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How Prime Numbers Are Interconnected and Built with Two Equations: Addition and Subtraction Rules the Function (6μ)

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Abstract

Are all prime numbers linked by four simple functions? Can we predict when a prime will appear in a sequence of primes? If we classify primes into two groups, Group 1 for all primes that appear before ζ (such that $p_1 = \zeta - 1$, for instance 5, $\zeta = 6$), an even number divisible by 3 and 2, and Group 2 for all primes that are after ζ (such that $p_2 = \zeta + 1$, for instance 7), then we find a simple function: for each prime in each group, $p_a - p_b = 6n$, where n is any natural number. If we start a sequence of primes with 5 for Group 1 and 7 for Group 2, we can attribute a μ value for each prime. The μ value can be attributed to every prime greater than 7. Thus $p_1 = 6\mu + 5$ for Group 1, and $p_2 = 6\mu + 7$. Using this formula, all the primes appear for $\mu = 1,2,3$, where μ is any natural number.

Keywords

Number Theory, Prime Number Groups, Twin Primes, Prime Structure and Sequence, Prime Subtraction and Addition

1. Introduction

When we print out a list of primes, we have the impression that the primes appear at random, with no regular rhythm. However, on closer inspection, a fascinating structure emerges when we classify the primes into two distinct groups according to their position relative to an even number divisible by 6. Group 1 comprises the primes appearing before an even number divisible by 6, while, on the other hand, Group 2 is made up of the primes that follow a number divisible by 6. We can first subtract one prime from an other in the same group and the result is a multiple of 6. For example, in Group 1, which begins with 5, we find 23 − 11 = 12 = 2 × 6. For Group 2, which begins with 7, 19 − 13 = 6. We shall see how we can deal with primes, one from Group 1, the other from group 2.

Subtraction and addition are different in this context. We do not have the same results when we add from the same group and when we add one prime and an other from different groups. A prime from Group 1 must be added to a prime from Group 2 to have a result divisible by 6. For instance, 5 + 13 = 18 = 3 × 6, but 5 + 11 = 16. Note that 5 belongs to Group 1 while 13 belongs to Group 2. Both 5 and 11 belong to the same group.

However, we can notice that the prime system as presented by the separation into two groups is coherent and somewhat predictable. Are the sets of primes in each group infinite? All primes are either above a multiple of 6 or beneath it. As multiples of 6 are infinite, we may suppose that if there exists an infinite number of primes, they will be either in Group 1 or Group 2.

2. Method

If we examine a sequence of three consecutive integers, with one being a prime, we have either $\eta\ p\ \xi$ or $\xi\ p\ \eta$, where $\xi \equiv 0 \pmod 6$, and η is not divisible by 6. When examining the difference between twin prime pairs, we discovered that this difference was always a multiple of 6, denoted as $6n$, where $n \in \mathbb{N}$. As all primes are encompassed by two even numbers, and, of course, one of them must be divisible by 6, as one number in a succession of 3 natural numbers is, our distinction between two groups holds for all primes.

From this intuition, we set out to verify our hypothesis empirically. First through subtraction, we found that in Group 1, $p_1 - p_2 = 6n$. So, on the number line of prime numbers with the form $\eta\ p\ \xi$ (such as 4, 5, 6), prime numbers must be increasing by multiples of 6. The same goes for Group 2, the number line of prime numbers with the form $\xi\ p\ \eta$ (such as 6, 7, 8). We then decided to introduce μ to differentiate between subtraction and the arithmetic progression beginning either with 5 or with 7. Then we realised that when we subtract a prime from one group with a prime from the other, the difference excludes 6 and corresponds to $6n \pm 2$, i.e., 4, 8, 10, etc.

Our study began with Euclid [1], and his proof that prime numbers are infinite [2]. I endeavoured to pair off primes using what I called the Euclidean prime factorial (EPF) beginning with 2, 6, 30, 210, 2310, ... and found that there was a prime number attached to the first one by subtracting 6, 30, 210, etc. from it. I then realized that all the numbers in the EPF were divisible by 6, so the result of the subtraction was too.

The primes seem completely random as shown in [3], with, when one prime is subtracted from the following prime, we find 3 − 2 = 1, 5 − 3 = 2, …, 23 − 19 = 6, and so on. Nothing seems rational nor coherent. However, I believe there may be a coherent system, and this is what I am trying to put forward in this paper. By creating two groups of primes, those below a multiple of six, for example, 5, 11, 17, …, and a group for those above, 7, 13, 19, 31, etc., I found the solution by examining the difference between pairs of twin primes.

3. Results

We use the following equations to build two prime number lines, one for each group:

$$\text{Group 1}: p = 6\mu + 5 \quad (1)$$

$$\text{Group 2}: p = 6\mu + 7 \quad (2)$$

The sequence begins either with 5 or 7 depending on the group, and the results alternate from one line to another. The two lines differ by 2, as we begin with a pair of twin primes. All primes should be either in Group 1 or Group 2, and appear only at the same time when the primes are twins. The μ value is the same for each member of the twin pair. If a prime does exist when we input a value for μ, it will appear, otherwise there is no prime for such and such a value, nor is there a prime on that position of the number lines for each group.

When we subtract primes of the same group, we find a multiple of 6, whereas if we subtract a prime from each group, the value varies and has no number divisible by 3. Subtraction of prime numbers between groups gives:

$$\text{Primes in the same group}: p_1 - p_2 = 6n \quad (3)$$

$$\text{Primes in different groups}: p_1 - p_2 = 6n \pm 2 \quad (4)$$

Addition is the inverse of subtraction when it comes to the results in this context.

$$\text{Group 1} + \text{Group 2}: p_1 + p_2 = 6n \quad (5)$$

$$\text{Group 2} + \text{Group 1}: p_2 + p_1 = 6n \quad (6)$$

We can verify those equations by selecting any prime, finding its group by adding or subtracting 1, then either we subtract 5 (Group 1) or 7 (Group 2) depending on the group, and dividing by 6, and we find the μ value. With the group number, μ is the prime's signature. When we add or subtract two primes we use n, the natural number, whereas μ is used for Equations 1 and 2. We use zeta, ζ, for numbers divisible by 6 as ζ is the sixth letter of the Greek alphabet, and not the ζ of the ζ function of Euler [4], which implies the "infinitude of primes".

3.1. Addition and Subtraction

When we subtract one prime from another, either they belong to the same group and the difference is a multiple of six, or we subtract primes from different groups and the difference excludes 6. This is why we have the impression of utter chaos when we look at a list of primes in increasing order. We shall examine the two forms of subtraction.

3.1.1. Primes in the Same Group

For primes in the same group:

- Group 1: $p_1 - p_2 = \xi_1 - 1 - (\xi_2 - 1) = \xi_1 - \xi_2 = 6n$.

- Group 2: $p_1 - p_2 = \xi_1 + 1 - (\xi_2 + 1) = \xi_2 - \xi_1 = 6n$.

  Subtraction: $p_1 - p_2 = 6n$ for primes in the same group (Equation 3).

3.1.2. Subtraction for Primes in Different Groups

Subtraction for primes in different groups: $p_1 + p_2 = 6n \pm 2$. (4) The two groups progress from 5, G1, and 7 G2, so they are out of phase by 2. We notice first that with the twins, separated by 2, if we subtract say 11 from 19, we find 8, but if we subtract 13 from 19, we find 6. This is true for all subtractions. However, we find that if ξ in the first prime taken is above the prime, and beneath the prime in the second prime selected, then: For 23 – 13 = 10, that is 12 – 2. So, we find (2 × 6) – 2. If ξ in the first prime taken is below the prime, and above the prime in the second prime selected, then we find 43 – 17 = 26 = (4 × 6) + 2.

In general, as we shall see below:

- $p_1 - p_2 = 6n \pm 2$ (Equation 4).
- Group 1 - Group 2: $p_1 - p_2 = \xi_1 - 1 - (\xi_2 + 1) = \xi_1 - \xi_2 - 2 = 6\mu - 2$.
- Group 2 - Group 1: $p_2 - p_1 = \xi_2 + 1 - (\xi_1 + 1) = \xi_2 - \xi_1 + 2 = 6\mu + 2$.

Consequently $p_a - p_b = 6n \pm 2$, in order to take into account the above-mentioned difference. If we build a model going from one group to another, the system is unpredictable because of the above equation. We could present this model graphically with two parallel lines, one above the x axis, $y = 1$, and one line below with $y = -1$. We can then plot the position of the primes, some on the above line, some below, often in clusters. We can use an arrow when a cluster of primes stops and a new cluster in the other group begins, as shown in **Figure 1**.

3.1.3. Addition of Two Primes

For addition, if the primes are in the same group, then $p_1 + p_2 = 6n \pm 2$ as we can see that 17 + 23 = 40 = 42 − 2 = (7 × 6) − 2 and n = 7. When the primes come from different groups, then $p_1 + p_2 = 6n$, as for instance, 19 + 11 = 30. So, addition is the inverse of subtraction when it comes to the results.

- Group 1 + Group 2: $p_1 + p_2 = \xi_1 - 1 + (\xi_2 + 1) = \xi_1 + \xi_2 = 6n$.
- Group 2 + Group 1: $p_2 + p_1 = \xi_2 + 1 + (\xi_1 - 1) = \xi_2 + \xi_1 = 6n$.

3.2. Twin Primes

There are many papers on twin primes and their infinity maynard 2019 twin. [5] [6] proved that "there are infinitely many pairs of distinct primes (p, q) with $|p - q| < 70$ million".[1] And [7] and [8] on the generating function of Mersenne primes and Fermat primes. We shall show how primes are related in a sequence from 5 or 7 to ∞. We studied twin primes first and we found that if we subtract the larger primes in each set of twins, then the smaller primes, we find 6n, pair 1 (19, 17) and pair 2 (13, 11) we find 19 − 13 = 6, 17 − 11 = 6, and with the even numbers between, 18 − 12 = 6. The sets of twin primes are separated by 6n. We show in **Table 1** how the primes progress by multiples of 6 in each group.

[1] $\liminf_{n\to\infty}\left(p^{n+1} - p^{n}\right) < 7\times 10^{7}$, where p^{n} is the n-th prime.

Table 1. How primes develop in their respective group.

G2: 7	13	19	*	31	37	43	*	*	61	67	73	79	*	*
97	103	109	*	*	127	*	139	*	151					
G1: 5	11	17	23	29	*	41	47	53	59	*	71	*	83	89
101	*	107	113	*	*	131	137	*	149					

The asterisks indicate places where there are no primes in Table 1. Primes can either be in one group or the other, and when it comes to twins, both. The twin primes are underlined. However, we may have a sort of wave when we alternate between each group. Progression alternating from Group1 to Group2: We see the progression by subtracting, thus, 13 − 11 = 2, 17 − 13 = 4, 19 − 17 = 2, 23 − 19 = 4, which is accounted for in equation 4. We take the last number in the sequence when we have several primes in their group. We found that the difference in this case was a function of $2n$, usually 2, 4, and 8 or $6n \pm 2$ as we also found 10 for the first primes chosen. So, we seem to have another progression here although somewhat unpredictable because extremely variable. In Group 1 and Group 2, the numbers progress by multiples of 6; in Group1 to Group 2, the sequences by multiples of 2 excluding 6. After those observations, we proceed to formulate rules describing the above groups.

3.3. Equations for Creating Primes or Finding the μ Value Attributed to a Prime

We can now formulate two equations for creating primes. For Group 1 primes, $p_1 = 6\mu + 5$, and for Group 2 primes $p_2 = 6\mu + 7$. We start with Group 1, and begin from 5, then Group 2, beginning with 7, and we begin with $\mu = 1$, then 2… ∞ (a recursive function). So, it is possible to calculate the μ value of a prime for very high numbers. Table 2 shows how the primes progress in each group with the μ value increasing by 1 from $\mu = 1$ to $\mu = 50$.

There are some values of μ that are not primes because 6μ increases by multiples of 6 and consequently some of the primes are separated by 12, 18, or more. One of the reasons for this is that with Group 1, μ cannot be a multiple of 5, and for Group 2, a multiple of 7. This is the reason why:

- If $p = 6\mu + 5$ and $\mu = 5$, then (6 × 5) + 5 = 5 (6 + 1) = 5 × 7, thus a composite number.
- If $p = 6\mu + 7$, and $\mu = 7$, then (6 × 7) + 7 = 7 (6 + 1) = 7 × 7.

Therefore, 5 and 7 (and their multiples) for each group respectively exclude the formation of primes. Are there other cases where μ is not suitable? That will be for further investigation. If we look at Table 1, we see that when $\mu = 5$ or $\mu = 7$ there are sometimes no primes in either Group 1 or Group 2 for the reason explained above.

Primes can be on line 1 or line 2, and both lines when we come across twins with the same value for μ. And sometimes there is a hole where there are no primes to be found. But all primes should be found with the above equations.

3.4. Labelling Primes

We can now label a prime with the following information: its group, and its μ value. We can write 43 (G2, μ:6) and 101 (G1, μ:16) as 43 belongs to Group 2, and $\mu = 6$ and 101 belongs to Group 1, with $\mu = 16$. This can be useful if we are trying to identify a pattern of prime emergence, for instance, we know the value of μ that does not produce primes, and, is there a relationship between μ values with specific factors and certain primes. This may help us to group primes in a specific fashion.

Let us now take some examples. 743 − 5 = 738 = 6 × 123, so we write 743 (G1, μ:123).

Now we take 1663 is in Group 2, so 1663 − 7 = 1656 = 6 × 276. So, we write 1663 (G2, μ:276).

Can we predict primes using this method? We can choose a value of μ, and with either 5 or 7 as starting points, look to see which number that appears is a prime. This method can be easily programmed. For example, we take $\mu = 100$. $6\mu = 600$. Group 1, 600 + 5 = 605 so not a prime. Group 2,600 + 7 = 607, which is a prime, 607 (G2, μ:100). Now if we find primes, one in each group, we have twins. For example, if we choose $\mu = 31$, we find $6\mu = 186$, and we have the twins 193 (G2, μ:31) and 191 (G1, μ:31).

Whatever the prime, we find its group then its μ value, and $\mu \in \mathbb{N}$. Twin primes have the same μ value.

The μ values for twins in Table 2 are: (1, 2), (4, 6), (9, 11), (16, 17), (22, 24), (29, 31), 32, (37, 39), (44, 46). In this sample, μ values for twins are separated by 2μ in all cases, so by 12. This could be interesting to study how this develops over a large sample.

There are some μ values that have no primes, 49, 47, 35, 33, 30, 19. However in general sometimes the μ value increase by 2 or 3, and in these cases the prime numbers are separated by 12 or 18. We call these places where there are no primes holes. We can notice that twin primes punctuate the sequence or progression of primes, and the twins are the nearest point between the two groups, viz. 2 which separates them. We notice that Group 1 or Group 2 primes appear but only together for a specific value of μ when we come across twin primes. When 6μ ends with a zero, Equation 1 does not yield a prime, but Equation 2 can, for example, 105 for (1) or 107 for (2) as shown above.

Table 2. Values of $f(6\mu)$ for Group 1 and Group 2 for $\mu = 1$ to 50.

μ	G1: $6\mu + 5$	G2: $6\mu + 7$	Twins (T)
	-- Continued from previous page		
μ	Group 1: $6\mu + 5$	Group 2: $6\mu + 7$	Twins (T)
	Continued on next page		
1	11	13	T
2	17	19	T
3	23		
4	29	31	T

Continued

5		37	
6	41	43	T
7	47		
8	53		
9	59	61	T
10		67	
11	71	73	T
12		79	
13	83		
14	89		
15		97	
16	101	103	T
17	107	109	T
18	113		
19			none
20		127	
21	131		
22	137	139	T
23			
24	149	151	T
25		157	
26		163	
27	167		
28	173		
29	179	181	T
30			
31	191	193	T
32	197	199	
33			none
34		211	
35			none
36		223	
37	227	229	T
38	233		
39	239	241	T
40			none
41	251		
42	257		
43	263		
44	269	271	T
45		277	
46	281	283	T
47			none
48	293		
49			none
50		307	

3.5. Chaotic or Random Progression of Primes

If we do not separate primes into two groups, we have apparently a random series of primes [9]. "*We used the inverse distribution and the Brody distribution for investigating the regular-chaos mixed systems. The distributions are made up of sequences of prime numbers from one hundred to three hundred and fifty million prime numbers. The prime numbers are treated as eigenvalues of a quantum physical system. We found that the system of prime numbers may be considered regular-chaos mixed system and it becomes more regular as the value of the prime numbers largely increases with periodic behaviour at logarithmic scale.*"

For example, **227**, 229, **233**, **239**, 241, **251**, **257**, **263**, **269**, 271, 277, **281**, 283, **293**, 307 correspond to the primes in their order on the prime number line. We have put Group 1 numbers in bold, and underlined twin primes. Numbers increase by 2, 4 or 6, so we have no clear picture of their evolution.

However, 233 – 227 = 6, in Group 1, whereas 101 – 31 = 70 = 72 – 2 = 12 × 6 – 2, that is Group1 - Group2 primes correspond to Equation 4. We now separate the primes and put them in their respective groups and order appears.

Table 3. Progression of the two groups of primes and their interconnection.

G2			97	103	109		127	
Twins				103 – 101 =2	109 – 107 =2			
Subtraction		89 – 83 =6	97 – 89 =8	101 – 97 =4	107 – 103 =4	113 – 109 =4	127 – 113 =14	131 – 127 =4
G1	83	89		101	107	113		131

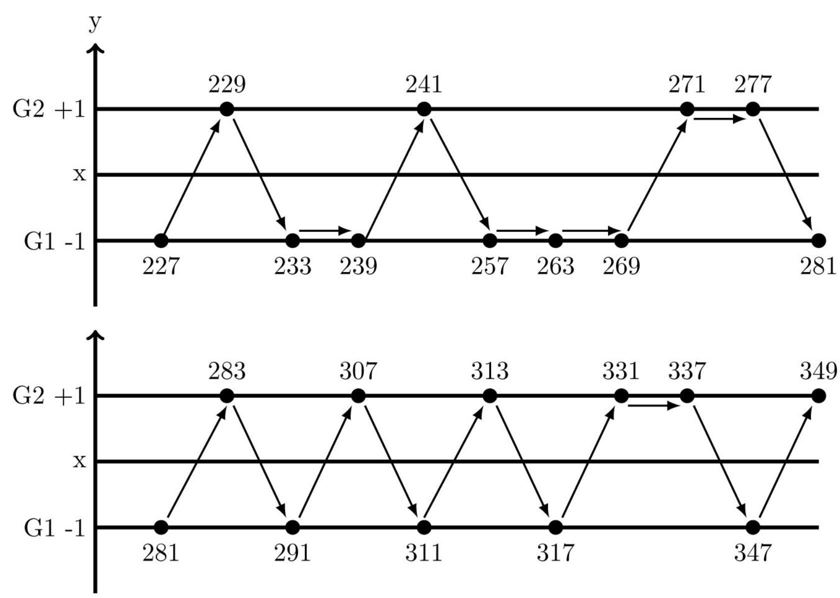

Figure 1. How the two groups interconnect.

Table 3 indicates how the prime line progresses as we pass from one group to another when the prime sequence ends in one group and begins in another. The prime number line becomes chaotic and unpredictible. We can clearly see that once we have put each prime into its respective group, the sequence is organized around multiples of 6 and Equations 1 and 2.

It is clear that the nearest point is with the twins, and between the groups, the primes are separated by values of 2 excluding 3 as a factor. Figure 1 shows how the primes progress and alternate between the two groups.

The graph in Figure 1 shows how the primes in the sequence chosen at random progress chaotically when we go from one group to the other. But observed separately each group progresses by multiples of six.

3.6. Special Types of Primes and Their Groups

There are many papers and articles on Fermat and Mersenne primes [8] [9]. The largest prime known is a Mersenne prime, $2^{82589933} - 1$. There are only six Fermat primes known. We find Pythagorean primes of the form $4n + 1$, which are primes of the sum of two squares.[2] Many more are to be found [9], Ramanujan primes and their connection with the prime counting function.[3] Here we shall examine two types of primes with respect to our conjecture, Mersenne and Fermat primes.

3.6.1. Mersenne Primes

Do Mersenne primes belong to Group 1 or Group 2? One way of proving this is that a Mersenne prime is of the form $2^n - 1$, and as 2^n is not divisible by 3, the prime number is above ζ, consequently it belongs to Group 2. We can define a Mersenne prime as $M_p = 2^p - 1$, where p is a prime number. We have shown the first 8 Mersenne primes in Table 4. Mersenne primes belong to Group 2.

The exponents (n) which produce Mersenne primes are 2, 3, 5, 7, 13, 17, 19, 31, and the resulting Mersenne primes are 3, 7, 31, 127, 8191, 131071, 524287, 2147483647, ...

Group 2 primes are: 7, 31, 127, 8191, 131071, 524287, 2147483647….

Let us look at Group 1. Can a Mersenne prime be in group 1?

$M_p = 2^p - 1 = 6n + 5 = 2^p - 6 = 6n$. For n to be an integer, 6 must divide $2^p - 6$, however 2^p does not have 3 as a factor, so $2^p - 6$ is not divisible by 3. Consequently, all Mersenne primes belong to Group 2.

Let us find μ for $M_6 = 131071$. We have 131071 − 7 = 131064 = 6μ. Therefore $\mu = 21844$. We can write 131071 as {G2, μ:21844}.

Let us take the example of 2147483640, a Mersenne prime. 2147483647 − 7 = 2147483640. Then $\mu = \frac{1}{6} \times 2147483642 = 357913940$, which is in Group 2.

[2]Group 1: 5, 17, 29, 41, 52, 89, 101, 113. Group 2: 12, 37, 61, 73, 97, 109.
[3]The whole list is given on this site: https://en.wikipedia.org/wiki/Category:Classes_of_prime_numbers.

Table 4. Mersenne primes and their group.

Exponent (p)	Primes	2^{p-3} exponent	$2^{p-3}-1$	$\left(2^{p-3}-1\right)/3$	Group
2	3				
3	7	0			
5	31	2	1		
7	127	4	15	5	2
13	8,191	10	1,023	341	2
17	131,071	14	16,383	5,461	2
19	524,287	16	65,535	21,845	2
31	2,147,483,647	28	268,435,455	89,478,485	2

Table 5. Values of F_n and corresponding μ values.

F_n	value	μ
F_0	$2^1 + 1 = 3$	0
F_1	$2^2 + 1 = 5$	0
F_2	$2^4 + 1 = 17$	2
F_3	$2^8 + 1 = 257$	42
F_4	$2^{16} + 1 = 65737$	10,972

3.6.2. Fermat Primes

Fermat primes are of the form $2^{2n}+1$. **Table 5** shows the five known Fermat primes. There μ value is given from 17 on.

Those primes are in Group 1: 5, 17, 257, 65,537. We find $\mu = 1/6\left(F_p - 5\right)$ for each prime.

Proof. Let $6\mu + 5 = 2^{2n} + 1$, then $\mu = \frac{1}{6}\left(2^{2n} + 4\right)$. We notice that $\mu = 1$ when $n = 1$, $\mu = 2$ when $n = 2$, and $\mu = 42$ when $n = 3$. Similarly, $65537 - 5 = 6\mu$, with $\mu = 10922$. No primes were found in Group 2. The reason is as follows: Let $6\mu + 5 = 2^{2n} + 1$, so $6\mu = 2^{2n} - 4 = 4\left(2n - 1\right)$. Note that $2n - 1$ is divisible by 3 for the exponents $n = 1, 2, 4$, and 8, i.e., 3,15,255.

3.7. Algorithm for Finding Primes in Each Group

Finding the primes in each group can be done with four steps:

- Step 1, choose a value for μ.
- Step 2, calculate 6 μ.
- Step 3, calculate 5 + 6μ: is the result a prime?
- Step 4, calculate 7 + 6μ: is the result a prime?

Examples: Let $\mu = 99$, $6\mu = 594$, 594 + 5 = 599, and 594 + 7 = 601. These are twin primes.

Next example, let $\mu = 111$, $6\mu = 666$, and 666 + 5 = 671 which is not a prime, and 666 + 7 = 673, which is a prime.

Third example: $\mu = 201$, $6\mu = 1206$ and 1206 + 5 = 1211, which is not a prime, and 1206 + 7 = 1213, which is a prime.

We can find the μ value for all primes using one or other of the above equations.

4. Discussion

Our hypothesis allows us to label any prime number with its group and μ values, and observe the rhythm of how the sequence progresses in one group or the other. No prime number is excluded from one group or the other. Primes in arithmetic progression are any sequence of at least three consecutive prime numbers.[4] If we take (3, 7, 5) we find $a_n = 3 + 4n$ for $0 \le n \le 2$.

According to the Green-Tho theorem greenprime [10] [11], we can find an arbitrarily long sequence of primes. For integer $k \ge 3$, an AP-k (Arithmetic Progression - k) is a sequence of k primes in arithmetic progression.[5] **Table 6** shows the different AP-ks.

"For integer $k \ge 3$, an AP-k (also called PAP-k) is any sequence of k primes in arithmetic progression. An AP-k can be written as k primes of the form $a \cdot n + b$, for fixed integers a (called the common difference) and b, and k consecutive integer values of n. An AP-k is usually expressed with $n = 0$ to $k - 1$. This can always be achieved by defining b to be the first prime in the arithmetic progression". The function $an + b$ corresponds to our function, with $a = 6$, $n = \mu$, and $b = 5$ or 7 according to the group.

In **Table 6** we find multiples of 6 in the equations given among many others. We have proposed only two equations and we have developed the importance of only two groups, which comprise all primes. The notion of segments of primes allows us to raise a few problems. In **Table 7**, we notice that the segments are numerous but short because we alternate between groups. However, all the existing primes are present in the list. Could we find longer progressive segments (or sequences) or is this simply impossible because of the way primes are organized, and in each group develop by 6μ, and they are out of phase by $6\mu \pm 2$? So, any sequence is broken. We also notice that the number of primes in each group in our sequence of primes in this table is 14 in Group 1, and 11 in Group 2. Over a large number of primes, could there be as many primes in one group as there are in the other?

Table 6. AP primes: any sequence of k primes in arithmetic progression.

k	Primes for $n = 0$ to $k - 1$	k	Primes for $n = 0$ to $k - 1$
3	$3 + 2n$	10	$199 + 210n$
4	$5 + 6n$	11	$110437 + 13860n$
5	$5 + 6n$	12	$110437 + 13860n$
6	$7 + 30n$	13	$4943 + 60060n$
7	$7 + 150n$	14	$31385539 + 420420n$
8	$199 + 210n$	15	$115453391 + 4144140n$
9	$199 + 210n$	16	$53297929 + 9699690n$

[4]More information about primes in arithmetic progression can be found at https://en.wikipedia.org/wiki/Primes_in_arithmetic_progression.
[5]Op. cit.

Table 7. Prime segments in Group 1 and Group 2.

μ	6μ	$6\mu+5$	$6\mu+7$	Twins/none
27	162	167		
28	168	173		
29	174	179	181	T
30	180			none
31	186	191	193	T
32	192	197	199	T
33	198			none
34	204		211	
35	210			none
36	216		223	
37	222	227	229	T
38	228	233		
39	234	239	241	T
40	240			none
41	246	251		
42	252	257		
43	258	263		
44	264	269	271	T
45	270		277	
46	276	281	283	T
47	282			none
48	288	293		
49	294			none
50	300		307	

A segment is an uninterrupted sequence of primes in **Table 7**, in ones, threes, or fours.

The length of each segment is 1, 2, 3, or 4 in G1 or G2 1 and 2 primes in G2. Consequently, it is possible that the segments will be short, and not long in some cases. This is because primes alternate between groups.

5. Further Avenues of Research

There are several problems that are raised by our hypothesis. Could the μ value help us to understand how twin primes progress? Could we use the model to find large primes that are not Mersenne primes but may lie between two of them and difficult if not impossible to detect?

It would be interesting to study the primes in sets of Pythagorean triples, observing to which group they belong, and finding an equation relating those primes to the Euclid identity, and finding the μ values that engender those primes. When $z > y$, and x is even, is there a reason why there are fewer primes for y than z? Also, how do triples relate to the odd or even number line?

We shall conceive a program that separates primes into the above-mentioned groups. We intend to develop our work on the Euclidean prime factorial (EPF). We have found that when $p + 1$ is not a prime, then we can add any prime greater than p, the largest prime in the product.6 We have $p_1 :! + p_2 = p_3$ where :!

is the prime factorial and $p_2 > p_1$, which is the highest prime in the factorial group. Many attempts have been made using the Euclid function, notably with the Euclid-Mullin sequence [12]. This function should allow us to find many large prime sequences.

6. Conclusions

We believe we have found a function which may show the rhythm of development of all primes in two distinct groups, (1) $p = 6\mu + 5$ for the first, and (2) $p = 6\mu + 7$ for the second. How can this formula be used to predict primes unknown to us, and the spaces we find where μ does not correspond to any prime, will these holes become more and more frequent as primes become fewer and fewer? Probably. We have seen how to add or subtract primes and the rules that can be applied. Addition is the inverse of subtraction. We can find which group a prime is in by adding or subtracting 1 and dividing by 3. The equations we have given show how primes of different groups can be separated by 2, 4, 8 and so on. So, what looks chaotic when we look at the list of primes, can be shown to be an ordered dynamic system. *Ordo ab chao*. The progression in each group is given by Equation 1 and Equation 2.

Primes alternate between each group, or we find twins with the same μ value, so sometimes we have a sequence of primes in one group, then the primes appear in the other. We find subtraction for primes in the same group, Equation 3, and subtraction for primes from one group to another, Equation 4.

We believe that every prime must be in one group or the other as they lie either above or below a multiple of 6. As the multiples of 6 are infinite, our conjecture might be used to find additional proof of the infinite number of primes.

We can try to explain the holes where we do not find a prime for Equation 1 or Equation 2. Can this help us to study prime density, which we could call the proportion of primes to odd numbers in a specific zone with a starting point and a closing point or upper and lower limits?[7] Can we draw up a network or lattice of primes to show how they are interconnected, as we suggested above using a graph in **Figure 1**? When we move from one group to another, Equation 44 applies, we return to a chaotic sequence. Data mining and AI might reveal relationships a human might not be aware of. For twin primes, it may be possible to understand if there is some repetition of 6μ which might be significant. The originality of our hypothesis is that it is extremely simple to verify empirically. What appears to be a system that is completely unpredictable and fortuitous can actually be described as an arithmetic progression with multiples of 6 when we subsume the primes in two groups.

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[6]For $p = 13$, $13:! + 1 = 59 \times 509$. If we try $13:! + 17$, we find 30047.

[7]Between 1 and 100, there are 25 primes and 50 odd numbers, so the density would be 50% of primes for 50 odd numbers. For 1 to 1000, the density would be 33.6%. Our definition.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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