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Why Is an Integral an Accurate Value?

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Abstract

The derivative and integral in calculus are both exact values. To explain this reason, the integration interval can be infinitely subdivided. The difference in area between curved trapezoids and rectangles can be explained by the theory of higher-order infinitesimal, leading to the conclusion that the difference between the two is an infinitesimal value. From this, it can be inferred that the result obtained by integration is indeed an accurate value.

Keywords

Integral, Infinitesimal, Curved Trapezoid, Accurate Value

1. Introduction

Definite integral is a type of integral, which is the limit of the sum of integrals of function $f(x)$ on the interval $[a, b]$. Here, attention should be paid to the relationship between definite integral and indefinite integral: If definite integral exists, it is a specific numerical value, while indefinite integral is a functional expression, and they only have a mathematical calculation relationship (Newton Leibniz formula). A function can have indefinite integrals, but not definite integrals; There can also be definite integrals, without indefinite integrals. A continuous function must have definite and indefinite integrals; If there are only a finite number of breakpoints, then the definite integral exists; If there is a breakpoint between jumps, then the original function must not exist, that is, the indefinite integral must not exist [1] [2] [3].

The concept of definite integral originated from finding the area of a planar figure and other practical problems. The idea of definite integral has already sprouted in the work of ancient mathematicians. For example, in ancient Greece, Archimedes used the method of summation to calculate the area of parabolic arches and other shapes around 240 BC. In 263 AD, Liu Hui of China proposed the same idea of circumcision. In history, the concept of integration was formed earlier than differentiation. However, until the emergence of the work of Newton and Leibniz (the second half of the 17th century), various results related to definite integrals were still isolated and scattered, and a relatively complete theory of definite integrals had not yet been formed. It was not until the establishment of the Newton Leibniz formula that computational problems were solved and definite integrals were rapidly established and developed.

The microelement method refers to a method of solving problems by analyzing the very small parts (microelements) of things, in order to achieve the overall goal of solving things. It is commonly used in solving physics problems, with the idea of "breaking the whole into parts". First, analyze the "microelements", and then analyze the whole through the "microelements".

As a beginner in calculus, there is always the question of neglecting the area of some triangles when calculating definite integrals. Why is the result of integration not an approximation but an exact value? This article provides a preliminary explanation of this issue.

1.1. The Calculation Method of Riemann Integral

When using the microelement method to solve problems, it is necessary to decompose them into numerous small "meta processes", and each "meta process" follows the same rules. Therefore, we only need to analyze these "meta processes", and then apply necessary mathematical or physical methods to the "meta processes" to solve the problem. Using this method will strengthen our rethinking of known laws, thereby leading to the consolidation of knowledge, deepening of understanding, and improvement of abilities.

From Figure 1, it can be seen that integral is actually finding the sum of infinitesimals. According to the methods in general textbooks, the calculation of integrals is divided into four steps:

1) Segmentation; 2) Approximation; 3) Summation; 4) Take the limit.

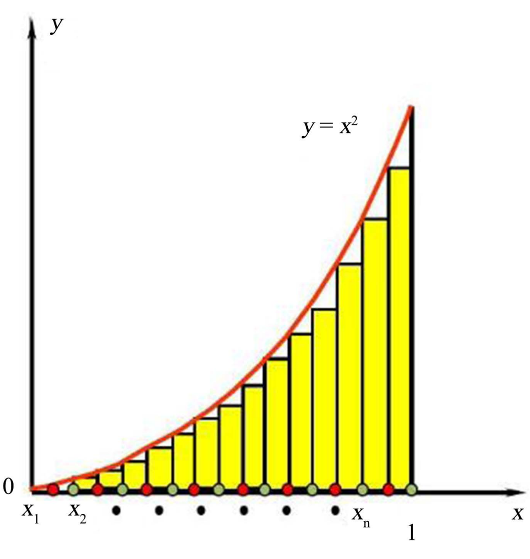

Figure 1. Integral.

1.2. Explanation of Why Integrals Are Exact Values

The above figure is an explanation of integral segmentation. Move the curved trapezoid of point x below:

The above figure is an explanation of integral segmentation [4] [5] [6] [7]. **Figure 2** is an enlarged version of the curved trapezoid located at point x in **Figure 1**: assuming point c is any adjacent point to point x, and Δx is infinitesimal. Due to the fact that there is always a difference between any two different numbers on the number axis, the number axis can be seen as composed of multiple points with intervals. The point d corresponding to point c in the figure is any adjacent point to point $f(x)$. According to the definition of infinitesimal, Δx is already too small to be represented by any number. To express the idea that Δx is not a number, point c and position $x + \Delta x$ in the figure above are intentionally disconnected, indicating that point c is the number closest to point x. Note that point c can be arbitrary. Similarly, point d corresponding to point c adopts the same method, representing points $f(x)$ to $f(x + \Delta x)$ on the curve, that arc between them is not a number either. Because $\Delta Y = f(x + \Delta x) - f(x)$ is also an infinitesimal, the difference between the area of a curved trapezoid and a rectangle is one triangle's area: $1/2\Delta x\Delta y$. This area is a high-order infinitesimal. Rectangular area $f(x)$ Δx is an infinitesimal. As long as it is infinitesimal or higher-order infinitesimal, it cannot be represented by numbers. For example, 1 + ΔX can only be equal to 1 because Δx is not a number, so $1 + \Delta x = 1$ is not an approximation, but an exact equality.

According to **Figure 1**, the sum of infinitesimals may not still be infinitesimal but may become a number. This result indicates that there is a gap between infinitesimals and a specific number, which requires infinitesimals to cross. Similarly, there is also a gap between high-order infinitesimals and infinitesimals. Infinite high-order infinitesimals may not still be high-order infinitesimals, but they can at most become infinitesimals, but infinitesimals are not yet a number.

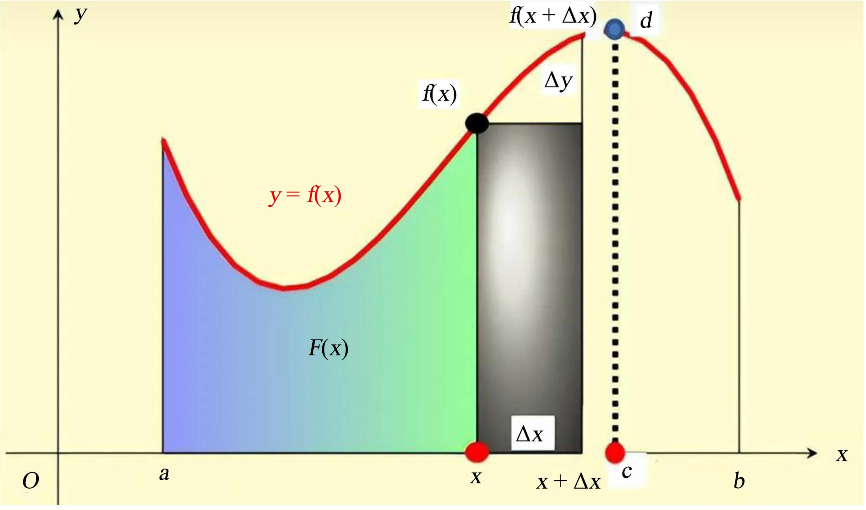

Figure 2. Integral of one curved trapezoid.

2. Conclusion

Based on the above analysis, the sum of infinitely many triangle areas omitted between the curved trapezoidal area and the trapezoidal area in **Figure 2** is the higher-order infinitesimal, which is equivalent to omitting at most one infinitesimal. Therefore, it does not affect the conclusion that the integral value is an accurate value.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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