

Photoconductivity under Pulsed Excitation

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Abstract

Ever since its mid nineteenth century inauguration, the logistic function and its numerous applications have received a great deal of attention from engineers, and natural and social scientists. In particular, its discrete relative, the logistic map, has proven to be a principal and indispensable tool of scientists in their effort to describe the dynamics of a variety of physical and biological systems. Our purpose in this paper is to describe one such application, namely, photoconductivity under pulsed excitation and show that the solution of the energy-independent kinetic rate equation for electron density

$$\frac{dn}{dt} = Q - BNn - Bpn \quad \text{can be expressed as a logistic map.}$$

Keywords

Photoconductivity, Logistic Map, Band Structure, Fermi Level, Recombination Coefficient, Trap Density

1. Introduction and Basic Terminology

The *logistic function* is a function of the form

$$f(t) = \frac{K}{1 + e^{-c(t-t_0)}} \quad (1)$$

Obviously, $f(t)$ approaches K as $t \rightarrow \infty$ and to 0 as $t \rightarrow -\infty$, and is, therefore, bounded. Here, K denotes the maximum value of the function and c denotes the logistic growth rate, that is, the steepness of the curve. The midpoint of the curve is at $\left(t_0, \frac{K}{2}\right)$.

The logistic function was first investigated (and named) by the Belgian mathematician Pierre François Verhulst (1804-1849), who used it to model population growth in his papers [1] [2] and [3]. In fact, in his 1845 paper, Verhulst writes.

Nous donnerons le nom de logistique à la courbe [We will give the name logistic to the curve].

Since then, it has found applications in neural networks, ecology, chemistry, physics, economics, probability, linguistics, sociology, and statistics.

It is customary to talk about the standard logistic function where $K = 1$, $c = 1$, and $t_0 = 0$. Thus, the standard logistic function is

$$f(t) = \frac{1}{1 + e^{-t}} \quad (2)$$

The graph of the standard logistic function is given in **Figure 1**.

The derivative of the standard logistic function is

$$f'(t) = f(t)[1 - f(t)] = f(t)f(-t) \quad (3)$$

Of course, this implies $f'(t) > 0$ for all t , and that, consequently $f(t)$ is an increasing function.

The maps of the logistic function have received great deal of attention in the literature: [4]-[9], to name a few. Their popularity can be traced back to two reasons. First, the logistic function is a paradigm exhibiting much of the behavior displayed by a very large class of noninvertible, nonlinear, discrete maps. Secondly, since the dynamics of many potentially chaotic systems can be cast into the logistic form, this particular map enables us to give a quantitative as well as a qualitative description of the transition to chaos—a property we will hinge on in this paper.

The *logistic map* is a second-degree recurrence relation, often cited as a typical example of how complex, chaotic behavior can arise from simple non-linear dynamical equations. The map was popularized in [10]. If we put $x = f(t)$ in the derivative of the general logistic function, we have

$$\frac{dx}{dt} = rx(1-x) \quad (4)$$

for some parameter r . Now, we can discretize this equation and write it as a difference equation to obtain the logistic map:

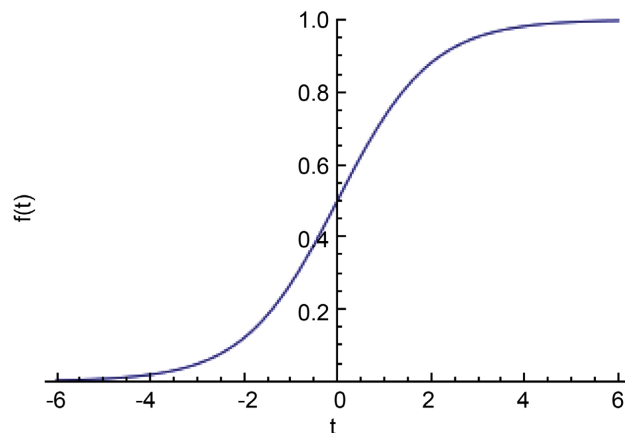


Figure 1. The graph of the Standard Logistic Function.

$$x_{j+1} = rx_j(1 - x_j) \quad (5)$$

Of course, we must have $0 \leq x_j \leq 1$ for all $j = 0, 1, \dots$.

The first couple of iterations are

$$x_1 = rx_0(1 - x_0) \quad (6)$$

$$x_2 = r^2(1 - x_0)x_0(1 - rx_0 + rx_0^2) \quad (7)$$

In general, this recurrence equation cannot be solved in closed form.

Of course, the behavior of the sequence depends on the value of the parameter r . In particular, for r between 2 and 3, the values eventually approach $\frac{r-1}{r}$. The rate of convergence is linear, except for $r = 3$, when it is much slower. For more information, see [11].

Let us now introduce another concept. A system is called *dissipative* if it loses energy to waste-heat. In the past decades, thanks to mostly the works of Prigogine (see, for example, [12]), the study of dissipative systems and dissipative structures has gained added momentum.

We will assume that the universe being a union of bounded subspaces, within the event horizon, all systems are dissipative, for there would always be a finite flow of energy across the boundaries. Consequently, in finite time, a portion of this energy would be dissipated in an irreversible process, implying there are no truly isolated systems in the observed universe.

By a *free electron*, we mean an electron which is not attached to the nucleus of an atom and is free to move when external energy is applied.

In solid-state physics, the *electronic band structure* (or simply *band structure*) of a solid describes the range of energy levels that electrons may have within it. The valence band and the *conduction band* are the bands closest to the *Fermi level*, the thermodynamic work required to add one electron to the body.

For more information on these physical concepts, the reader is referred to the excellent works by Holgate [13] and Kittel [14].

2. Photoconductivity under Pulsed Excitation

Photoconductivity is the phenomenon evinced by the increase in the electrical conductivity of a material by the absorption of light or other electromagnetic radiation. Many materials exhibit a marked change in electrical conductivity when irradiated; this change, which is insignificant in highly conductive substances, becomes quite noteworthy in semiconductors.

In a semiconductor, there are certain number of free electrons occupying energy levels in the conduction band and a certain number of positively charged holes left in the valence band by the free electrons that enter the conduction band due to natural vibration of atoms. If this substance is irradiated, the creation of free charge carriers (*i.e.*, the electrons and the holes) increases, which produces an increased conductivity. This process continues until the carriers generated by the photon absorption can no longer move through the

material.

Assuming the electrons are the majority carriers, the energy-independent kinetic rate equation for electron density n is represented by the equation

$$\frac{dn}{dt} = Q - BNn - Bpn \quad (8)$$

Here, Q is the volume rate of carrier generation due to irradiation and p is the volume density of the holes. The constant B is the *recombination coefficient* and N , the *trap density*. Consequently, the term- BNn denotes the *electron impurity capture rate* and the term- Bpn represents the *bimolecular recombination rate* per unit volume.

Since in uncharged matter $p = n$, this equation reduces to the form

$$\frac{dn}{dt} = -Bn^2 - BNn + Q \quad (9)$$

Let us now complete the square on the right-hand side and put

$$g = n + \frac{N}{2}$$

and

$$K^2 = \left(\frac{Q}{B} + \frac{N^2}{4} \right)$$

The above equation becomes

$$\frac{dg}{g^2 - K^2} = -Bdt \quad (10)$$

By direct quadrature, we get

$$\frac{g - K}{g + K} = e^{-2BKt} \quad (11)$$

that is,

$$g = \frac{K + Ke^{-2BKt}}{1 - e^{-2BKt}} \quad (12)$$

Hence,

$$n(t) = \frac{\left(K - \frac{N}{2} \right) + \left(K + \frac{N}{2} \right) e^{-2BKt}}{1 - e^{-2BKt}} \quad (13)$$

Let n_s be the steady state solution, in other words, the solution that exhibits negligible change over an arbitrary long period (that is, $\frac{dn_s}{dt} = 0$). This implies

$$K - \frac{N}{2} = n_s$$

that is,

$$2KB = (2n_s + N)B$$

Putting

$$\sigma = \frac{2n_s + N}{N}$$

and

$$\tau = \frac{1}{BN}$$

we can write the solution in terms of the steady-state solution as

$$n(t) = \frac{n_s \left[1 + \left(\frac{\sigma N}{2n_s} \right) e^{-2Bkt} \right]}{1 - e^{-2Bkt}} \quad (14)$$

The pulsed beam can be modeled in a given interval $[0, p]$ as a square wave Q that has a fixed amplitude, say Q_0 , for part of the interval, and 0 throughout the rest. So, for some $0 \leq \beta \leq 1$, we define Q as

$$Q(t) = \begin{cases} Q_0 & \text{for } 0 \leq t < \beta p \\ 0 & \text{for } \beta p \leq t \leq p \end{cases} \quad (15)$$

We extend $Q(t)$ over the real line as

$$Q(t + kp) = Q(t)$$

for any integer k .

The net increment in $n(t)$ per pulse is equal to the number of electrons liberated minus the number of electrons recombined, that is

$$n(t + p) - n(t) = Q_0 \beta p - Bn(t)[n(t) + N]p \quad (16)$$

or equivalently,

$$n(t + p) = Q_0 \beta p + \left(1 - \frac{p}{\tau}\right)n(t) - \left(\frac{p}{\tau N}\right)n^2(t) \quad (17)$$

Putting

$$t = jp \\ n(jp) = n_j$$

and

$$p\tau = b$$

our equation can be written in a recursive form as

$$n_{j+1} = Q_0 \beta p + (1 - b)n_j - \left(\frac{b}{N}\right)n_j^2 \quad (18)$$

We claim (18) can be written as the logistic equation. To this end, let us apply the homogenous affine transformation

$$n_j = \gamma x_j - \omega$$

Now, (18) can be written as

$$\gamma x_{j+1} - \omega = Q_0 \beta p + (1 - b)(\gamma x_j - \omega) - \left(\frac{b}{N}\right)(\gamma x_j - \omega)^2 \quad (18')$$

Rearranging terms we get

$$x_{j+1} = \frac{Q_0\beta p - (1-b)\omega - \left(\frac{b}{N}\right)\omega^2 + \omega}{\gamma} + \left[1 - b + \frac{2b\omega}{N}\right]x_j - \left(\frac{b}{N}\right)\gamma x_j^2 \quad (19)$$

Recalling,

$$\frac{b}{N} = pB$$

we obtain

$$x_{j+1} = \left(\frac{p}{\gamma}\right)\left[Q_0\beta - B\omega(\omega - N)\right] + [1 + pB(2\omega - N)]x_j - pB\gamma x_j^2 \quad (20)$$

To put (20) in logistic form, we must choose the undetermined coefficients γ , ω , and r such that

$$\begin{aligned} Q_0\beta - B\omega(\omega - N) &= 0 \\ 1 + pB(2\omega - N) &= r \\ pB\gamma &= r \end{aligned}$$

The solutions are

$$\begin{aligned} \gamma &= N\left(\frac{\tau}{p} + \sqrt{1 + \frac{4Q_0\tau\beta}{N}}\right) \\ \omega &= \frac{N}{2}\left(1 + \sqrt{1 + \frac{4Q_0\tau\beta}{N}}\right) \\ r &= 1 + \frac{p}{\tau}\sqrt{1 + \frac{4Q_0\tau\beta}{N}} \end{aligned} \quad (21)$$

Consequently, the affine transformation

$$n_j = \gamma x_j - \omega \quad (22)$$

that is, the affine transformation

$$n_j = N\left(\frac{\tau}{p} + \sqrt{1 + \frac{4Q_0\tau\beta}{N}}\right)x_j - \frac{N}{2}\left(1 + \sqrt{1 + \frac{4Q_0\tau\beta}{N}}\right) \quad (23)$$

puts Equation (20) in logistic form.

By studying the onset of bifurcation as a function of b and Q_0 , it is possible to determine the recombination coefficient B and the trap density N . From

$$r = 1 + \frac{p}{\tau}\sqrt{1 + \frac{4Q_0\tau\beta}{N}} \quad (24)$$

we get

$$\left(\frac{r-1}{b}\right)^2 = 1 + \frac{4Q_0\tau\beta}{N} \quad (25)$$

Putting $r = 3$ and $y = \frac{4}{b^2}$, we get

$$y = 1 + mQ_0 \quad (26)$$

i.e., a linear function of Q_0 with slope $m = \frac{4\tau\beta}{N}$.

The quantities b , τ , and β are measurable. As given in [15], the quantity Q_0 can be calculated as the product of quantum efficiency and incident radiation intensity. Hence, we now have

$$N = \frac{4\tau\beta}{m}$$

and

$$B = \frac{m}{4\tau^2\beta}$$

implying N and B can be determined experimentally.

For some applications of photoconductivity we refer the reader to the excellent papers [16] [17] [18].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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