

# A PID Tuning Approach for Inertial Systems Performance Optimization

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## Abstract

In the practice of control the industrial processes, proportional-integral-derivative controller remains pivotal due to its simple structure and system performance-oriented tuning process. In this paper are presented two approaches for synthesis the proportional-integral-derivative controller to the models of objects with inertia, that offer the procedure of system performance optimization based on maximum stability degree criterion. The proposed algorithms of system performance optimization were elaborated for model of objects with inertia second and third order and offer simple analytical expressions for tuning the PID controller. Validation and verification are conducted through computer simulations using MATLAB, demonstrating successful performance optimization and showcasing the effectiveness PID controllers' tuning. The proposed approaches contribute insights to the field of control, offering a pathway for optimizing the performance of second and third-order inertial systems through robust controller synthesis.

## Keywords

PID Control Algorithm, Inertial Systems, System Performance Optimization, Maximum Stability Degree

## 1. Introduction

Proportional-integral-derivative (PID) control algorithm has gained a wide use in control of the technological processes of heavy and light industries, due to its simple structure, easy implementation and the advantages that it offers to the automatic control systems [1] [2]. PID control algorithms are sufficient for many control problems, ensuring elimination of steady-state offsets and anticipation the future change of signal.

PID controllers have a long history, with the proportional-integral-derivative

algorithm introduced by Minorsky in 1927, and the first industrial equipment being a pneumatic one produced by the Foxboro company in 1931. Contemporary PID controllers differ significantly from those functional ones produced 80 years ago. Today, they can be found in combinations with logic and sequential controllers, selectors, and other functional blocks with incorporation the principles of artificial intelligence [3] [4]. The practice of the automation demonstrates that the main challenge in the control processes is the tuning of the PID controllers, due to inadequate choice of sampling period, nonlinear behavior of the control process or actuators, or wrong estimation of the mathematical model, that approximates the dynamics of the control process.

Many methods and algorithms have been proposed and developed based on the classic PID control concept over the last 80 years, where the main problem is the problem of synthesis the PID control algorithm, which supposes calculation of the tuning parameters according to the dynamics of the control process. Solving this problem is related to various factors, such as the operating principles of industrial processes, types of exogenous signals acting on the controlled process, technical characteristics, and more [5] [6].

The incorrect tuning of the PID controller can lead to the bad performance of the automatic control system and in the worst case can lead to the instability of the system [7] [8] [9] [10]. In this case the ensuring the closed loop system stability is one of the most important aspect in the synthesis of the control algorithm. Another aspect is related with ensuring the high performance to the automatic control system, which supposes the fast response, small overshoot, no oscillation and the existence of the procedure that permits to vary the performance of the system, which will offer the benefits and flexibility in the tuning of the controller [11] [12] [13].

For the last decades the artificial intelligence (AI) becomes to play an important role in the domain of control systems, namely the tuning the PID controllers for the case of system performance optimization, rejection of disturbances, control of nonlinear and complex processes. AI in control systems harnesses the power of real-time decision making, adaptability, and optimization, resulting in substantial enhancements in performance, cost reduction, and safety improvements across a diverse spectrum of applications. This transformative potential is realized by combining traditional AI techniques with cutting-edge metaheuristic algorithms. Evolutionary algorithms (EAs) are a subset of meta-heuristic algorithms that are particularly inspired by the process of natural selection and evolution. They are used to find solutions to complex problems by mimicking the principles of biological evolution. In the context of control systems, they can be used in various ways to optimize the system performance. Although, this strategy has drawbacks as: lack of guaranteed optimality or slow convergence to the optimal solutions, hardly can be used as auto-tuning methods, requires a significant amount of computational resources [14].

This paper presents an approach for synthesis a PID controller for models of

objects with inertia, offering the possibility of optimization the system performance in terms of settling time and overshoot. The main contributions of research are as follows:

1) The analytical expressions for synthesizing the PID controller to the model of object with inertia second order, in dependency of the model object parameters and imposed value of settling time, offering the high stability degree of the system.

2) The methodology for synthesis the PID control algorithm to the model of object with inertia third order, which offers the possibility of variation the system overshoot.

3) By the computer simulation was verified the tuning algorithms to the models of object with inertia second and third order.

The rest of this paper is organized as following: Section 2 presents algorithm for Synthesis the PID controller with procedure of system performance optimization. Simulation results are provided in Section 3, and the conclusions are presented in Section 4.

## 2. Synthesis of PID Control Algorithm Based on System Performance Optimization Procedure

### 2.1. Synthesis of the Control Algorithm to the Second-Order Inertial Systems

In **Figure 1**, it is presented the structural scheme of the automatic control system, where  $H_{PID}(s)$  is transfer function of the PID controller. The typical structure of the PID controller is described by the following transfer function [15]:

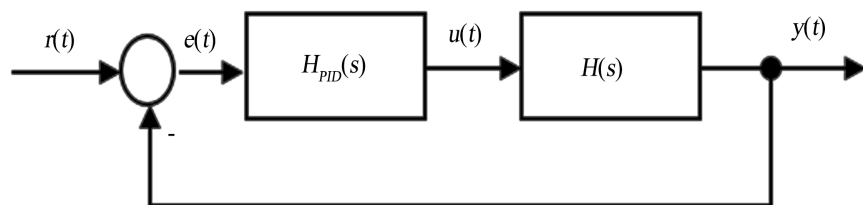
$$H_{PID}(s) = k_p + \frac{k_i}{s} + k_d s, \quad (1)$$

where  $k_p$ —is the proportional tuning parameter,  $k_i$ —integral tuning parameter,  $k_d$ —derivative tuning parameter of the PID controller [3] [15].

The control object is described by the following transfer function with inertia:

$$H(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)} = \frac{k}{a_0 s^2 + a_1 s + a_2}, \quad (2)$$

where  $k$  is the transfer coefficient of the control object,  $T_1$ ,  $T_2$ —time constants;  $a_0$ ,  $a_1$ ,  $a_2$ —parameters of the control object.



**Figure 1.** Structural scheme of the automatic control system.

The characteristic equation of the closed-loop control system is:

$$A(s) = \frac{1}{k} (a_0 s^3 + a_1 s^2 + a_2 s) + k_d s^2 + k_p s + k_i \quad (3)$$

One of the method, that is used for tuning PID controllers is maximum stability degree method with iterations (MSDI) [16]. This method offers the analytical expressions for calculation the tuning parameters that offers the maximum displacement in the complex half-plane of the nearest characteristic equation's roots of the designed system to the imaginary axe  $Re p_i \leq 0$ .

In the paper [17], it was proposed the modified maximum stability degree method, which offers the simple analytical expressions for calculation the tuning parameters as:

$$k_p = \frac{a_1}{a_0} k_d = \frac{a_1^2}{2ka_0} = \frac{(T_1 + T_2)^2}{2kT_1T_2}, \quad (4)$$

$$k_i = \frac{a_2}{a_0} k_d = \frac{a_1}{2ka_0} = \frac{T_1 + T_2}{2kT_1T_2}, \quad (5)$$

$$k_d = \frac{a_1}{2k} = \frac{T_1 + T_2}{2k}. \quad (6)$$

In the work [17], was proposed the expression for calculation the maximum stability degree of the system for the case when number of the tuning parameters is equal or less then the characteristic equation order:

$$J = \frac{a_1}{2a_0}. \quad (7)$$

Based on Equation (7) for calculation the value the stability degree, Equations (4) - (6) can be rewritten as

$$k_p = \frac{a_1}{k} \cdot J \quad (8)$$

$$k_i = \frac{1}{k} \cdot J, \quad (9)$$

$$k_d = \frac{a_0}{k} \cdot J. \quad (10)$$

The approximately dependency between stability degree of the system  $J$  and the settling time of the automatic control system  $t_s$  is presented by the following relationship [15] [18]:

$$t_s \approx \frac{1}{J} \ln \frac{1}{\varepsilon_{st}}, \quad (11)$$

where  $\varepsilon_{st}$  is steady state error of the system.

For the case, when steady state error will be  $\varepsilon_{st} = 0.02$ , the settling time is equal with:

$$t_s \approx \frac{4}{J}. \quad (12)$$

In this way, to the system can be imposed the value of the settling time, and

knowing this value from Equation (12), it can be recalculated the value of stability degree:

$$J \approx \frac{4}{t_s}.$$

Equations (8) - (10) can be rewritten as:

$$k_p = \frac{4 \cdot a_1}{k \cdot t_s} \quad (13)$$

$$k_i = \frac{4}{k \cdot t_s}, \quad (14)$$

$$k_d = \frac{4 \cdot a_0}{k \cdot t_s}. \quad (15)$$

According to the (13) - (15) expressions the tuning parameters depend on model object parameters and imposed value of settling time:

$$k_p, k_i, k_d = f(a_0, a_1, a_2, k, t_s). \quad (16)$$

## 2.2. Synthesis of the Control Algorithm to the Third-Order Inertial Systems

It is considered, that control object is described by the following transfer function:

$$H(s) = \frac{k}{(T_1s+1)(T_2s+1)\cdots(T_ns+1)} = \frac{k}{a_0s^n + a_1s^{n-1} + \cdots + a_{n-1}s + a_n} = \frac{B(s)}{A(s)}, \quad (17)$$

where  $T_1, T_2, \dots, T_n$  are time constants;  $a_0, a_1, \dots, a_{n-1}, a_n$  are the parameters of the characteristic equation;  $k$  is the transfer coefficient of the control object;  $n$  is the order of the characteristic equation  $A(s)$ .

According to the transfer function of the PID controller (1) and transfer function of the control object (17), the characteristic equation of the closed loop system with PID controller is following:

$$A(s) = \frac{1}{k}(a_0s^{n+1} + a_1s^n + \cdots + a_{n-1}s^2 + a_n s) + k_d s^2 + k_p s + k_i. \quad (18)$$

It is considered that the characteristic equation has among the roots a pair of complex dominant roots, in this way it is proposed into characteristic Equation (18) to be done the substitution  $s = -J + j\omega$ , where  $J$  is stability degree of the system and  $\omega$  is imaginary part of the dominant complex root. In this way the characteristic Equation (18) will become:

$$A(-J + j\omega) = \frac{1}{k}(a_0(-J + j\omega)^{n+1} + a_1(-J + j\omega)^n + \cdots + a_{n-1}(-J + j\omega)^2 + a_n(-J + j\omega)) + k_d(-J + j\omega)^2 + k_p(-J + j\omega) + k_i = 0. \quad (19)$$

In conformity with maximum stability degree method [19], Equation (19) derives two times and there are obtained the analytical expressions for calculation the tuning parameters of PID controller:

$$k_p = \frac{1}{k} \left( (-1)^{n+1} (n+1) a_0 (-J + j\omega)^n + (-1)^n n a_1 (-J + j\omega)^{n-1} + \dots \right. \\ \left. + 2a_{n-1} (-J + j\omega) - a_n \right) + 2k_d (-J + j\omega); \quad (20)$$

$$k_i = \frac{1}{k} \left( (-1)^n a_0 (-J + j\omega)^{n+1} - (-1)^n a_1 (-J + j\omega)^n + \dots - a_{n-1} (-J + j\omega)^2 \right. \\ \left. + a_n (-J + j\omega) \right) - k_d (-J + j\omega)^2 + k_p (-J + j\omega); \quad (21)$$

$$k_d = \frac{1}{2k} \left( (-1)^n n(n+1) a_0 (-J + j\omega)^{n-1} \right. \\ \left. - (-1)^n n(n-1) a_1 (-J + j\omega)^{n-2} + \dots - 2a_{n-1} \right). \quad (22)$$

From Equations (20) - (22), it can be observed that the tuning parameters of the PID controller depend on the values of the control object's parameters and the value of the real and imaginary part of the dominant complex roots:

$$k_p, k_i, k_d = f(a_0, a_1, \dots, a_n, k, J, \omega).$$

It is considered the case when control object is described by the transfer function with inertia third order:

$$H(s) = \frac{k}{(T_1s+1)(T_2s+1)(T_3s+1)} = \frac{k}{a_0s^3 + a_1s^2 + a_2s + a_3} = \frac{B(s)}{A(s)}, \quad (23)$$

where  $T_1, T_2, T_3$  are time constants;  $k$  is transfer coefficient of the system;

$$a_0 = T_1T_2T_3, \quad a_1 = T_1T_2 + T_1T_3 + T_2T_3, \quad a_2 = T_1 + T_2 + T_3, \quad a_3 = 1.$$

It is known from [20] that the value of maximum stability degree of the system is:

$$J = \frac{a_1}{4a_0}. \quad (24)$$

And from [20], it is known that tuning parameters of the PID controller can be calculated based on the following relationships:

$$\begin{cases} k_p = \frac{a_1 - a_3}{2a_0} k_d; \\ k_i = \frac{2a_3}{a_1 + a_3} k_d; \\ k_d = \frac{3a_1^2 - 8a_0a_2}{8ka_0}. \end{cases} \quad (25)$$

Based on Equations (22), (24) and (25), the expression for calculation the  $k_d$ -tuning parameter of the derivative component in case of tuning the PID controller to the model of object with inertia third order (23) can be presented in the following form:

$$k_d = \frac{3a_1^2 - 8a_0a_2}{8ka_0} + 12a_0\omega^2. \quad (26)$$

In this way, the tuning parameters of the PID controller for the case then control object is described by the transfer function with inertia third order are the following:

$$\begin{cases} k_p = \frac{a_1 - a_3}{2a_0} k_d; \\ k_i = \frac{2a_3}{a_1 + a_3} k_d; \\ k_d = \frac{3a_1^2 - 8a_0a_2}{8ka_0} + 12a_0\omega^2. \end{cases} \quad (27)$$

Based on the system of Equations (27) for calculation the tuning parameters of the PID controller, it is ensured the high stability degree of the automatic control system. If the value of  $\omega$  is equal with zero, the tuning parameters are calculated based on the system of Equations (25) in case of model of object with inertia third order—(23), in this care the transient response of the automatic control system will be critically damped.

The approximately dependency between stability degree of the system  $J$  and the settling time of the automatic control system  $t_s$  is presented by the following relationship [15] [18]:

$$t_s \approx \frac{1}{J} \ln \frac{1}{\varepsilon_{st}}, \quad (28)$$

where  $\varepsilon_{st}$  is steady state error.

For the case, when steady state error will be  $\varepsilon_{st} = 0.02$ , the settling time is equal with:

$$t_s \approx \frac{4}{J}. \quad (29)$$

And according to Equations (24) and (29), the settling time can be calculated according to the following expression:

$$t_s \approx \frac{16a_0}{a_1}. \quad (30)$$

Due the fact that derivative component from Equation (27) depends on the values of the control object parameters and the value of  $\omega$ , it is possible to vary the value of overshoot, by the changing the value of  $\omega > 0$ . In this way, by the changing the  $\omega$ , it is possible to change the performance of the system, namely the rise time and overshoot, keeping the settling time unchangeable.

### 3. Applications and Computer Simulations

#### 3.1. Tuning the PID Controller to the Model of Object with Inertia Second Order with Imposed Settling Time

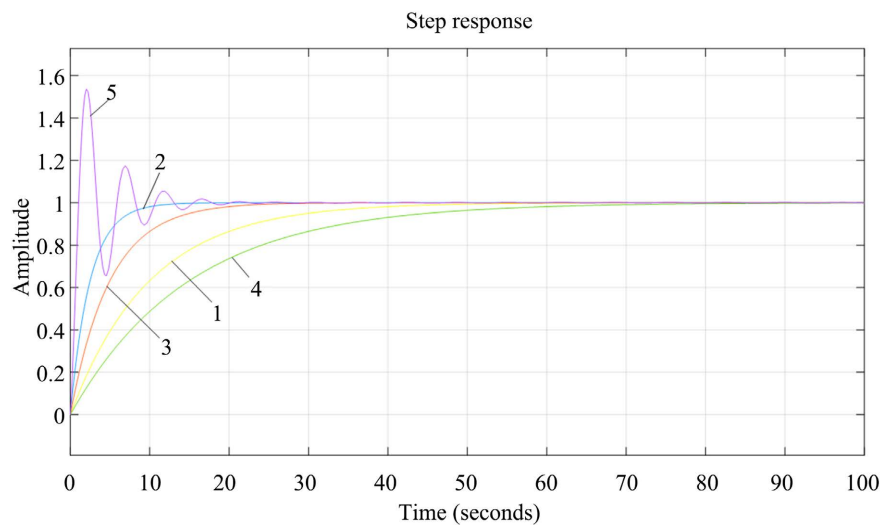
It is considered, that control object is described by the transfer function with inertia second order:

$$H(s) = \frac{1}{10s^2 + 2s + 1} = \frac{B(s)}{A(s)}. \quad (31)$$

To the model of object (31), it was tuned the PID controller based on Equations (13) - (15) for the case of imposing the value of settling time as:  $t_s = 10$  s,  $t_s$

= 20 s and  $t_s = 60$  s and the obtained tuning values are presented in **Table 1** (No. 1 - 4). For comparison of the obtained results, it was used the genetic algorithm, where the fitness function was settled according to the imposed settling time to the system  $t_s = 15$  s, the obtained results of tuning are presented in **Table 1** (No. 5). The computer simulation of the automatic control system with PID controller is presented in **Figure 2**, where the curves numbering correspond with numbering from **Table 1**.

From **Table 1** and **Figure 2**, it can be concluded that the proposed methodology of tuning the PID controller to the model of object with inertia second order (31) offers high precision in tuning controller with imposed settling time— $t_s$  and without overshoot— $\sigma$ , that was demonstrated through computer simulation (curves 1 - 4). In case of using genetic algorithm, the value of imposed settling time was satisfied, but it was obtained the oscillated transient response of the system with high overshoot (curve 5).



**Figure 2.** Transient responses of the automatic control system: 1—PID controller tuned by Equations (4) - (6); 2—PID controller tuned by Equations (13) - (15), with  $t_s = 10$  s ; 3—PID controller tuned by Equations (13) - (15), with  $t_s = 20$  s ; 4—PID controller tuned by Equations (13) - (15), with  $t_s = 60$  s ; 5—PID controller tuned by the genetic algorithm, with  $t_s = 15$  s .

**Table 1.** Tuning parameters of the PID controller and automatic system performance.

No.	Imposed settling time, s.	$k_p$	$k_i$	$k_d$	$t_s$ s.	$t_p$ s.	$\sigma$ %
1	-	0.2	0.1	1	39.143	39.143	-
2	10	0.8	0.4	4	9.86	9.86	-
3	20	0.4	0.2	2	19.68	19.68	-
4	60	0.133	0.0667	0.667	59.86	59.86	-
5	15	17.782	5.833	6.108	14.9	0.803	53.5



### 3.2. Tuning the PID Controller to the Model of Object with Inertia Third Order with Optimization the Overshoot Value

It is supposed that control object is described by the transfer function with inertia third order:

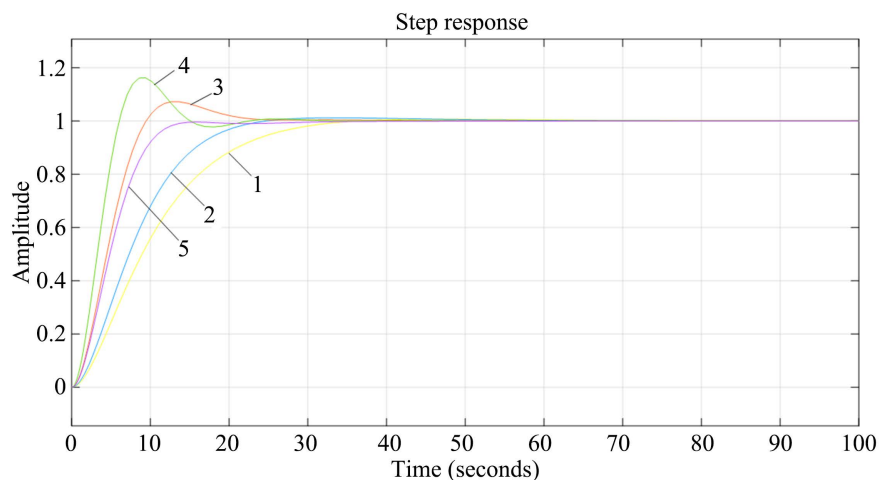
$$H(s) = \frac{1}{108.7s^3 + 80.43s^2 + 18.39s + 1} = \frac{B(s)}{A(s)}. \quad (32)$$

Next, to the model of object (32) is proposed to be tuned the PID controller based on the relationships (25):

$$\begin{cases} k_p = \frac{a_1 - a_3}{2a_0} k_d = 1.4313 \\ k_i = \frac{2a_3}{a_1 + a_3} k_d = 0.0963 \\ k_d = \frac{3a_1^2 - 8a_0a_2}{8ka_0} = 3.9193. \end{cases}$$

Next, it was proposed to be optimized the performance of the automatic control system. Due to this, it was proposed to be used relationships (27) and by varying  $\omega$ , it was possible to obtain the different values of the rising time ( $t_r$ ) and percentage of the system overshoot ( $\sigma$ ).

The obtained values of the tuning parameters of the PID controller are presented in **Table 2** and in **Figure 3**, there are presented the transient responses of the automatic control system with PID controller for different values of the  $\omega$ , where: curve 1—the PID controller with  $\omega = 0$ ; curve 2—the PID controller with  $\omega = 0.03$ ; curve 3—the PID controller with  $\omega = 0.07$ ; curve 4—the PID controller with  $\omega = 0.1$ . Curve 5 from **Figure 3** was obtained for the case of using genetic algorithm, where fitness function was designed so as to be obtained aperiodic transient response.



**Figure 3.** Transient responses of the automatic control system: 1—PID controller tuned by Equations (27), with  $\omega = 0$ ; 2—PID controller tuned by Equations (27), with  $\omega = 0.03$ ; 3—PID controller tuned by Equations (27), with  $\omega = 0.07$ ; 4—PID controller tuned by Equations (27), with  $\omega = 0.1$ ; 5—PID controller tuned by the genetic algorithm, with  $\sigma = 0$ .

**Table 2.** Tuning parameters of the PID controller and automatic system performance.

No.	$\omega$	$k_p$	$k_i$	$k_d$	$t_s$	$t_r$	$\sigma, \%$
1	0	1.43	0.096	3.91	30.5	30.5	0.0
2	0.03	1.86	0.125	5.093	21.17	21.2	1.21
3	0.07	3.76	0.253	10.31	20.03	6.25	7.37
4	0.1	6.19	0.41	16.968	18.86	4.083	16.57
5		3.157	0.184	9.703	12.5	12.5	0

For  $\omega \geq 0.12$ , the system has overshoot bigger than 20%, that it is not favorable for the performance of the automatic control systems.

#### 4. Conclusions

In this paper, two approaches for synthesis the PID controller are presented, that offer the system performance optimization.

The first approach of tuning PID controllers to the second-order inertial models, allows the imposition of a settling time on the automatic control system, ensuring that the system operates without overshoot. This method was developed based on the maximum stability degree criterion, providing the optimal stability degree for the system. The proposed method was verified through computer simulation for the case of tuning the PID controller with different imposed values of settling time and it demonstrated good results and high precision in performance ensuring. The obtained results were compared with genetic algorithm, that permitted to obtain the transient response of the system with imposed settling time, but gave high overshoot in comparison with proposed algorithm of tuning, which permits to vary the settling time without overshoot.

Another approach to tuning the PID controller allows for the variation of the percent overshoot. The obtained results were verified by computer simulation for the case of tuning the PID controller to the model of object with inertia third order, so as this method permits to control the oscillation degree of the system.

The proposed methodology for tuning PID controllers offers the possibility, through simple analytical expressions, to tune PID controllers in such a way that the system achieves a desired settling time, or degree of oscillation. By employing this methodology, engineers can easily and analytically determine the appropriate tuning parameters, providing a systematic and efficient way to achieve the desired performance in terms of settling time or oscillation degree for the controlled system.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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