, 5⌒7, 5⌒8, and 5⌒9**), are drawn to **radials** (**M-6, M-7. M-8, and M-9** respectively) of the base circle. For further details on ***the involute profile***, see references [5] [6].

3. Example Problem

Given a circle of arbitrary radius, as shown in Figure 2, construct a square that is equal in area to this circle, ***using an unmarked straightedge and compass only***.

Procedure

STEP 1: Determine the Circumference of the Circle from the Involute Profile

Figure 2 shows a rolling wheel that represents the given circle as it rolls to the right to complete one revolution, in three positions, namely: **Start**, **Midway**, and **End** positions. Hence, the distance covered defines the circumference of the given circle.

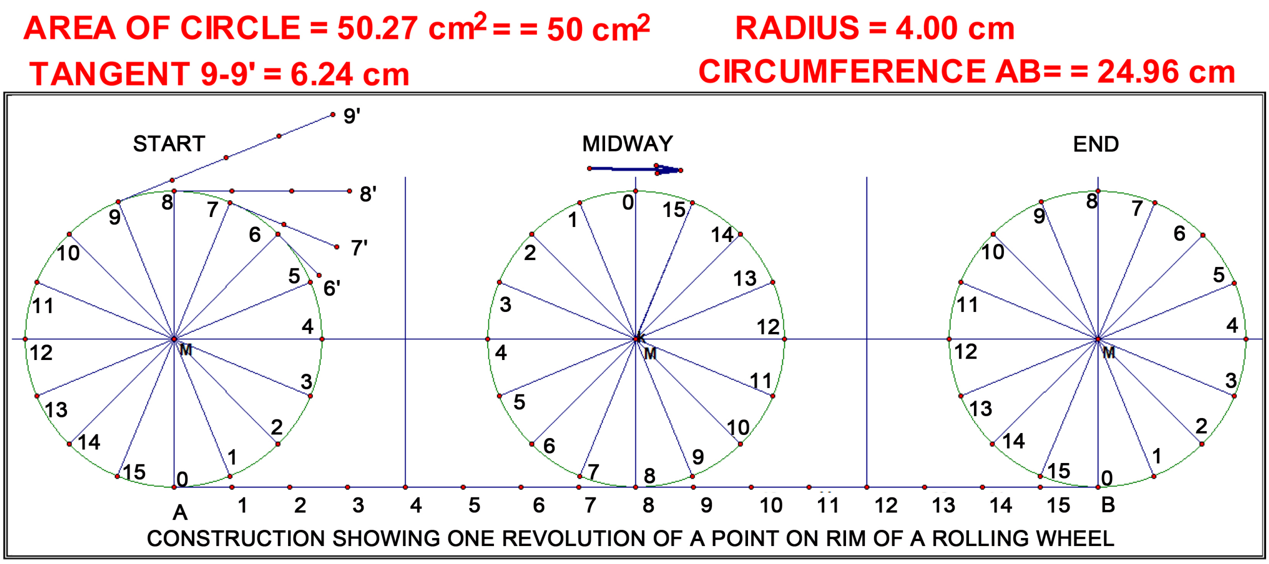

Figure 2. Revolution of a point on rim of a rolling wheel.

It should be noted that while **points 6’, 7’, 8’ and 9’** of the **involute profile** are necessary for the laying out the path of the involute curve, in this construction, they are required ***only*** for determining the distance AB covered by points of contact of the wheel on a flat surface in one revolution, which is the circumference of the wheel. Hence, there is no need for any additional tool besides the straightedge and a compass for the construction.

Knowing that the circumference of a circle is $2\pi r$ and the area of a circle is πr^2, then we can think of this area as being represented by a rectangle ABCD where one side AB is known to be $2\pi r$ and the other side BC is unknown, but can be determined, since this distance AB is related to the circumference of the circle.

STEP 2: Determine BC (See Figure 3)

BC is found by equating the area of the rectangle to the area of the circle

$$\mathrm{AB} \times \mathrm{BC} = \pi r^2 \quad \text{which yields} \tag{1}$$

$$\mathrm{BC} = \frac{\pi r^2}{2\pi r}$$

$$\mathrm{BC} = r/2 \quad \text{half radius} \tag{2}$$

STEP 3: Determine the Equivalent Rectangular Area to the Given Circle

From Equation (1), the rectangle ABCD can be written as

$$\text{Area of given circle} = \mathrm{AB} \times r/2 \tag{3}$$

STEP 4: Convert the Rectangle ABCD to a Square

With the dimensions of the rectangle defined, all that is required is to convert the rectangle to a square, using The Mean Proportional Method [7] [8] (see Figure 4).

1) Referring to rectangle ABCD, with center at C and radius CB, describe an arc cutting DC (ext'd) at K.

2) With KD as the base, construct a semicircle cutting BC (extended) at a point F.

This point will define the segment CF as one side of the required square.

3) Complete the required square C I J F.

4) Measure and compare area of the given circle to that of the square.

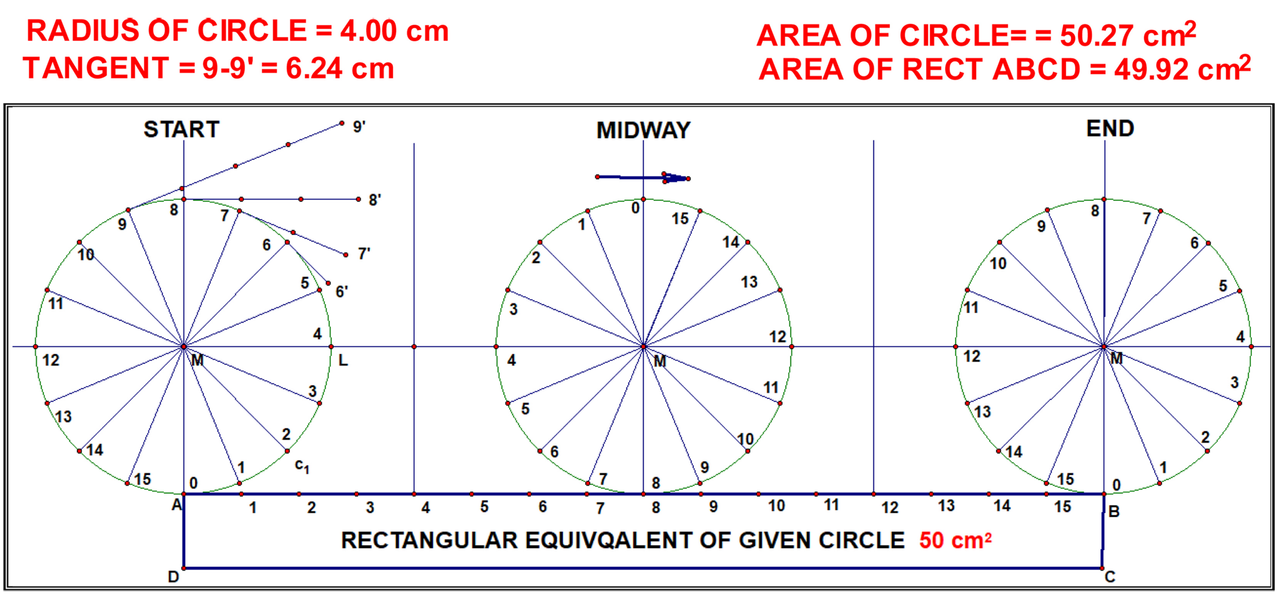

Figure 3. Rectangular equivalent of given circle.

4. Proof

The proof of this construction is in the final measurements of the rectangle ABCD and the square.

C I J F results (see **Figure 4**), which are as follows:

Area of circle = 50.27 cm^2 = **50 cm^2**.

Area of rectangle = 49.90 cm^2 = **50 cm^2**.

Area of square = 49.94 cm^2 = **50 cm^2**.

Note that in comparing these results, it should be understood that, as with all graphical techniques, the degree of accuracy depends on precision of line work, measurements and a wise choice of scales.

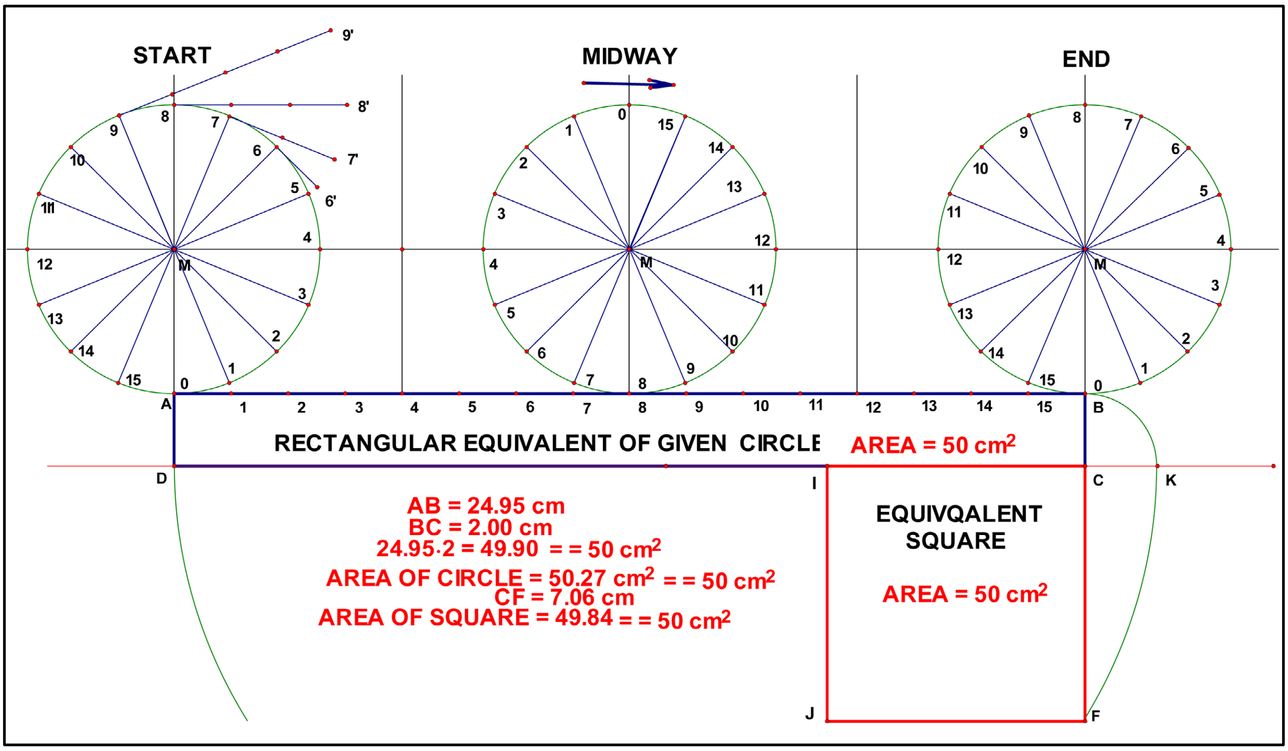

Figure 4. The quadrature.

5. Practical Benefits

Apart from the mere satisfaction of an academic interest that the quadrature of a circle problem has generated for centuries, there are several practical benefits that the construction presented here offers.

As with all other graphical solutions, it offers an alternative to the analytical approach normally used to solve the problem and, in so doing, it provides additional insight into a problem solution that otherwise may be too theoretical or abstract.

In the field of engineering mechanics, particularly that of kinematic analysis and synthesis, many graphical solutions rely on geometric constructions. In this context, the quadrature construction could be a part of a more complex graphical procedure, without which such a procedure cannot be considered totally graphical. Also, in the field of engineering mechanics, construction can provide a means to make useful comparisons between area-dependent relationships such as those involving stress, flow, and moment of inertia.

In addition, the construction provides a means whereby the number pi (π) can be generated as a precision scale for making or checking measurements that contain pi (π) as a factor.

Finally, what is significant to note is that it is difficult to foresee at this time the various ramifications that the solution to this problem could bring. But already one could see that the area of a circle needs not to be expressed solely as a function of pi (π). That is, once the equivalent square for the circle is found, one can quickly determine the other side of an area-equivalent rectangle once one side is specified.

6. Summary

A graphical procedure for constructing a square that is equivalent in area to a given circle (of any radius), ***using only an unmarked straightedge and a compass***, has been presented. The procedure, which is based on a novel application ***of the involute profile***, when applied to a circle of arbitrary radius, produced a square that is equivalent in area (i.e. 50 cm^2) to the given circle. This equivalency clearly validates the logic of the procedure, which like any mathematical formula, it guarantees that the required quadrature is achievable, once the construction is carried out with precision.

To be specific, despite the formidability of this age-old challenge, the construction presented has produced a square that is equivalent in area to a given circle (of any radius), ***using an unmarked straightedge and compass alone***. Also, by circumventing the use of the number pi (π), the construction has made it possible to achieve a complete solution within a finite number of steps (actually 4 steps), unlike other attempted solutions that this author has encountered in the literature. Therefore, one can only conclude that, finally, the age-old challenge of squaring the circle has been met [1].

Note

The use of the Geometer's Sketch Pad [9] was solely for the layout of arcs and lines involved in building a presentable construction and not for measurements except for determining final results. Hence, its use is not a violation of the unmarked straightedge and compass rule.

Also, based on this author's research, nowhere in the literature, has he encountered the application of the involute profile in solving the quadrature problem.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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