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Time Discretized Variational Iteration Method for the Stochastic Volatility Process with Jumps

Henrietta Ify Ojarikre, Ebimene James Mamadu

Department of Mathematics, Delta State University, Abraka, Nigeria
Email: emamadu@delsu.edu.ng

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Abstract

A model for both stochastic jumps and volatility for equity returns in the area of option pricing is the stochastic volatility process with jumps (SVPJ). A major advantage of this model lies in the area of mean reversion and volatility clustering between returns and volatility with uphill movements in price asserts. Thus, in this article, we propose to solve the SVPJ model numerically through a discretized variational iteration method (DVIM) to obtain sample paths for the state variable and variance process at various timesteps and replications in order to estimate the expected jump times at various iterates resulting from executing the DVIM as n increases. These jumps help in estimating the degree of randomness in the financial market. It was observed that the average computed expected jump times for the state variable and variance process is moderated by the parameters ϱ (variance process through mean reversion), Θ (long-run mean of the variance process), σ (volatility variance process) and λ (constant intensity of the Poisson process) at each iterate. For instance, when $\varrho = 0.0, \Theta = 0.0, \sigma = 0.0$ and $\lambda = 1.0$, the state variable cluttered maximally compared to the variance process with less volatility cluttering with an average computed expected jump times of 52.40607869 as n increases in the DVIM scheme. Similarly, when $\varrho = 3.99, \Theta = 0.014, \sigma = 0.27$ and $\lambda = 0.11$, the stochastic jumps for the state variable are less cluttered compared to the variance process with maximum volatility cluttering as n increases in the DVIM scheme. In terms of option pricing, the value 52.40607869 suggest a better bargain compared to the value 20.40344029 due to the fact that it yields less volatility rate. MAPLE 18 software was used for all computations in this research.

Keywords

Volatility, Equity Returns, Wiener Process, State Variable, Variance Process, Variational Iteration Method

1. Introduction

The stochastic volatility process with jumps (SVPJ) is governed by the stochastic differential equation (SDE) [1] [2] [3]

$$\left.\begin{aligned} \mathrm{d}S(t) &= \left[\left(-\lambda\mu+\kappa\right)\mathrm{d}t+\sqrt{V(t)}\,\mathrm{d}W_1(t)+\left(j-1\right)\mathrm{d}M(t)\right]S(t), \\ \mathrm{d}V(t) &= \varrho\left(\Theta-V(t)\right)\mathrm{d}t+\sigma\sqrt{V(t)}\,\mathrm{d}W_2(t)+\delta\mathrm{d}M(t), \end{aligned}\right\} \tag{1.1}$$

where parameters are detailed in **Table 1**, and $\mathrm{d}M(t)$ being the Poision process.

The SVPJ was introduced by Andrew Matytsin in 1999 [4]. Special cases of the SVPJ are the Heston's stochastic volatility and Merton's jump diffusion models. The classical Black-Scholes model is the foundation of the Merton's model which has the advantage of replicating asset returns that are discontinuous. The jumps, however, do not depend on the diffusion. Bate [5] was first to merge the above two powerful models in order to ascertain the risk neutral mean for the variances and asset value. Thus, Bate's stochastic volatility with jump is obtained by setting $\delta=0$, with Brownian motion being correlated as

$$E\left[\mathrm{d}W_1(t)\mathrm{d}W_2(t)\right]=\alpha\mathrm{d}t \tag{1.2}$$

Eraker *et al.*, [6] following the Bate's procedure developed a model introducing jumps in volatility as given below:

$$\left.\begin{aligned} \mathrm{d}S_t &= \left[\mu\mathrm{d}t+\sqrt{V_t}\,\mathrm{d}W_t^s+\left(j_t\right)\mathrm{d}M_t^s\right]S_t, \\ \mathrm{d}V_t &= \varrho\left(\Theta-V_t\right)\mathrm{d}t+\sigma\sqrt{V_t}\,\mathrm{d}W_t^v+\delta_t\mathrm{d}M_t^v \end{aligned}\right\} \tag{1.3}$$

The authors considered the strategy called the "likelihood—base estimation" to provide estimates of jump times, parameters and spot volatility [7] [8].

Over the years, many works by mathematicians on the numerical evaluation of these models have been on the increase. However, the variational iteration method in a discretized setting has not been implemented in literature in the evaluation of these models.

Thus, the objective of this paper is to seek the numerical solution of the SVPJ through a discretized variational iteration method (DVIM) to obtain sample paths for the state variable and variance process at different timesteps and replications

Table 1. Parameters description.

Parameters	Description
κ	Risk-neutral drift.
ϱ	Variance process through mean reversion.
δ	Volatility jump step size.
σ	Volatility variance process.
Θ	Long-run mean of the variance process.
$M(t)$	Poisson process with intensity λ.
λ	Constant intensity of the Poisson process $M(t)$.
μ	Wiener process of two-dimension with correlation *a*.
$W(t)$	Lognormal random variable with variance β^2 and mean α.

in order to estimate the expected jump times. In this approach, the standard variational iteration method as proposed by He [9] [10] [11] [12] is being discretized in time within the time interval $[0,T]$ with equidistant step size δ_T in order to estimate the expected jump times.

The method is quite robust in its analysis as it discretizes the state and variance parameters in a stochastic jump process. The method is easy to implement without making any weak assumptions or perturbation to produce convergent solutions that depict the degree of randomness in the area of option price in real time.

The parameters α, β and μ are related by [13] [14]

$$\mu = \mathrm{e}^{\alpha+\frac{1}{2}\beta^2} - 1 \tag{1.4}$$

2. Materials and Methods

Discretized Variational Iteration Method

Early numerical studies coupled with theoretical investigations by Wright [15], Clements and Anderson [16] and others [17] [18] [19] [20] [21] revealed that not every heuristic discrete time approximation of a stochastic differential equation converges absolutely to the solution process as the maximum step size tends to zero. As revealed, one cannot simply employ deterministic solvers of ordinary differential equations for stochastic differential equations. Thus, the need for a careful and systematic investigation through the time discretized variational iteration method for the solution process of (1.1) cannot be over-emphasized.

Now, we consider a discretization $(\gamma)_\delta$ with

$$0 = \gamma_0 < \gamma_1 < \gamma_2 < \cdots < \gamma_N = T, \quad N \geq 0, \tag{2.1}$$

over the time interval $[0,T]$ with

$$\delta = \frac{T}{N}, \tag{2.2}$$

being the equidistant step size.

An Ito process $S = \{S_t, t \geq 0\}$ with initial condition $S_0 = s_0$, has the differential form

$$\mathrm{d}S_t = a(S_t)\mathrm{d}t + b(S_t)\mathrm{d}W_t, \tag{2.3}$$

is called the Ito stochastic differential equation [22], where $a(S_t)$ and $b(S_t)$ are the drift and diffusion coefficients respectively.

Now, the mathematical formulation of the proposed method for the SVPJ process is as follows:

We construct a correction functional for the SVPJ process as

$$\begin{aligned} S_{n+1}(t) &= S_n(t) + \int_0^T \lambda_1(s)\Big(dS_n(s) - \big[(-\lambda\mu + \kappa)\tilde{\Delta}_n + \sqrt{V_n(s)}\tilde{\Delta}W_n(s) \\ &\quad + (j-1)\tilde{\Delta}M_n(s)\big]S_n(s)\Big)\mathrm{d}s, \\ V_{n+1}(t) &= V_n(t) + \int_0^T \lambda_2(s)\Big(dV_n(s) - \big[\varrho(\Theta - V_n(t))\tilde{\Delta}_n \\ &\quad + \sigma\sqrt{V_n(s)}\tilde{\Delta}W_{n+1}(s) + \delta\tilde{\Delta}M_n(s)\big]\Big)\mathrm{d}s \end{aligned} \tag{2.4}$$

subject to the initial conditions

$$S_0 = s_0,\ V_0 = v_0, \tag{2.5}$$

with

$$\Delta_n = \gamma_{n+1} - \gamma_n, \tag{2.6}$$

$$\Delta W_n = W_{\gamma_{n+1}} - W_{\gamma_n} \tag{2.7}$$

and

$$\Delta M_n = M_{\gamma_{n+1}} - M_{\gamma_n}, \tag{2.8}$$

for $n = 0(1)(N-1)$.

The λ_1 and λ_2 are the Lagrange multipliers which can be obtained through the variational theory. Here, $\tilde{\Delta}_n, \tilde{\Delta}W_n(s), \tilde{\Delta}W_{n+1}$ and $\tilde{\Delta}M_n(s)$ are restricted variables. We take the variation δ on both sides of (2.4) as follows:

$$\begin{aligned}\delta S_{n+1}(t) &= \delta S_n(t) + \delta\int_0^T \lambda_1(s)\Big(dS_n(s) - \Big[\left(-\lambda\mu + \kappa\right)\tilde{\Delta}_n \\ &\quad + \sqrt{V_n(s)}\tilde{\Delta}W_n(s) + (j-1)\tilde{\Delta}M_n(s)\Big]S_n(s)\Big)\mathrm{d}s \\ &= 0, \\ \delta V_{n+1}(t) &= \delta V_n(t) + \delta\int_0^T \lambda_2(s)\Big(dV_n(s) - \Big[\varrho\left(\Theta - V_n(t)\right)\tilde{\Delta}_n \\ &\quad + \sigma\sqrt{V_n(s)}\tilde{\Delta}W_{n+1}(s) + \delta\tilde{\Delta}M_n(s)\Big]\Big)\mathrm{d}s \\ &= 0.\end{aligned} \tag{2.9}$$

Solving (2.9), we have,

$$1 + \lambda_1\big|_{t=s} = 0 \quad \text{and} \quad 1 + \lambda_2\big|_{t=s} = 0.$$

$$\Rightarrow \lambda_1 = \lambda_2 = -1. \tag{2.10}$$

The scheme (2.4) only produces approximations at the times of discretization. For intermediate iterations, values from previous discretization point could be used. It should be noted that ΔW_n in (2.7) are normally distributed random variables $N(0, \Delta_n)$ with means and variances given as [23] [24] [25]

$$E(\Delta W_n) = 0,\ E\left((\Delta W_n)^2\right) = 0,\ n = 0(1)(N-1). \tag{2.11}$$

3. Sample Paths Approximation

Usually, solutions of stochastic differential equations are difficult to achieve, so often times simulations are used to illustrate the trajectories of the solution process at the different time discretization for $n = 0(1)(N-1)$. If the solution is known explicitly, then, mathematical software can be used to compute the absolute error using the absolute error criterion, which is given as [26]

$$\epsilon = \left(\left|S_T - S(T)\right|\right) \quad \text{and} \quad \epsilon = \left(\left|V_T - V(T)\right|\right), \tag{3.1}$$

which quantify the sample path closeness at the extreme of the time interval $[0, T]$. To this end, the discretized variational iteration method for the SVPJ model corresponding to N different simulations of the Wiener and Ito processes

are implemented.

4. Numerical Simulations

Here, we use the scheme (2.4) for $n=0(1)(N-1)$ to construct an SVPJ process and estimate the expected jump times with the given variable parameters in line with [7] [8] as follows:

$$\varrho=3.99,\Theta=0.014,\sigma=0.27,\kappa=0.0319,\delta=0.1,M=10000,$$
$$N=100,T=5.0,\alpha=0.1,\beta=0.15,a=-0.79 \text{ and } \lambda=0.11.$$

The sample paths as shown in **Figure 1(a)** and **Figure 1(b)** respectively, generates 10 replications with 30 timesteps for the state variable and variance process. The average computed expected jump times for the state variable and variance process is 20.40344029.

Similarly, using different parameters $\varrho=0.0,\Theta=0.0,\sigma=0.0$ and $\lambda=1.0$, with every other parameters remains the same as in the previous case. The sample paths for the state variable and variance process are shown in **Figure 2(a)** and **Figure 2(b)**, respectively. It generates 10 replications with 30 timesteps. The average computed expected jump times is 52.40607869.

5. Discussion of Results

The DVIM has been applied successfully for the SVPJ process. Results were generated and presented graphically via MAPLE 18 software. The sample paths were generated with 10 replications and 30 timesteps to quantify the closeness at the extreme of the time interval $[0,T]$. It was observed that the average computed expected jump times, which is 20.40344029, for the state variable and variance process is moderated by the parameters ϱ,Θ,σ and λ. For instance, at each

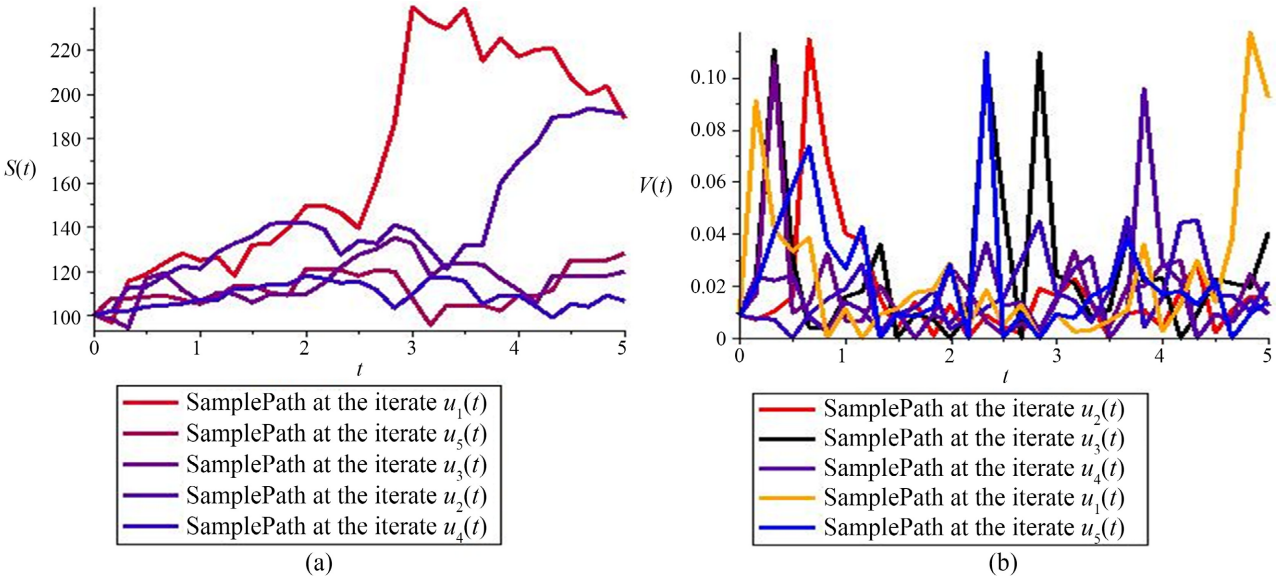

Figure 1. (a) Sample path for the state variable with 10 replications and 30 timesteps at different iterates; (b) Sample path for the variance process with 10 replications and 30 timestep at different iterates.

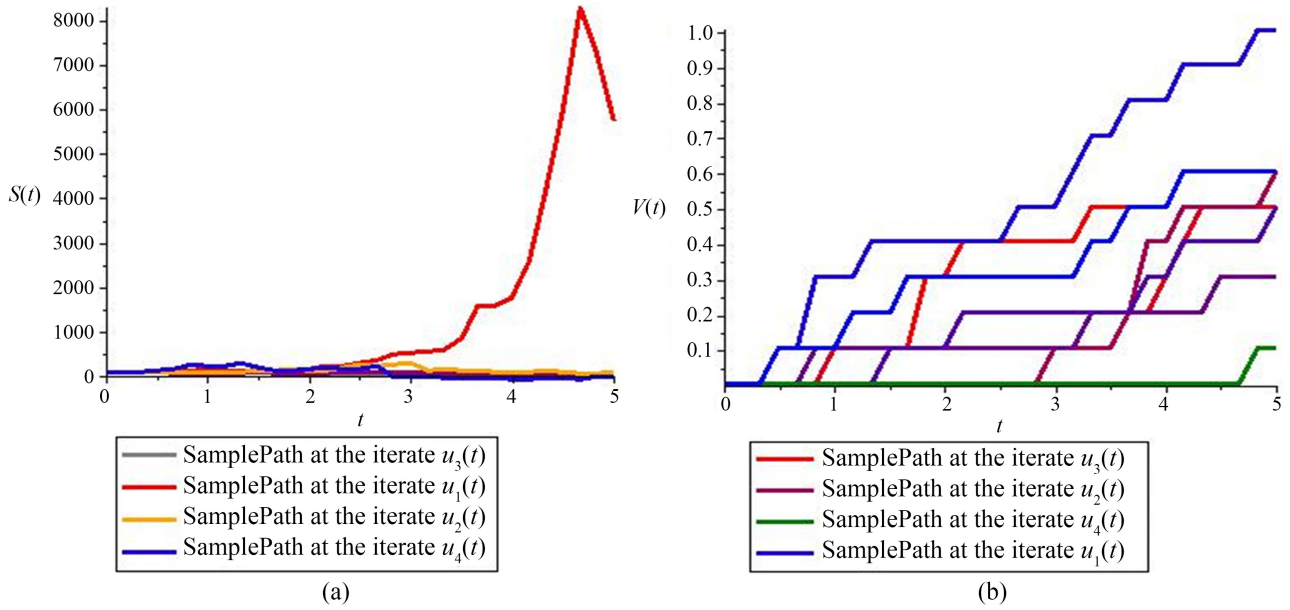

Figure 2. (a) Sample path for the state variable with 10 replications and 30 timesteps at different iterates. (b) Sample path for the variance process with 10 replications and 30 timestep at different iterates.

iterate for $n \to \infty$, with parameters $\varrho = 3.99, \Theta = 0.014, \sigma = 0.27$ and $\lambda = 0.11$, the stochastic jumps for the state variable are less cluttered compared to the variance process with maximum volatility cluttering as shown in **Figure 1(a)** and **Figure 1(b)**. The value 20.40344029, in terms of option pricing implies that volatility cluttering and mean reversion between equity returns experiences less uphill movement in price assert. In contrast, at each iterate for $\to \infty$, with parameters when $\varrho = 0.0, \Theta = 0.0, \sigma = 0.0$ and $\lambda = 1.0$, the state variable cluttered maximally compared to the variance process with less volatility cluttering as shown in **Figure 2(a)** and **Figure 2(b)**, respectively. The average computed expected jump time is 52.40607869. In terms of option pricing, the value 52.40607869 suggests a better bargain compared to the value 20.40344029 due to the fact that it yields less volatility rate. We tend to point out that the negativity of the correlation parameter does not imply the curves in the figures must appear negative (see [1] [3] for details).

6. Conclusion

This paper focused on the numerical application of the SVPJ model. We have successfully discretized the variational iteration method for the purpose of generating sample paths with different replications and timesteps for computing the average expected jumps times experience in the volatility cluttering. Results obtained are satisfactory and are in line with those available in literature [13] [14] and very relevant in the area of option pricing and Forex trading.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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