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Influence of Prandtl Number on Thin Film Condensation in Forced Convection in an Inclined Wall Covered with a Porous Material

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Abstract

The numerical study of thin film type condensation in forced convection of a saturated pure vapor in an inclined wall covered with a porous material is presented. The generalized Darcy-Brinkman-Forchheimer (DBF) model is used to describe the flow in the porous medium while the classical boundary layer equations have been exploited in the case of a pure liquid. The dimensionless equations are solved by an implicit finite difference method and the iterative Gauss-Seidel method. The objective of this study is to examine the influence of the Prandtl number on the hydrodynamic and thermal fields but also on the local Nusselt number and on the boundary layer thickness. For $Pr \leq 0.7$ (low) the velocity and the longitudinal temperature increase with the Prandtl number. On the other hand, when $Pr \geq 2$ (high) the Prandtl number no longer influences the velocity and the longitudinal temperature. The local Nusselt number increases as the Prandtl number increases and the thickness of the hydrodynamic boundary layer increases as the Prandtl number decreases.

Keywords

Condensation, Thin Film, Forced Convection, Inclined Porous Wall, Generalized Darcy-Brinkman-Forchheimer Model, Prandtl Number

1. Introduction

The study of steam condensation is today the subject of much interest from scientific researchers because of its use in several technological fields of industry, such as refrigeration, air conditioning, energy storage, desalination, exchangers, etc. Since the work of Nusselt [1] who was one of the precursors and who had studied the case of condensation in laminar film of a pure vapor stagnant on a vertical plate, several other authors had worked in this field and made several publications of their analytical, experimental or numerical results. Nowadays, the relatively recent numerical studies are the most popular because they do not require a lot of means for their realization. They are generally based on the simultaneous resolution of hydrodynamic and thermal equations. For this study several numerical, analytical and experimental works have been consulted. Among them there are: Ndiaye M. *et al.* [2] proposed a numerical model for the study of saturated pure vapor condensation of thin film type in forced convection on a wall covered with porous material. The transfers in the porous medium and the liquid film are described by the Darcy-Brinkman model and the classical boundary layer equations respectively. The dimensionless equations are solved by an implicit finite difference method combined with an iterative Gauss-Seidel method. They analyzed the influences of the Prandtl and Froude numbers on the transfers in the liquid phase. The parameters related to the thermal problem (Prandtl number) have no influence on the adimensional velocity although the thermal and hydrodynamic problems are coupled via the heat balance equation. The study showed that heat transfers increase when $Pr \leq 0.2$, however for values of $Pr > 0.2$ there is no change on heat transfers. However, values of $Fr_K \geq 0.00$ have no effect on the hydrodynamic field, for $Fr_K < 0.001$ the velocity increases significantly in the liquid film and inertial effects can no longer be ignored (a reduction in Froude number leads to an increase in the permeability K of the porous medium thus the substance become more and more permeable). PT Ndiaye *et al.* [3] presented the numerical study of the condensation in thin layers in forced convection of a saturated pure vapor in a channel whose walls are covered with a porous material. The generalized Darcy-Brinkman-Forchheimer (DBF) model is used to describe the flow in the porous medium. Porous medium, while the classical boundary layer equations have been exploited in the case of a pure liquid. The dimensionless equations are solved by an implicit finite difference method and the iterative Gauss-Seidel method. In this study, the role of parameters such as Reynolds number and Prandtl number on the longitudinal velocity and temperature in the porous medium and in the pure liquid, and the rate of heat transfer (local Nusselt number) were investigated and highlighted. Increasing Reynolds number and Prandtl number results in increasing longitudinal velocity and temperature. The tangents of the velocity curves at the porous interface on the medium side are smaller than those obtained on the liquid side with low Reynolds number and Prandtl number. It is also noted that increasing the Reynolds number and Prandtl number improves the thermal performance of the interface. Prandtl improves the heat exchange at the interface of the porous medium and the film liquid. Ndiaye M. *et al.* [4] using the same numerical model have analyzed the influence of Reynolds number and the dimensionless thickness of the porous layer on the transfers in the liquid phase and the porous medium as well as the thickness of the liquid film. The thickness of the condensate film is determined by the heat balance equation at the liquid-vapor interface which is solved by an iterative Gauss-Seidel procedure. Their results showed that the Reynolds number does not influence the thermal field, the temperature increases with a decrease in the thickness of the adimensional porous layer. Increasing the Reynolds number and the adimensional thickness of the porous layer leads to an increase in the longitudinal velocity and a decrease in the thickness of the liquid film. Increases in Reynolds number and dimensional thickness of the porous layer have adverse effects on condensation (decrease in liquid film thickness). The study also showed that inertial effects can no longer be ignored when the Reynolds number calculated from the permeability coefficient is higher than 7. Ndiaye M. *et al.* [5] proposed a numerical model for the study of saturated pure vapor condensation of thin film type in forced convection on a wall covered with porous material and they analyzed the influences of Reynolds and Jacob numbers, the adimensional thickness of the porous layer and the adimensional heat transfer conductivity on the transfers in the liquid phase. They used the same resolution as before. Their results showed that the thermal field is not influenced by the dynamic problems despite their coupling at the interface. The parameters related to the thermal problem (thermal conductivity ratio and Jacob number) have no effect on the longitudinal velocity. It also appears that the Reynolds number has little influence on the thermal field. The longitudinal velocity increases with the thickness of the porous layer. The decrease of the adimensional thickness of the porous layer and the thermal conductivity ratio and an increase of the Jacob number leads to an increase of the temperature. R. Chaynane *et al.* [6] presented an analytical and numerical study of the condensation of a film on a wall inclined to the vertical and covered with a material. The Darcy-Brinkman model is used to describe the flow in the porous medium, while the classical boundary layer equations have been exploited in the pure liquid taking into account inertia and enthalpic convection terms. The problem posed has been solved by analytical and numerical means. The results are mainly presented in the form of the dimensionless thickness of the liquid film, the velocity and temperature profiles and the heat exchange coefficients represented by the Nusselt number. The effects of various parameters such as angle (φ), effective viscosity (Reynolds number), dimensionless thickness of the porous substrate, and dimensionless thermal conductivity on the flow and heat transfers are presented. SANYA S. A.O. *et al.* [7] proposed an analytical study of condensation in laminar film on a vertical surface, bordering a thick anisotropic porous medium saturated by a pure vapor by using the Darcy-Brinkman model. This study aims at obtaining information necessary for the characterization and the precision of the dynamics of the phenomena of condensation in order to seek, for a silo of food grains, the maximum humidities to be recommended according to the products and the climatic zones. Boundary layer approximations in the region of the liquid film allows to deduce analytical solutions showing the effect of the permeability of the porous medium and of the angle of inclination of the main axes on the thickness of the liquid film, the mass flow rate of the liquid film and the surface heat flux at the wall. The resulting Nusselt number depends on the square root of the Rayleigh number and the dimensionless film thickness. The latter is a function of the permeability anisotropy constant K^* and the dimensionless parameter related to the wall subcooling. Nasr A. [8] numerically studied the problem of heat and mass transfer enhancement during liquid film-like condensation along a vertical channel covered with a porous layer on one of its walls. The liquid film falls down a wall of the channel under mixed convection. The wall is covered with thin porous layer and externally subjected to a uniform cooling flow of heat while the second wall is dry and isothermal. The effects of porosity, porous layer thickness and ambient conditions on heat and mass transfer efficiency and liquid film condensation were examined in detail. The numerical results show that the heat and mass transfer performance at the liquid-vapor interface during liquid film condensation is enhanced by the presence of the porous layer. A decrease in the porosity ε and thickness of the porous layer increases the heat and mass transfer performance at the liquid-vapor interface. Increases in porosity, porous layer thickness, ambient pressure, humidity, and cooling heat flux increase water film condensation in contrast to increasing inlet temperature. L. Merouani *et al.* [9] have also worked on the numerical study of laminar film condensation of vapor-gas mixtures in an inclined channel with an insulated top wall and an isothermal bottom wall lined with a thin porous material. A two-dimensional model two-dimensional model is developed using a set of complete boundary layer equations for the liquid film and vapor-air mixture, while the Darcy-Brinkman-Forchheimer approach is used for the porous material. The governing equations are discretized with an implicit finite difference scheme. The resulting systems of algebraic equations are solved numerically using the Gauss and Thomas algorithms. The numerical results are used to determine the velocity, temperature, and vapor concentration profiles in the vapor-air mixture, the liquid film, and the porous substrate. The axial evolution of the condensate flow rate and wall heat flux is also presented and analyzed for different operating conditions. It is found that the tilt angle, relative humidity input values and Reynolds number have a much greater influence on the condensation process than the tilt angle. Condensation process is much more important than that resulting from a change in the properties of the porous layer. A numerical study of heat and mass transfer during laminar film condensation of water vapor in an inclined channel with a porous wall was presented by El Hammami Y. *et al.* [10]. In the porous medium, they described the flow by the Darcy-Brinkman-Forchheimer model. The results are obtained for two positions of the channel, horizontal and vertical. The analysis of the effects of the porous layer thickness H, inlet temperature, inlet pressure, and inlet Reynolds number are studied. The simulation results show well the influence of the porous layer thickness on the heat and mass transfer performance. Asbik M. *et al.* [11] analytically studied the laminar thin-film condensation of pure and saturated vapor flowing in forced convection over a vertical porous plate. The Darcy-Brinkman model and the classical boundary layer equations describe the transfers in the porous medium and the pure liquid respectively. Thermal dispersion is taken into account in the heat equation for heat transfer in the porous layer. The analysis of velocity, temperature and local Nusselt number profiles shows the influence of thermal dispersion on the condensate flow and heat transfer. The results show that increasing the thermal dispersion coefficient leads to a considerable increase in heat exchange. Renken K. J. *et al.* [12]-[18], were the first to show numerically via a finite-difference scheme the effect of the porous layer thickness on the transfers by studying laminar thin film condensation on a vertical surface with a porous coating. Their model simulates two-dimensional condensation inside a highly permeable, thinly conductive porous layer. The local volume averaging technique is used to establish the energy equation. The Darcy-Forchheimer model is employed to describe the flow field in the porous layer while the classical boundary layer equations are used in the pure condensate region. They also developed experiments on forced convection condensation in thin and porous coatings. This study presents the results of forced convection heat transfer experiments of condensation on plates with a thin porous coating. The composite system consists of a porous, thin, highly conductive and permeable material bonded to a cold surface in isothermal condensation that runs parallel to the saturated vapor flow, were interested in studying forced convective condensation on a thin porous layer led plate. The system consists of a relatively thin permeable highly conductive composite placed parallel to the vapor flow. The pores have thicknesses ranging from 0 to 254 micrometers which are used as a passive technique for heat transfer enhancement. The use of a thin porous layer on isothermal surfaces under the condition of forced convection shows that the thinner the porous layer the more heat transfer is increased. Thus, the experiments demonstrated the advantages of using a thin porous coating in a forced convection heat transfer environment. Sellami K. *et al.* [19] proposed a numerical study of coupled heat and mass exchange. They found that the evaporative cooler is more efficient for high porosity and thick, porous medium, with an improvement reaching 23% for high porosity. Abdelaziz Nasr [20] presented a numeric model of heat and mass transfer enhancement of liquid film condensation by covering a porous layer on one of channel vertical plates has been numerically investigated. He has showed numerically that the heat and mass transfer performance at the liquid-gas interface during the liquid film condensation is enhanced by the presence of the porous layer. Xuehu Ma *et al.* [21] presented a numerical study of film condensation on a vertical porous plate coated with a porous/fluid composite system based on the dispersion effect. The mathematical model improves the corresponding conventional conditions by considering the stress force at the porous coating/fluid interface. Numerical results show the effects of the thickness of the porous medium, the effective thermal conductivity, the permeability of the condensate film thickness and the local Nusselt numbers.

2. The Physical Model

We consider a saturated porous medium confined on a vertical plate, of thickness H, permeability K and porosity ε (**Figure 1**). This flat plate tilted by an angle ϕ of length L is placed in a pure, saturated vapor flow of longitudinal velocity u_0. The vapor condenses on the wall of the plate held at temperature T_w lower than the saturation temperature T_s of the vapor. The condensate film flows under the effect of gravity and viscous frictional forces. There are three zones: Zone (1) is the porous medium saturated by the liquid. The zone (2) corresponds to the liquid film while the zone (3) is relative to the saturating vapor. Let (x, y) and (u, v) be respectively the Cartesian coordinates and the components of the velocity in the porous medium and the liquid in the reference frame associated with the model.

For the rest of our study, we have taken the following simplifying assumptions:

1) The porous substrate is isotropic and homogeneous.

2) The fluid saturating the porous medium is Newtonian and incompressible.

3) The flow generated is laminar and two-dimensional.

4) The work, induced by viscous and pressure forces, is negligible.

5) The thermo-physical properties of the fluids and those of the porous matrix are assumed constant.

6) The Darcy-Brinkman-Forchheimer model is used to describe the flow in the porous layer.

7) The inertia and enthalpy convection terms are taken into account in both phases.

8) The effective dynamic and kinematic viscosities of the porous material are equal to those of the condensate film.

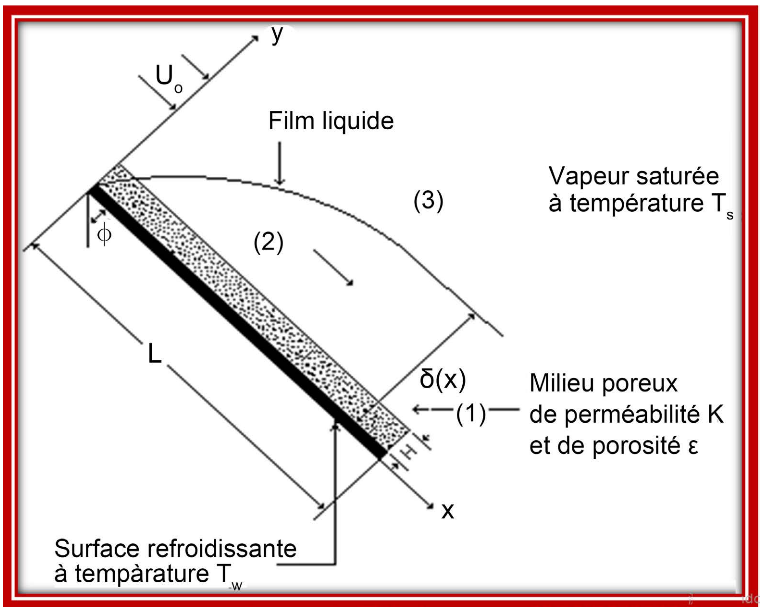

Figure 1. Physical model geometry and coordinate system.

9) Condensation occurs as a thin film.

10) The porous matrix is in local equilibrium with the condensate.

11) The liquid-vapor interface is in thermodynamic equilibrium and the shear stress is assumed to be negligible.

12) The vapor and the film are separated by a distinct boundary.

13) The flow is considered of Bernoulli type in the pure vapor phase.

3. Dimensionless Equations

The equations governing the transfers in the domains (1) and (2) defined above as well as the boundary conditions associated with them, have been dimensionless using the following parameters and variables, we pose:

$$H^* = \frac{H}{\sqrt{K}} \tag{1}$$

$$x^* = \frac{x}{\sqrt{K}} \tag{2}$$

$$y^* = \frac{y}{\sqrt{K}} \tag{3}$$

$$L^* = \frac{L}{\sqrt{K}} \tag{4}$$

$$\delta^* = \frac{\delta}{\sqrt{K}} \tag{5}$$

$$u^* = \frac{u}{u_r} \tag{6}$$

$$u_r = \frac{K}{\nu_{eff}} g \cos\phi \tag{7}$$

$$\theta = \frac{T - T_w}{T_s - T_w} \tag{8}$$

$$\lambda^* = \frac{\lambda_l}{\lambda_{eff}} \tag{9}$$

$$\nu^* = \frac{\nu_{eff}}{\nu_l} \tag{10}$$

To reduce the resulting physical domain (x^*, y^*) that has a curvilinear interface (liquid/vapor interface) into a rectangular domain (X, η), we perform the following variable change:

$$X = x^* \tag{11}$$

$$\eta = coef \cdot \frac{y^*}{H^*} + (1 - coef) \cdot \left\{ \frac{y^* - H^*}{\delta^* - H^*} \right\} \tag{12}$$

with the *coef*, a coefficient equal to 1 if we are in the porous layer and 0 in the pure liquid.

Thus, the (x^*, y^*) plane is transformed into a rectangular (X, η) field and the porous medium/liquid and liquid/saturating vapor interfaces are respectively parameterized by the coordinate lines $\eta = 1$ and $\eta = 2$. In both media, dimensionless equations are written:

Porous medium: $0 \le \eta \le 1$

Continuity equation

$$\frac{\partial u_p^*}{\partial X} + \frac{1}{H^*}\frac{\partial v_p^*}{\partial \eta} = 0 \tag{13}$$

Equation of the following momentum balance X

$$\begin{aligned} & u_p^* \frac{\partial u_p^*}{H^*} + \frac{v_p^*}{H^*}\frac{\partial u_p^*}{\partial \eta} \\ & = -\frac{\varepsilon^2}{Re_K} u_p^* + \frac{\varepsilon^2}{Fr_K}\cos\phi\left(\frac{\rho_l - \rho_v}{\rho_l}\right) + \frac{1}{H^{*2}}\frac{\varepsilon^2}{Re_K}\frac{\partial^2 u_p^*}{\partial \eta^2} - \varepsilon^2 F \cdot u_p^{*2} \end{aligned} \tag{14}$$

Heat equation

$$u_p^* \frac{\partial \theta_p}{\partial X} + \frac{v_p^*}{H^*}\frac{\partial \theta_p}{\partial \eta} = \frac{1}{H^{*2}\lambda^* Pr Re_K}\frac{\partial^2 \theta_p}{\partial \eta^2} \tag{15}$$

Liquid medium: $1 \prec \eta \prec 2$

Continuity equation

$$\frac{\partial u_l^*}{\partial X} - \frac{\eta - 1}{\delta^* - H^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}X}\frac{\partial u_l^*}{\partial \eta} + \frac{1}{\delta^* - H^*}\frac{\partial v_l^*}{\partial \eta} = 0 \tag{16}$$

Equation of the following momentum balance X

$$\begin{aligned} & u_l^*\left(\frac{\partial u_l^*}{\partial X} - \frac{\eta - 1}{\delta^* - H^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}X}\frac{\partial u_l^*}{\partial \eta}\right) + \frac{v_l^*}{\delta^* - H^*}\frac{\partial u_l^*}{\partial \eta} \\ & = \frac{1}{\left(\delta^* - H^*\right)^2}\frac{1}{v^* Re_K}\frac{\partial^2 u_l^*}{\partial \eta^2} + \left(\frac{\cos\phi}{Fr_K}\right)\left(\frac{\rho_l - \rho_v}{\rho_l}\right) \end{aligned} \tag{17}$$

Heat equation

$$u_l^*\left(\frac{\partial \theta_l}{\partial X} - \frac{\eta - 1}{\delta^* - H^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}X}\frac{\partial \theta_l}{\partial \eta}\right) + \frac{v_l^*}{\delta^* - H^*}\frac{\partial \theta_l}{\partial \eta} = \frac{1}{\left(\delta^* - H^*\right)^2}\frac{1}{Pr Re_K}\frac{\partial^2 \theta_l}{\partial \eta^2} \tag{18}$$

3.1. Boundary and Interface Conditions

The basic equations described above are solved taking into account the boundary conditions specific to our problem. These are the following.

Boundary condition at wall level $\eta = 0$

$$u_l^* = v_p^* \tag{19}$$

$$\theta_p = 0 \tag{20}$$

At the liquid interface and porous medium: $\eta = 1$

$$u_l^* = u_p^* \tag{21}$$

$$\theta_l = \theta_p \tag{22}$$

Continuity of thermal flows:

$$\frac{1}{H^*}\frac{\partial\theta_p}{\partial\eta}=\frac{\lambda^*}{\delta^*-H^*}\frac{\partial\theta_l}{\partial\eta} \tag{23}$$

Continuity of constraints:

$$\frac{1}{H^*}\frac{\partial u_p^*}{\partial\eta}=\frac{\mu^*}{\delta^*-H^*}\frac{\partial u_l^*}{\partial\eta} \tag{24}$$

At the vapor/liquid interface $\eta=2$

$$\theta_l=1 \tag{25}$$

$$\frac{\partial u_l^*}{\partial\eta}=0 \tag{26}$$

3.2. Mass and Heat Balance Equations

Since the velocity and dimensionless temperature depend on the thickness of the liquid film *, the heat balance is expressed by the following relation, respectively: The heat and heat balance (27), (29) satisfy the following relationship.

$$\begin{aligned}&\frac{Ja}{Pe_{eff}\nu^*H^*}\left(\frac{\partial\theta_p}{\partial\eta}\right)_{\eta=0}\\&=\frac{\mathrm{d}}{\mathrm{d}X}\int_0^1 H^*\left\{1+Ja\left(1-\theta_p\right)\right\}u_p^*\mathrm{d}\eta+\frac{\mathrm{d}}{\mathrm{d}X}\int_2^1\left(\delta^*-H^*\right)\left\{1+Ja\left(1-\theta_l\right)\right\}u_l^*\mathrm{d}\eta\end{aligned} \tag{27}$$

with:

$$Pe_{eff}=\lambda^*PrRe_K \tag{28}$$

The mass flow rate:

$$H^*\int_0^1 u_p^*\mathrm{d}\eta+\left(\delta^*-H^*\right)\int_1^2 u_l^*\mathrm{d}\eta=\frac{\rho_v}{\rho_l}\delta^* \tag{29}$$

The following relations define the dimensionless numbers:

$$Ja=\frac{C_{pL}\left(T_s-T_w\right)}{h_{fg}} \tag{30}$$

Represents Jacob's number that compares sensible and latent heat.

$$Fr_K=\frac{\left(u_r\right)^2}{g\sqrt{K}}: \tag{31}$$

the Froude number which characterizes the ratio between the forces of inertia and gravity.

$$Re_K=\frac{u_r\sqrt{K}}{\upsilon_{eff}}: \tag{32}$$

the Reynolds number which compares the effects of inertia and viscosity.

$$Pr=\frac{\upsilon_l}{\alpha_l}: \tag{33}$$

the number of Prandtl.

$$Pr_{eff} = \frac{\upsilon_{eff}}{\alpha_l}: \tag{34}$$

the Prandtl number calculated on the basis of the effective viscosity.

3.3. Determination of the Local Nusselt Number

The Nusselt number is the heat transfer rate at the porous medium/pure liquid interface.

$$Nu = \frac{1}{\delta^*(X) - H^*} \frac{\partial \theta_l}{\partial \eta}\bigg|_{\eta=1} \tag{35}$$

4. Numerical Resolution

The boundary layer equations are solved according to the implicit scheme given in **Figure 2**. In this scheme, the partial derivative at notes (*i*, *j* of cell *i*, *j*). The uniform mesh is not rectangular and the geometric progression. The domain mesh of the real problem is converted into a numerical mesh of the next rectangular domain. The transfer equations are discretized by a finite difference method. The advection and diffusion terms are discretized with a backward decentered and centered scheme respectively in order to make the main diagonals of the matrices as dominant as possible. The coupled algebraic equations are solved numerically by a line of iterative Gauss-Seidel relaxation method.

5. Results and Discussion

Validation of Our Calculation Model

To validate our model in which the inertia terms are considered and the tilt angle $\Phi = 30^\circ$ we compared it with the results of the work M. Ndiaye *et al.* [2] [3] [4]. We subsequently find an acceptable correspondence as shown in **Figure 3** and **Figure 4**.

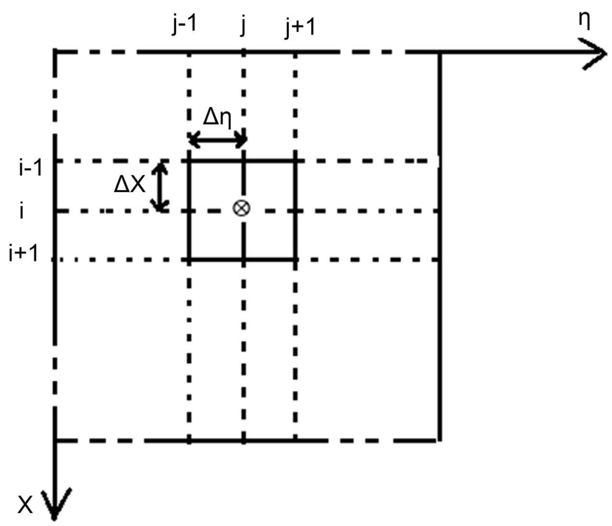

Figure 2. Mesh. * Nodes involved in discretization.

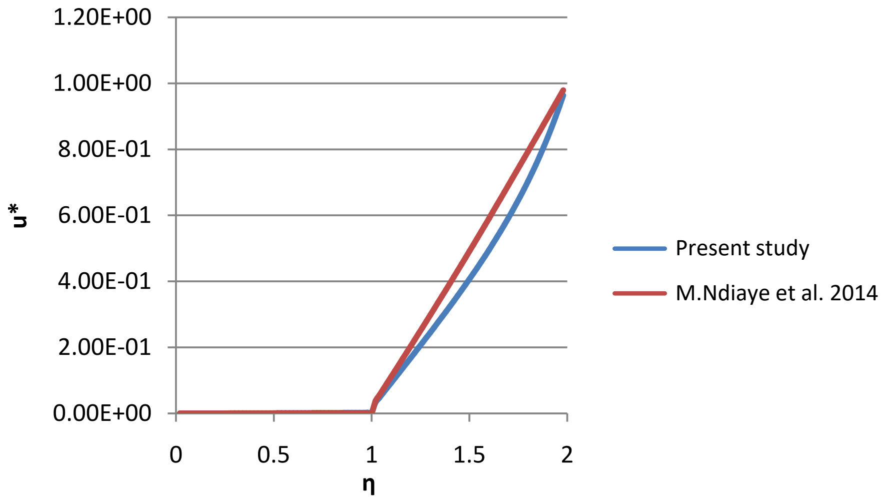

Figure 3. Comparison of the velocity profile of our study with that of Ndiaye M. (2014) $Re_K = 50; Pr = 2.5; Fr_K = 10^{-6}; Ja = 10^{-8}; \lambda^* = 1,25; H^* = 10^{-7}; \nu^* = 1$.

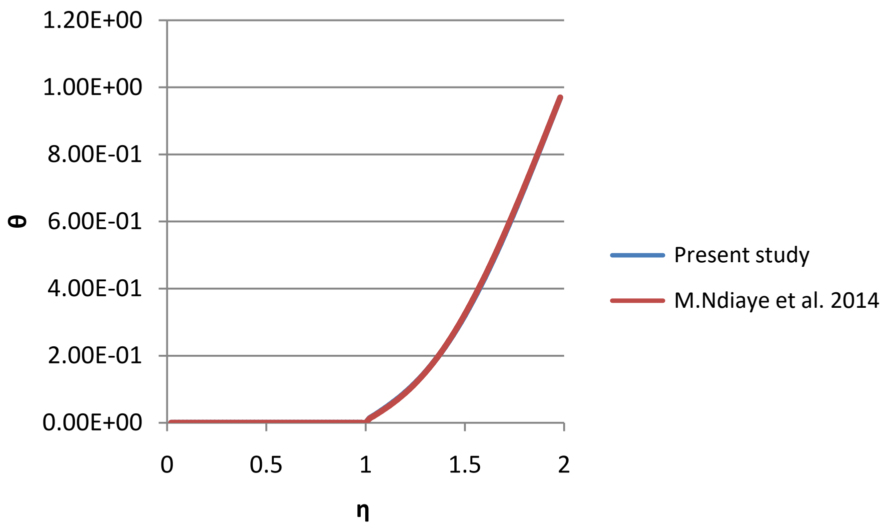

Figure 4. Comparison of the temperature profile of our study with that of Ndiaye M. (2014) $Re_K = 50; Pr = 2.5; Fr_K = 10^{-6}; Ja = 10^{-8}; \lambda^* = 1,25; H^* = 10^{-7}; \nu^* = 1$.

Our results are then compared with those of M. Ndiaye *et al.* [1] and P.T. Ndiaye *et al.* [2] and are shown in **Figures 5-8** below.

$$\mathcal{E} = 0.4; Re_K = 5; Pr = 2; \upsilon^* = 1; H^* = 2\times10^{-2}; Fr_K = 10^{-2}; Ja = 10^{-5}; \lambda^* = 5$$

The mesh sensitivity study led us to choose ΔX = 0.05 and $\Delta\eta$ = 0.02. The convergence criterion in the iterative process is set to 10^{-6}.

In our study we found an increas in the velocity and longitudinal temperature on both porous and liquid media for $Pr \le 0.7$ (**Figure 5**). This is contrary to the work of M. Ndiaye *et al.* [2] for whom there is no influence of Prandtl number on the velocity and longitudinal temperature when $Pr \le 0.7$ although the thermal and hydrodynamic problems are coupled via the heat balance equation. However, when $Pr \ge 2$ (**Figure 5**), we can see that the variation of the velocity and the

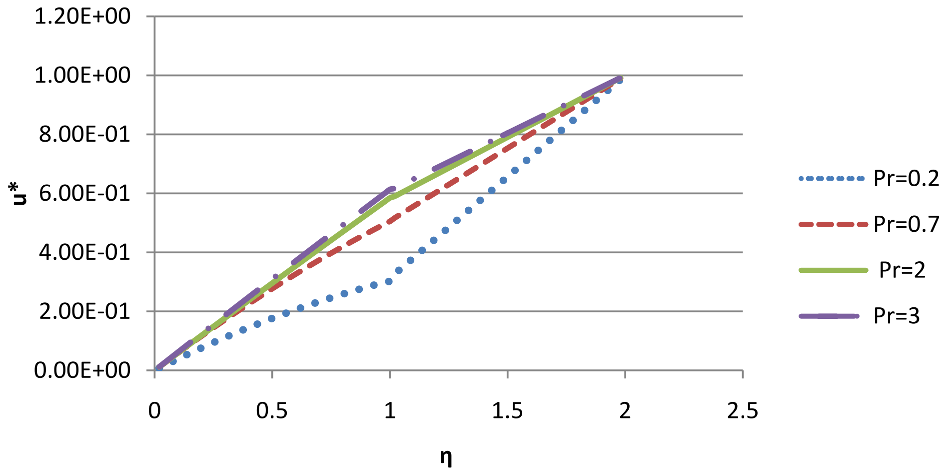

Figure 5. Variation of the longitudinal velocity as a function of the ordinate η for different values of the Prandtl number $Re_K = 5; Fr_K = 10^{-2}; Ja = 10^{-5}; \lambda^* = 5; H^* = 2\times10^{-2}; \nu^* = 1$.

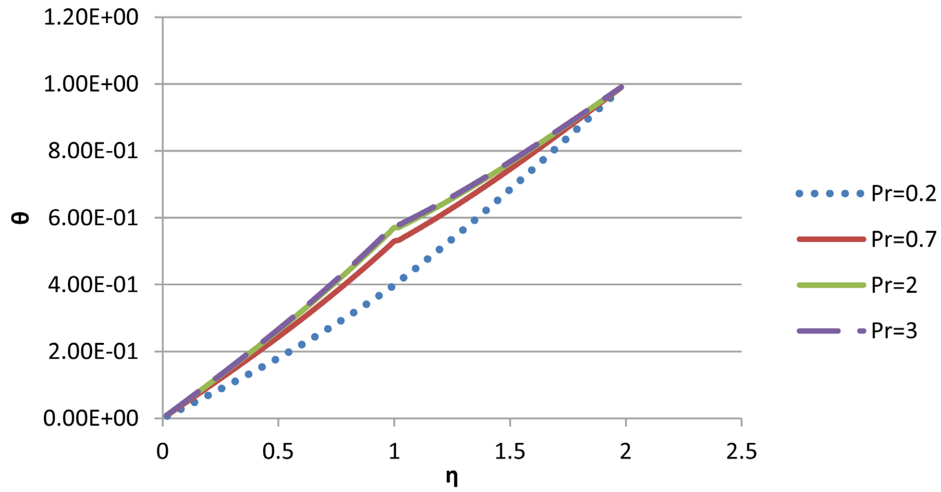

Figure 6. Variation of the longitudinal temperature as a function of the ordinate η for different values of the Prandtl number $Re_K = 5; Fr_K = 10^{-2}; Ja = 10^{-5}; \lambda^* = 5; H^* = 2\times10^{-2}; \nu^* = 1$.

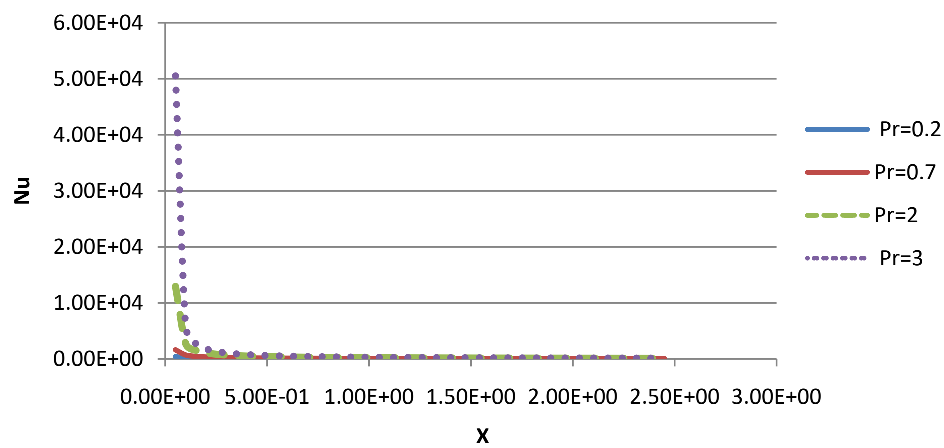

Figure 7. Variation of the Nusselt number as a function of the abscissa X for different values of the Prandtl number $Re_K = 5; Fr_K = 10^{-2}; Ja = 10^{-5}; \lambda^* = 5; H^* = 2\times10^{-2}; \nu^* = 1$.

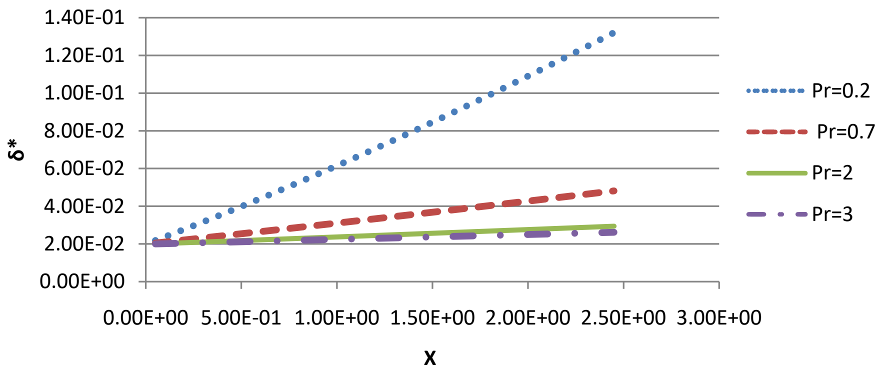

Figure 8. Variation of the liquid film thickness as a function of the abscissa X for different values of the Prandtl number $Re_K = 5; Fr_K = 10^{-2}; Ja = 10^{-5}; \lambda^* = 5; H^* = 2\times10^{-2}; \nu^* = 1$.

longitudinal temperature does not evolve with the increase of the Prandtl number but the temperature romains strongly influenced by the velocity profile for low value of the Prandtl number ($Pr \leq 0.7$) which would mean that the effects of viscosity become negligible in front of the thermal diffusion effects. Thus we have similar velocity and temperature profiles with M. Ndiaye *et al.* [2] than when $Pr \geq 2$ (**Figure 5**, **Figure 6**). We can explain this difference by the fact that the hydrodynamic effects are accentuated by the inertia forces with a predominance of the Forchheimer term which is taken into account in this study.

Compared to the work of PT Ndiaye *et al.* [3] we also find that the velocity and longitudinal temperature profile increases with the Prandtl number when $Pr \leq 0.7$ (low Prandtl values) (**Figure 5**) which is the opposite of the results of PT Ndiaye *et al.* [3] for them the Prandtl number has a weak influence on the longitudinal velocity more particularly on the porous medium when $3 \leq Pr \leq 10$ (high Prandtl value) in spite of a high Reynolds number ($Re = 200$). For values of the Prandtl number of PT Ndiaye *et al.* [3] between $7 \leq Pr \leq 30$ (very high value) our two temperature profiles become similar (**Figure 6**). This difference is due to the high aspect ratio of the channel (considered by PT Ndiaye *et al.* [3]) ($L/A = 100$) which is inversely proportional to the velocity and the longitudinal temperature.

We found that the local Nusselt number decreases exponentially along the x-axis (**Figure 7**). The conductive transfer gradually increases along the porous wall over the convective heat transfer. The increase in Prandtl number leads to an increase in Nusselt number. Similar profile to that of PT Ndiaye *et al.* [3].

The thickness of the hydrodynamic boundary layer increases as the Prandtl number decreases (**Figure 8**). This means that the greater the thermal diffusivity, the greater the thickness of the boundary layer. In other words, the variations of the boundary layer thickness depend a lot on the thermal parameters. Similar profile to that of M. Ndiaye *et al.* [2].

6. Conclusion

We have presented a numerical study of thin film condensation in forced convection on a wall inclined to the vertical covered with a porous material. The influence of the Prandtl number on the parameters such as: the longitudinal velocity, the longitudinal temperature, the local Nusselt number and the thickness of the boundary layer were presented. The results showed that considering the generalized Darcy-Brinkman-Forchheimer (DBF) model to describe the flow in the porous medium that there is an influence of the Prandtl number on the velocity and temperature for low values ($Pr \leq 0.7$) because of the Forchheimer term. Then it is necessary to have very high values of Prandtl number: $7 \leq Pr \leq 30$ to have velocity and temperature profiles similar to ours because of the high aspect ratio $L/A = 100$ of the channel. Finally, the local Nusselt number increases when the Prandtl number increases and the thickness of the hydrodynamic boundary layer increases when the Prandtl number decreases. The limit of application of the formulas used has not been determined during this work. In perspectives it would be interesting to see the effects of anisotropy and inhomogeneity of the porous medium and the dispersion in the model.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Nomenclature

ϕ: Angle d'inclinaison
α: thermal diffusivity, $m^2 \cdot s^{-1}$
δ thickness of condensate, m
ε: Porosity
η: dimensionless coordinate in the transverse direction
θ: temperature dimensionless
λ: thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
μ: viscosité dynamique, $kg \cdot m^{-1} \cdot s^{-1}$
ν: Viscosité cinématique, $m^2 \cdot s^{-1}$
ρ: density, $kg \cdot m^{-3}$
Latines letters:
Cp: specific heat, $J \cdot kg^{-1} \cdot K^{-1}$
F: Forchheimer coefficient
Fr_K: Froude number
g: gravitational acceleration, $m \cdot s^{-2}$
H: thickness of the porous layer, m
h_{fg}: enthalpy of evaporation, $J \cdot kg^{-1}$
Ja: Jacob number
K: hydraulic conductivity or permeability, m^2
Nu: local Nusselt number
Pe: Peclet number
Pr: Prandtl number
Re_K: Reynolds number
T: temperature, K
u: velocity along x, $m \cdot s^{-1}$
U_0: velocity of the free fluid (at the entrance of the channel), $m \cdot s^{-1}$
v: velocity along y, $m \cdot s^{-1}$
x, y: Cartesian coordinates, m
X: dimensionless coordinate in the longitudinal direction
Subscripts:
eff: effective value
int: porous substrate/pure liquid interface
l: liquid
p: porous
s: saturation
v: steam (vapeur)
w: wall
*: dimensionless quantity