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Type 2 Possibility Factor Rotation in No-Data Problem

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Abstract

Uemura [1] discovered a mapping formula that transforms and maps the state of nature into fuzzy events with a membership function that expresses the degree of attribution. In decision theory in no-data problems, sequential Bayesian inference is an example of this mapping formula, and Hori *et al.* [2] made the mapping formula multidimensional, introduced the concept of time, to Markov (decision) processes in fuzzy events under ergodic conditions, and derived stochastic differential equations in fuzzy events, although in reverse. In this paper, we focus on type 2 fuzzy. First, assuming that Type 2 Fuzzy Events are transformed and mapped onto the state of nature by a quadratic mapping formula that simultaneously considers longitudinal and transverse ambiguity, the joint stochastic differential equation representing these two ambiguities can be applied to possibility principal factor analysis if the weights of the equations are orthogonal. This indicates that the type 2 fuzzy is a two-dimensional possibility multivariate error model with longitudinal and transverse directions. Also, when the weights are oblique, it is a general possibility oblique factor analysis. Therefore, an example of type 2 fuzzy system theory is the possibility factor analysis. Furthermore, we show the initial and stopping condition on possibility factor rotation, on the base of possibility theory.

Keywords

Type 2 Fuzzy Events, Quadratic Mapping Formula, Stochastic Differential Equation in Fuzzy Event, Possibility Principal Factor Analysis, Possibility Oblique Factor Analysis, Initial and Stopping Condition

1. Introduction

Okuda *et al.* [3] constructed the decision rule under the fuzzy environment; however, this is an example of Bayes Decision Rule. Otherwise, Uemura [1] [4] [5] and Hori *et al.* [2] constructed another decision making on vague events. This decision-making is a special case for Bayes Decision Theory in No Data problem. In this paper, we obtain a mention of the system theory to possibility factor rotation according to the type 2 vague events.

Uemura [1] found a mapping formula that maps and transforms the state of nature in no information problem, (a no-data problem), in which no observable information can be observed in Bayesian statistics, to a fuzzy event by a membership function representing its degree of attribution. Note that the no-data problem can be attributed to Bayesian statistics, where the causality law between the state of nature and the observed information is uniform. Now, the fuzzy in this paper is sometimes called Vague to distinguish it from the Fuzziness of Zadeh [6]. Therefore, the extension of our study is named Vague Sets and Theory (Hori, Takemura, and Matsumoto [2]). Zadeh's fuzzy deals with vertical ambiguity, e.g., possibility interval type regression modeling, while our Vague deals with horizontal ambiguity, e.g., α-level cut of fuzzy sets. Also, Zadeh's modeling is conceptually very close to the interval-type modeling of subjective Bayesian theory, and the rotation based on our quadratic mapping formula is very relevant for factor analysis or independent component analysis (Hori [7]). First, Uemura [1] defined a mapping function from the state of nature for fuzzy events. Next, Hori *et al.* [2] showed that this definition is a formula. When the formula for the mapping function for fuzzy events is adapted to the theory of utility functions and developed into a decision-making method based on the utility function in fuzzy events, it can be applied to the case of the two-choice question (Uemura [1]). This is because a nondiscriminatory state in decision withholding arises, and Hori *et al.* [2] imposed an ergodic condition between the previous and next nondiscriminatory states and developed it into a Markov (decision) process in fuzzy events and derived a stochastic differential equation of it in fuzzy events. In this paper, Type 2 Fuzzy Events in which both horizontal and vertical ambiguity are considered at the same time, are discussed. Here, the quadratic mapping formula transforms a non-mapping function by relating it to two mapping functions and provides an orthogonal rotation to the function after the quadratic transformation. For Type 2 Fuzzy Events, orthogonal rotations from 0 to 180 degrees can be interpreted as cases where the longitudinal possibility error model and the transverse necessity error model are considered. The orthogonal rotation from 180 to 360 degrees can be interpreted as the case where the longitudinal necessity error model and the transverse possibility error model are taken into account. Here, what is measured by the possibility and necessity measures can be regarded as a kind of information content, and the possibility main factor rotation is provided to increase this information content. Note that Type 2 Fuzzy Events are attributed to a multidimensional possibility multivariable error model that takes into account the possibility and necessity of the longitudinal variable error model and the transverse variable error model. Furthermore, we show the initial and stopping conditions on possibility general factor rotation.

2. Mapping Formulas in Fuzzy Events

Uemura [1] defined the formula for mapping a function $f(x)$ by $g_1(x)$ as a formula (1). However, the definition is provided as a system theory, while as an example of application, a two-choice question is provided with respect to determinism. Later, it is shown that this definition is a formula (Hori, Takemura, and Matsumoto [2])

$$\mathrm{SUP}_{y=f(x)} g_1(x) = g_1\left(f^{-1}(y)\right) \tag{1}$$

The stochastic differential equation for a fuzzy event that represents the transition of the nondiscriminatory state regarding the decision withholding in the two-choice question based on the state of nature in sequential Bayesian inference is formulated as in Equation (2), and the Markov process in the fuzzy event is obtained as its solution as in Equation (3). The pole of the *S*-Markov process in the fuzzy event is the mapping formula for the fuzzy event in Equation (1). Here, ergodic conditions between each natural state are assumed. In addition, the monotonicity of the function f is a condition because it requires the existence of the first rank of the inverse function f^{-1} (Hori, Takemura, and Matsumoto [2]).

$$\frac{\mathrm{d}F}{\mathrm{d}t} = b\left(t, f_t^{-1}(y_t)\right) + \sigma\left(t, f_t^{-1}(y_t)\right) \cdot W_t \tag{2}$$

It is assumed that b is the mean term, σ the variance term, and W the error term in the equation of state for normal events.

$$F_t = L^{-1}\left(t, g_1\left(f^{-1}(y_t)\right)\right) \tag{3}$$

Here, L is the transition matrix of the Markov process of normal events.

Although this formula is subject to strict condition between Ergodic Conditions and monotonicity conditions of the function f, in natural states, it is able to be applied to the Go-Reserved Judgement Problem in the no-data problem, and is applicable to fuzzy stochastic differential equations for the transition of nondiscriminatory states concerning decision withholding in sequential Bayesian inference.

3. A Simultaneous Stochastic Differential Equation for Type 2 Fuzzy Events

Hori [7] [8] formulated the quadratic mapping formula as in Equation (4).

$$\mathrm{SUP}_{\substack{y=f(x) \\ Z=g_1\left(f^{-1}(y)\right)}} g_2(Z) = g_2\left(g_1^{-1}\left(f^{-1}(y)\right)\right) \tag{4}$$

Here, Equation (4) is a quadratic mapping formula that maps Equation (1), once again, by $g_2(x)$. The special property of the quadratic mapping formula is that it inverts 180 degrees when the mapped functions are equivalent, as in Equation (5). This indicates that this is a type of principal factor analysis. In statistical principal factor analysis, a 180-degree rotation requires two rotations of every 90 degrees. However, note that the quadratic mapping formula reverses 180 degrees in one rotation.

$$\text{if } g_1(\cdot) = g_2(\cdot) \text{ then } x = f^{-1}(y) \tag{5}$$

In this paper, the concept of time t into this quadratic mapping formula and derive a type 2 Markov process and its simultaneous fuzzy stochastic differential equation, albeit inverse.

First, if the transition matrix is L, the Markov process D_t is formulated as follows. (Takahashi [9])

$$D_t = L(t, x_t) \tag{6}$$

The type 1 fuzzy Markov process, which introduces the concept of fuzzy events, is derived in Equation (7), and the type 2 fuzzy Markov process is derived in Equation (8).

$$F_t = L^{-1}(t, g_1(x_t)) \tag{7}$$

$$\begin{aligned} F_{F_t} &= \underset{y_t = L^{-1}(t, g_1(f(x_t)))}{\mathrm{SUP}} L^{-1}(t, g_2(x_t)) \\ &= L^{-1}\left(t, L^{-1}\left(t, g_2\left(g_1^{-1}\left(f^{-1}(y_t)\right)\right)\right)\right) \end{aligned} \tag{8}$$

Here, the Markov process in Equation (8) is derived from the following simultaneous fuzzy stochastic differential equations.

Equation (9) represents the change in the x-axis direction, and Equation (10) represents the change in the y-axis direction. x_t is a fuzzy variable in the horizontal direction and follows the fuzzy stochastic differential equation in Equation (9), and Z_t is a fuzzy variable in the vertical direction and follows the fuzzy stochastic differential equation in Equation (11).

$$\begin{cases} \dfrac{\mathrm{d}Z}{\mathrm{d}t} = m_1\left(t, g_{1t}\left(f_t^{-1}(Z_t)\right)\right) + \sigma_1\left(t, g_{1t}\left(f_t^{-1}(Z_t)\right)\right) \cdot W_{1t} & (9) \\ \dfrac{\mathrm{d}x}{\mathrm{d}t} = m_2\left(t, g_{2t}\left(f_t(x_t)\right)\right) + \sigma_2\left(t, g_{2t}\left(f_t(x_t)\right)\right) \cdot W_{2t} & (10) \end{cases}$$

Here, $Z_t = f_t(x_t)$, so Equation (10) is equivalent to Equation (11).

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = m_2\left(t, g_{2t}(Z_t)\right) + \sigma_2\left(t, g_{2t}(Z_t)\right) \cdot W_{2t} \tag{11}$$

In the simultaneous fuzzy stochastic differential Equations (9) and (10), when the sum of the weights of each equation is 1, the type 2 Markov process of Equation (8) is derived. This means that the fuzzy event is a direct sum, which is closely related to the main factor analysis in Section 5.

4. Type 2 Possibility Principal Factor Rotation

Type 2 Fuzzy Events simultaneously encompass a two-dimensional necessity variable error model that considers longitudinal and transverse possibility errors. The 180-degree orthogonal rotation is the case of Equation (5), where possibility theory is applied to these possibility variable error models. Note that since both longitudinal and transverse fuzzy variables are considered, possibility theory is able to be applied. In this paper, particular attention to the measure of the size relationship of the fuzzy set is paid. Here, the possibility measure (POS) and the necessity measure (NES) are defined as followed (D. Dubois and H. Parade [10]). In addition, M and N are assumed to be Orthogonal Fuzzy Events with orthogonal degrees of attribution.

$$\mathrm{POS}(M \geq N) \triangleq \underset{U \geq V}{\mathrm{SUP}} \min\left(\mu_M(U), \mu_N(V)\right) \tag{12}$$

$$\mathrm{POS}(M > N) \triangleq \underset{U}{\mathrm{SUP}} \inf_{V \geq U} \min\left(\mu_M(U), \mu_N(V)\right) \tag{13}$$

$$\mathrm{NES}(M \geq N) \triangleq \inf_{U} \underset{V \leq U}{\mathrm{SUP}} \max\left(1-\mu_M(U), \mu_N(V)\right) \tag{14}$$

$$\mathrm{NES}(M > N) \triangleq 1-\underset{U \geq V}{\mathrm{SUP}} \min\left(\mu_M(U), \mu_N(V)\right) \tag{15}$$

The possibility principal factor rotation matrix for type 2 fuzzy is as follows:

$$\begin{bmatrix} x_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} \mathrm{POS}(M \geq N) & \mathrm{NES}(M > N) \\ \mathrm{NES}(M > N) & \mathrm{POS}(M \geq N) \end{bmatrix} \begin{bmatrix} x_t \\ Z_t \end{bmatrix} \tag{16}$$

In particular, note that in (16), when the possibility measure is 1, it is the identity matrix, and when the necessity measure is 1, it is the inversion matrix. Therefore, the possibility main factor rotation matrix in Equation (16) indicates that the sum of the weights of Equation (9) and (10) in the simultaneous fuzzy stochastic differential equation is 1 (Hori [7] [11]).

5. Initial and Stopping Condition in Type 2 Possibility Principal Factor Rotation

The initial condition and stopping condition for a normal Markov process are shown in [12]. Since we deal with horizontal ambiguity, we introduce the concept of quadratic possibility theory to the rotation according to a complex Markov process. The initial and stopping condition are shown in Equation (17), (18) and (19), (20), respectively. Where the rotation can start from (18) satisfying the initial condition (17). And the rotation can stop under (20) satisfying the stopping condition (19).

1) $\left(F_{10t}, F_{20t}\right) = \left(Z_{10t}, Z_{20t}\right)$ (17)

2) $\mathrm{POS}\left(\left(F_{1t}, F_{2t}\right) \geq \left(Z_{1t}, Z_{2t}\right) \mid x_0\right) \leq \mathrm{POS}\left(\left(DF_{1t}x_0, DF_{2t}x_0\right) \geq \left(DZ_{1t}x_0, DZ_{2t}x_0\right)\right)$

3) $\mathrm{NES}\left(\left(F_{1t}, F_{2t}\right) \geq \left(Z_{1t}, Z_{2t}\right) \mid x_0\right) \geq \mathrm{NES}\left(\left(DF_{1t}x_0, DF_{2t}x_0\right) \geq \left(DZ_{1t}x_0, DZ_{2t}x_0\right)\right)$

Where $\left(DF_{1t}x_0, DF_{2t}x_0\right) = \left(DX_{1t}, DZ_{2t}\right)$

(Starting State) $F_{10}(\cdot)_t = F_{20}(\cdot)_t$ (18)

1) $F_{ti0} = Z_{ti0}\ (i = 1,2)$

2) $\mathrm{POS}\left(F_{it} \geq Z_{it} \mid x_{i0}\right) \leq \mathrm{POS}\left(DF_{xi0} \geq DZ_{xi0}\right)(i = 1,2)$

3) $\mathrm{NES}\left(F_{it} \geq Z_{it} \mid x_{i0}\right) \geq \mathrm{NES}\left(DFx_{i0} \geq DZx_{i0}\right)(i = 1,2)$

Where $DFx_{i0} = DZx_{i0}\ (i = 1,2)$

Where F_{i0t} and $DF_{i0t}\ (i = 1,2)$ represents 2 complex events, and the quadratic possibility theory is applied. If the mapping function is equivalent, they invert 180-degree, and the initial condition and stopping condition is reversed. Note that the complex event become also one in a simulation like this.

1) $\left(F_{1t0}, F_{2t0}\right)=\left(Z_{1t0}, Z_{2t0}\right)$ (19)

2) $\mathrm{POS}\left(\left(F_{1t0}, F_{2t0}\right) \geq\left(Z_{1t}, Z_{2t}\right)\right) \leq \mathrm{POS}\left(\left(DF_{10}, DF_{20}\right) \geq\left(DZ_{10}, DZ_{20}\right)\right)$

3) $\mathrm{NES}\left(\left(F_{1t}, F_{2t}\right) \geq\left(Z_{1t}, Z_{2t}\right)\right) \geq \mathrm{NES}\left(\left(DF_{10}, DF_{20}\right) \geq\left(DZ_{10}, DZ_{20}\right)\right)$

Where $\left(DF_{10}, DF_{20}\right)=\left(DZ_{10}, DZ_{20}\right)$

(Stopping State) $F_{10}(\cdot)_t=F_{20}(\cdot)_t$ (20)

1) $F_{ti0}=Z_{ti0}\ (i=1,2)$

2) $\mathrm{POS}\left(F_{it} \geq Z_{it}\right) \leq \mathrm{POS}\left(DF_{i0} \geq DZ_{i0}\right)(i=1,2)$

3) $\mathrm{NES}\left(F_{it} \geq Z_{it}\right) \geq \mathrm{NES}\left(DF_{i0} \geq DZ_{i0}\right)(i=1,2)$

Where $DF_{i0}=DZ_{i0}$

6. Type 2 Possibility Oblique Factor Rotation

Assume that the fuzzy variables in the *x*-axis direction and the fuzzy variables in the *y*-axis direction are transformed into fuzzy events *N* and *M* on the natural state *S* by the membership functions $\mu_N(S)$ and $\mu_M(S)$ Note that the membership functions $\mu_N(S)$ and $\mu_M(S)$ are derived by the quadratic mapping formula as follows

$$\mu_N(S)=\Pi_N\left(\Pi_M^{-1}\left(f_1^{-1}(x)\right)\right) \tag{21}$$

$$\mu_M(S)=\Pi_M\left(\Pi_N^{-1}\left(f_2^{-1}(y)\right)\right) \tag{22}$$

Π_N and Π_M are the prior possibility distributions of the fuzzy variables *N* along the *x*-axis and *M* along the *y*-axis. The system functions are $x=f_1(S)$ and $y=f_2(S)$, respectively.

After the transformation, fuzzy event *N* and fuzzy event *M* that are not direct sums, as shown in the image in **Figure 1** (Uemura [4]), are discussed. Note that the sum of the membership functions representing the degree of attribution of fuzzy event *N* and fuzzy event *M* to the state of nature is less than or equal to 1. Therefore, the nondiscriminatory event F_e is automatically derived as follows. In decision theory, when the sum of the membership functions is less than or equal to 1, it is better to use the probability of the fuzzy event, and when the sum is greater than 1, it is better to use the probability measure of the fuzzy event (Uemura [13]).

Suppose that the possibility distribution $\Pi_{FK}(K=1,\cdots,n)$ of two or more non-orthogonal Fuzzy Events is pre-set by Equation (21) and (22).

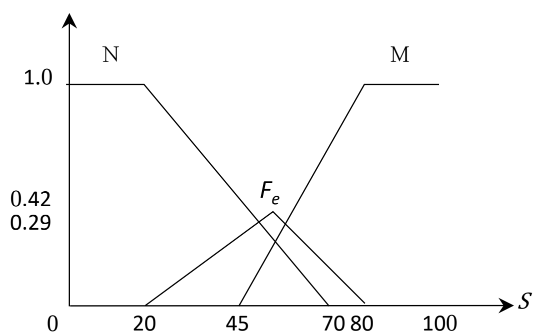

Figure 1. Indifferent event.

In this section, we consider the general case where $\sum_{k=1}^{n}\Pi_{Fk}(S)\leq 1 \quad \forall_S \in S$. Here, we introduce the concept of Indifferent Event F_e in order to avoid the risk of decision-making arising from the lack of information in Fuzzy Events. The possibility distribution of this Indifferent Event can be automatically derived by the following equation.

$$\Pi_{F_e}(S)=1-\sum_k \Pi_{Fk}(S) \tag{23}$$

In this paper, we pick up the fuzzy variables in the *x*-axis direction and the fuzzy variables in the *y*-axis direction. Note that we consider only two fuzzy events such that $N = F_1$ and $M = F_2$.

The Indifferent Event F_e is divided into zones of the state of nature to make sense of it. In Zone $X=\{0\leq s<20\}$, it is completely *N*, In Zone $a=\{20\leq s<45\}$, it is a conditional Indifferent Event known to be a fuzzy event *N* In Zone $b=\{45\leq s<70\}$, it is an Indifferent Event that is neither fuzzy event *N* nor fuzzy even *M*. However, the relationship between the magnitude of fuzzy event *N*. and fuzzy event *M*. Zone $c=\{70\leq s<80\}$ is a conditional Indifferent Event that is known to be fuzzy event *M*. Zone $Y=\{80\leq s\leq 100\}$ is completely *M*. Here, each zone has different characteristics, so it is necessary to analyze each zone individually. However, decomposing and recomposing the system is very risky. In this decision problem, *N*, *M* and F_e are orthogonal sum events, so there is no need to decompose and recompose the system.

The fuzzy event *N* is derived from the stochastic differential Equation (9) along the *x*-axis, while the fuzzy event *M* is derived from the stochastic differential Equation (10) along the *y*-axis. With respect to the weights W_1 and W_2 of those simultaneous stochastic differential equations, if the two fuzzy events are in direct sum, that is, if the sum of the weights of each differential Equation (9) and (10) is orthogonal ($W_1 + W_2 = 1$), then it is a possible principal factor analysis (Hori [7] [11]). On the other hand, if the sum of the weights is less than 1 ($W_1 + W_2 < 1$), it is a possibility oblique factor analysis. In the following, for each zone, we focus on the indicator of the fuzzy set size relationship in possibility theory and derive the possibility factor rotation matrix MM_i according to the definition of the probability measure that represents the size relationship of the fuzzy set.

1) Possibility factor rotation matrix in Zoon *X*:

$$MM_x=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{24}$$

2) Possibility factor rotation matrix in Zone *a*:

$$MM_1=\begin{bmatrix}\mathrm{POS}(N\geq F_e) & \mathrm{NES}(N>F_e)\\ \mathrm{NES}(N>F_e) & \mathrm{POS}(N\geq F_e)\end{bmatrix} \tag{25}$$

3) Possibility factor rotation matrix in Zone *b*:

$$MM_2=\begin{bmatrix}\mathrm{POS}(F_e\geq N) & \mathrm{NES}(F_e>M)\\ \mathrm{NES}(F_e>M) & \mathrm{POS}(F_e\geq N)\end{bmatrix} \tag{26}$$

4) Possibility factor rotation matrix in Zone *c*:

$$MM_3 = \begin{bmatrix} \mathrm{POS}(F_e \geq M) & \mathrm{NES}(F_e > N) \\ \mathrm{NES}(F_e > N) & \mathrm{POS}(F_e \geq M) \end{bmatrix} \tag{27}$$

5) Possibility factor rotation matrix in Zone *d*:

$$MM_4 = \begin{bmatrix} \mathrm{POS}(M \geq F_e) & \mathrm{NES}(M > F_e) \\ \mathrm{NES}(M > F_e) & \mathrm{POS}(M \geq F_e) \end{bmatrix} \tag{28}$$

6) Possibility factor rotation matrix in Zoon Y:

$$MM_y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{29}$$

The information content I_i in each zone is given by Equation (30). The final possibility oblique factor rotation MM is the weighted sum of the information content in each zone, the possibility factor matrix elements in each zone, and Equation (31).

$$I_i = \max_S \mu_{F_e i}(S) \times \log \mu_{F_e i}(S) \ (i = 1,2,3,4) \tag{30}$$

$$MM = \sum_{i=1}^{4} MM_i \times I_i \tag{31}$$

7. Initial and Stopping Condition on Type 2 Possibility Oblique Factor Rotation

In this section, we consider the possibility oblique factor rotation after the initial observation X_0. At first, in zone X and zone Y, the initial and stopping condition can be derived from the normal Markov process (Takahashi [12]). Second, N and F_e is direct sum in zone *a*. Furthermore, M and F_e is direct sum in zone *d*. In this case, these rotations are as well as the possibility principal rotations. At last, the initial and stopping conditions in zone *b* and zone *c* are derived from Equation (32) and (33). Note that the type2 fuzzy event N can rotate as same as the type2 fuzzy event M, because changing M to N in Equation (32) and (33) is similar to the original version.

(Initial Condition)

1) $(M_{0t}, N_{0t}) = (Z_{10t}, Z_{20t})$ (32)

2) $I_2 \mathrm{POS}((M_t, F_{et}) \geq (Z_{1t}, Z_{2t}) \mid x_0) \leq I_3 \mathrm{POS}((M_t X_0, F_{et} X_0) \geq (M_t X_0, N_t X_0))$

3) $I_2 \mathrm{NES}((N_t, F_{et}) \geq (Z_{1t}, Z_{2t}) \mid x_0) \geq I_3 \mathrm{NES}((N_t, F_{et}) \geq (M_t X_0, N_t X_0))$

(Stopping Condition)

1) $(M_{0t}, N_{0t}) = (Z_{10t}, Z_{20t})$ (33)

2) $I_2 \mathrm{POS}((M_t, F_{et}) \geq (Z_{1t}, Z_{2t})) \geq I_3 \mathrm{POS}((M_t, F_{et}) \geq (M_t, N_t))$

3) $I_2 \mathrm{NES}((N_t, F_{et}) \geq (Z_{1t}, Z_{2t})) \leq I_3 \mathrm{NES}((N_t, F_{et}) \geq (M_t, N_t))$

The starting and stopping state are obtained in zone b as follows:

(Stating State) $M_{10}(\cdot)_t = M_{20}(\cdot)_t$ (34)

1) $M_{ti0} = F_{ei0} \ (i = 1,2)$

2) $I_2 \mathrm{POS}(M_{ti} \geq F_{eti} \mid X_0) \leq I_3 \mathrm{POS}(N_{ti0} \geq F_{eti}) \ (i = 1,2)$

3) $I_2 \mathrm{NES}(M_{ti} \geq F_{eti} \mid X_0) \geq I_3 \mathrm{NES}(N_{ti0} \geq F_{eti}) \ (i = 1,2)$

(Stopping State) $M_{10}(\cdot)_t = M_{20}(\cdot)_t$ (35)

1) $M_{ti0} = F_{ei0}\ (i = 1,2)$

2) $I_2\mathrm{POS}(M_{ti} \geq F_{eti}) \leq I_3\mathrm{POS}(N_{ti} \geq F_{eti})\ (i = 1,2)$

3) $I_2\mathrm{NES}(M_{ti} \geq F_{eti}) \geq I_3\mathrm{NES}(N_{ti} \geq F_{eti})\ (i = 1,2)$

The starting and stopping state are obtained in zone c as follows:

(Starting State) $N_{10}(\cdot)_t = N_{20}(\cdot)_t$ (36)

1) $N_{ti0} = F_{ei0}\ (i = 1,2)$

2) $I_2\mathrm{POS}(N_{ti} \geq F_{eti} \mid X_0) \leq I_3\mathrm{POS}(M_{ti0} \geq F_{eti})\ (i = 1,2)$

3) $I_2\mathrm{NES}(N_{ti} \geq F_{eti} \mid X_0) \geq I_3\mathrm{NES}(M_{ti0} \geq F_{eti})\ (i = 1,2)$

(Stopping State) $N_{10}(\cdot)_t = N_{20}(\cdot)_t$ (37)

1) $N_{ti0} = F_{ei0}\ (i = 1,2)$

2) $I_2\mathrm{POS}(N_{ti} \geq N_{eti}) \leq I_3\mathrm{POS}(M_{ti} \geq F_{eti})\ (i = 1,2)$

3) $I_2\mathrm{NES}(N_{ti} \geq N_{eti}) \geq I_3\mathrm{NES}(M_{ti} \geq F_{eti})\ (i = 1,2)$

8. Approach Forward to Type 2 Possibility Factor Analysis

In this section, we mention the decision making rule for the possibility factor analysis such that can make a decision rotating the possibility factor rotation in the decision making problem. In the every zone shown by the imaging **Figure 1**, we propose the simple decision rule after the possibility factor rotating. In Equation (38), (39) and (40), D_1 and D_2 are the decisions. And $x = U_1(S \mid D_1)$, $y = U_2(S \mid D_2)$, are utility functions. $\mu_N(S)$ is the type 2 membership function in x-axis. $\mu_M(S)$ is the type 2 membership function in y-axis. And $\pi(S)$ is the possibility prior distribution. Note that the decision maker can obtain these utility functions by the lot method after deciding his type (Risk Aversion, Risk Neutral, Risk Proneness).

1) Zone x, y (Integral Transfer (maximizing Expected Utility)) (38)

$$E(D_1) = \int \pi(S) \times \mu_N\left(U_1^{-1}(x \mid D_1)\right)\mathrm{d}S$$

$$E(D_2) = \int \pi(S) \times \mu_M\left(U_2^{-1}(y \mid D_2)\right)\mathrm{d}S$$

2) Zone a, d (max-product method) (39)

$$\pi(D_1) = \max_s \pi(S) \times \mu_N\left(U_1^{-1}(x \mid D_1)\right)$$

$$\pi(D_2) = \max_s \pi(S) \times \mu_M\left(U_2^{-1}(y \mid D_2)\right)$$

3) Zone b, c (min-max principal) (40)

$$\wedge(D_1) = \min_S \max\left(\pi(S), \mu_N\left(U_1^{-1}(x \mid D_1)\right)\right)$$

$$\wedge(D_2) = \min_S \max\left(\pi(S), \mu_M\left(U_2^{-1}(y \mid D_2)\right)\right)$$

Here, in Equations (38), (39) and (40), after we pick up the bigger measure in the every zone, we can decide the individual optimal decision with these measures. Otherwise, because type 2 fuzzy event M, N and F_e are the direct sum, we can select the total optimal decision by the max the weighted sum of the information content (30). Note that we may need to obtain the individual optimal decision in the every zone. (Uemura, Inaida [11]).

9. Conclusion

In this paper, we mentioned that Type 2 Fuzzy for no-data problems is derived from simultaneous stochastic differential equations in fuzzy events and is closely related to artificial intelligence in multi-input nonlinear factor analysis. The Type 2 Fuzzy Event is a multidimensional possibility variable error model that simultaneously considers longitudinal and transverse errors introducing the concepts of possibility and necessity, and it is application to main factor analysis based on possibility theory is described. Focusing on the oblique rotation of general factor analysis, the possibility and inevitability measures were defined so that the sum of these measures does not satisfy 1, and the oblique factor rotation matrix was derived by automatically subtracting the sum from 1 for indiscriminate fuzzy events to obtain the possibility and inevitability measures for indiscriminate fuzzy events. If the sum of the weights of the simultaneous stochastic differential equation between the longitudinal and transverse fuzzy variables is greater than 1, no indiscriminate event occurs. For this reason, after normalizing the membership function, the possibility principal factor analysis is applied. Although determinism was not discussed in this paper, it has been proposed that sequential Bayesian inference be employed in cases involving nondiscriminatory events by adding the action of decision withholding (Uemura [1] [4]), because of the information risk of nondiscriminatory events. At last, Japanese call Japanese original traditional decision-making from this world to the other world around another world by the type 2 KIDOU (in Japanese) (Hori [14]). Our proposing theory can apply to deriver the sea wave from the sea wave equation. Specially, when the sea wave equation is regarded as Normal Possibility Process (*i.e.* Gauss Process), we can obtain the sea wave that is Gauss Process (Uemura [9]). Japanese can regard this sea wave process as Sea Goddess/God decision making (Hori [14]).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Uemura, Y. (1991) Decision-Making in Method in Fussy Events. *Journal of the Fussy Society of Japan*, **3**, 123-130. (In Japanese)

[2] Hori Jr., H., Takemura, K. and Matsumoto, Y. (2019) Complex Markov Decision Process. *Journal of Fuzzy Mathematics*, **27**, 957-972.

[3] Okuda, T., Tanaka, H. and Asai, K. (1976) Decision Problems and Information Content in Fuzzy Events. *Journal of the Society of Instrument and Control Engineers*, **12**, 63-68. (In Japanese) https://doi.org/10.9746/sicetr1965.12.63

[4] Uemura, Y. (1990) Fuzzy-Bayes Decision Rule with Indifferent Zone, IIZUKA-90.

[5] Uemura, Y. (1993) A Decision Rule on Fuzzy Events under an Observation. *Journal of Fuzzy Mathematics*, **1**, 39-52.

[6] Zadeh, L.A. (1965) Fuzzy Sets. *Information and Control*, **8**, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[7] Hori Jr., H. (2023) Initial and Stopping Condition in Possibility Principal Factor Analysis. *Journal of Applied Mathematics and Physics*, **11**, 1482-1486. https://doi.org/10.4236/jamp.2023.115097

[8] Hori Jr., H. (2023) Type 2 Fuzzy Stochastic Differential Equations in No-Data Problems and Its Application to Possible Principal Factor Analysis. *Journal of Biomedical Fuzzy Systems Society*, **25**, 65-69. (In Japanese)

[9] Uemura, Y. (2006) Fuzzy Decision Making and Fuzzy Statistics. Fukuro-Shuppan, Fukuyama. (In Japanese)

[10] Dubois, D. and Parade, H. (1988) Possibility Theory. Plenum Press, New York.

[11] Uemura, Y. and Inaida, J. (2023) Possibility State Crimination Method in No-Data Problem. FSS2023, Karuizawa. (In Japanese)

[12] Takahashi, Y. (2011) Kiyoshi Ito's Mathematics. Nippon-Hyoron-Sha, Tokyo. (In Japanese)

[13] Uemura, Y. (1995) The Limit of Using a Probability of a Fuzzy Event in a Fuzzy Decision Problem. *Control and Cybernetics*, **24**, 233-238.

[14] Hori Jr., H. (2022) Type 2 KIDOU and Its Applications. *Annual Conference of the Society for Biomedical Fuzzy Systems*, Himeji, 18 December 2002, 18-20. (In Japanese)