

# Effect of Viscous Dissipation ( $\phi$ ) on Temperature Distribution of Blood Plasma in Presence of a Magnetic Field

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## Abstract

Applications of heat transfer show the variations in temperature of the body which is helpful for the purpose of thermal therapy in the treatment of tumor glands. This study considered theoretical approaches in analyzing the effect of viscous dissipation on temperature distribution on the flow of blood plasma through an asymmetric arterial segment. The plasma was considered to be unsteady, laminar and an incompressible fluid through non-uniform arterial segment in a two-dimensional flow. Numerical schemes developed for the coupled partial differential equations governing blood plasma were solved using Finite Difference scheme (FDS). With the aid of the finite difference approach and the related boundary conditions, results for temperature profiles were obtained. The study determined the effect of viscous dissipation on temperature of blood plasma in arteries. The equations were solved using MATLAB softwares and results were presented graphically and in tables. The increase in viscous dissipation tends to decrease blood plasma heat distribution. This study will find important application in hospitals.

## Keywords

Magnetic Field, Heat Transfer, Finite Difference Method (PDE), Finite Difference Scheme (FDS), Blood Plasma, Asymmetric Segment

## 1. Introduction

Blood plasma flow (BPF) in the human circulatory system depends upon the pumping action of the heart which in turn produces a pressure gradient throughout the system [1]. BPF through the body is regulated by the size of

blood vessels, the action of smooth muscle and the fluid pressure of the plasma itself. Blood plasma flow is vital for maintaining life. The factors that govern plasma flow are pressure, resistance to flow, blood viscosity cardiac output, compliance, blood volume, blood viscosity, the length and diameter of the blood vessels [2]. These properties of blood plasma and the motion of the arterial wall play an important role in the physiology of the cardiovascular system.

In recent studies, blood plasma arterial flow has gained serious attention of researchers, physiologists, clinical persons and in theoretical mathematics because blood and blood vessels are substantial health risk factors and can substantially contribute to morbidity and mortality [3]. Majority of deaths reported in the developed countries result from cardiovascular diseases. Earlier, most of the cardiovascular diseases affected the aged group, but, that situation is different now. There are several other risk factors for heart diseases like age, gender, use of tobacco, high blood pressure and cholesterol causing the development of stenosis.

Magnetic field applied and transfer of heat affect the flow of blood which is useful for solving the problems that may arise in the cardiovascular system. The heart produces the greatest rhythmic electromagnetic field in all of the body organs, making it the strong source of electromagnetic energy in the human body. A major contributor to heart attacks and strokes, blood viscosity, can be reduced by magnetic fields [4]. The thickness and viscosity of BPF can be significantly reduced by powerful magnetic fields. Haemoglobin, a protein made of iron that is found inside red blood cells, is the sole cause of the magnetic effect. Red blood cells arrange themselves on the straight line of field of an electromagnet in a manner similar to how iron filings do around a bar magnet. This has numerous effects that lessen viscosity. For starters, the path of flow streamlines the cells. Additionally, the orientation helps the cells to form clumps of varied sizes by adhering to one another.

Heat transfer concerns the generation, use, conversion and exchange of thermal energy between physical systems. Circulation of blood transfers heat between tissues and the dimensions of the blood vessels are various, whenever more blood plasma is needed in some vessels of the body due to more activity, these blood vessels expand to supply the required blood while other vessels tight to compensate it [4]. The arterioles, capillaries and venules act as perfect heat exchangers in which the blood quickly reaches the tissue temperature.

Heat increases blood flow by expanding blood vessels. It increases blood plasma flow to a specific area and improves circulation. Increased blood flow can help relax a tight muscle, restore movement and reduce pain. Heat eases stiffness after inflammation has resolved. This is because heat on an inflamed area causes the blood vessels to dilate, enhancing blood flow to the injured area. Applying heat to an affected area can provide comfort and increase muscle flexibility, as well as heal damaged tissues. Heat boosts the flow of blood plasma and nutrients to an area of the body [5].

## 2. Literature Review

Impact of viscous dissipation and coriolis effects in heat and mass transfer analysis of the 3D non-Newtonian fluid flow [6]. The impacts of thermal radiation and mass transfer were considered during the fluid motion. The flow was modeled by a set of nonlinear coupled partial differential equations (PDEs) satisfying certain associated boundary conditions. Appropriate transformations were used to convert the coupled system of PDEs to ODEs. The resultant coupled ODEs systems were solved analytically by using the well-developed analytical procedure of Homotopy Analysis Technique (HAM) [6].

The influences of relevant parameters were investigated on the state functions of the Carreau fluid MHD motion by depicting 2D graphs. It was found that the increasing thickness of paraboloid surface of revolution augments the fluid velocity and shear stress components, whereas drops the gradient in the velocity distribution. The enhancing Grashof number increases the fluid velocity components as well as its gradients. The increasing magnetic field strength retards the fluid migration, and therefore drops the velocity gradients. The increasing rotation parameter augments the horizontal component of velocity and reduces the velocity vertical component. The fluid temperature enhances with the increasing strength of radiation source, internal heat source, and reflection parameters [7].

The enhancing chemical reaction parameter values drop the fluid concentration. The convective heat energy transport increases with the enhancing Brownian motion, while the convective mass transfer rises with the increasing radiation source strength. The results obtained are compared with a numerical technique by graphs and tables, where the accuracy and effectiveness of HAM is proved. The agreement of the obtained results and the published research validates the exactness of the applied analytical technique [7].

A study on the impact of temperature dependent viscosity and thermal conductivity on MHD blood flow through a stretching surface with ohmic effect and chemical reaction [7]. The viscous, incompressible electrically conducting MHD blood flow with temperature-dependent thermal conductivity and viscosity through a stretching surface in the presence of thermal radiation, viscous dissipation, and chemical reaction was carried out. The flow was subjected to a uniform transverse magnetic field normal to the flow. The governing coupled partial differential equations were converted into a set of non-linear ordinary differential equations (ODE) using similarity analysis [7].

The resultant non-linear coupled ordinary differential equations were solved numerically using the boundary value problem solver in MATLAB with a convincing accuracy. The effects of the physical parameters such as viscosity parameter, permeability parameter ( $\beta$ ), magnetic field parameter ( $M$ ), Local Grashof number ( $Gr$ ) for thermal diffusion, Local modified Grashof number for mass diffusion ( $Gm$ ), the Eckert number ( $Ec$ ), the thermal conductivity parameter on the velocity, temperature, concentration profiles, skin-friction coefficient, Nus-

selt number, and Sherwood number were presented graphically. The physical visualization of flow parameters that appeared in the problem was discussed with the help of various graphs to convey the real life application in industrial and engineering processes [7].

A comparison was made with previously published work and present study revealed the good agreement with the published work. This study was helpful in the clinical healing of pathological situations accompanied by accelerated circulation.

An analytical study on effects of viscous dissipation and suction on a steady MHD natural convection couette flow of heat generating or absorbing fluid [8]. This work concerns a theoretical investigation on the effects of suction or injection, magnetic field, permeability of porous materials and viscous dissipation on an electrically conducting incompressible fluid passes through a vertical porous channel filled with porous materials. One of the plates moves in the flow direction while the other is stationary [8]. The governing coupled flow equations were solved analytically using Homotopy Perturbation Method (HPM). The influences of the flow parameters on velocity and temperature were plotted on graphs while numerical values for rate of heat transfer and shear stress on the heated and cold plates were presented in tables. Excellent agreements were found when compared with the previous works [8].

It is noteworthy to mention that the hydrodynamic and thermodynamic distributions of the fluid increase with increase in viscous dissipation ( $Ec$ ). It was also found that the shear stress decreases with increase in the magnetic field ( $M$ ) while a reverse case was observed for growing the permeability of the porous materials ( $K$ ). It is further found that the velocity and temperature distributions decrease with increase in suction [9].

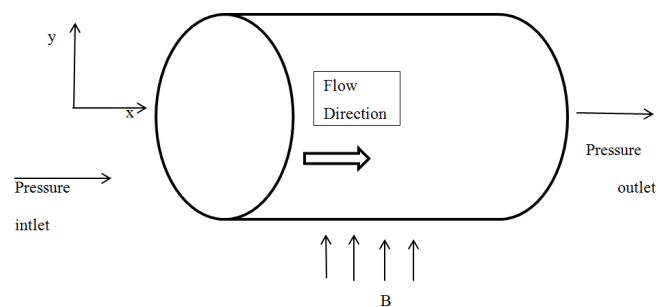
MHD effects on mixed convective nanofluid flow with viscous dissipation in surrounding porous medium [10]. In existence of concerning magnetic field, heat together with mass transfer features on mixed convective copper-water nanofluid flow through inclined plate was investigated in surrounding porous medium together with viscous dissipation. A proper set of useful similarity transforms was considered as to transform the desired governing equations into a system as ordinary differential equations which are nonlinear. The transformed equations for nanofluid flow included interrelated boundary conditions which were resolved numerically applying Runge-Kutta integration process of sixth-order together with Nachtsheim and Swigert technique. The numerical consequences were compared together with literature which was published previously and acceptable comparisons were found. The influence of significant parameters like as magnetic parameter, angle for inclination, Eckert number, fluid suction parameter, nanoparticles volume fraction, Schmidt number and permeability parameter on concerning velocity, temperature along with concentration boundary layers remains examined and calculated. Numerical consequences were presented graphically. Moreover, the impact regarding these physical parameters for engineering significance in expressions of local skin friction coefficient in addi-

tion to local Nusselt together with Sherwood numbers was correspondingly examined. In existence of growing magnetic field strength, the temperature gradient decreases at the wall which in turn leads to a reduction in rate of heat transfer. The gravitational effects is maximum at a vertical position whereas minimum at horizontal position. Consequently, momentum boundary layer nanofluid flow decreased to increase of  $\alpha$  [10]. Moreover, when, the angle regarding inclination  $\alpha$  was increased, velocity of nanofluid flow and shear stress also decreased.

Effect of brownian diffusion on squeezing elastic-viscous nanofluid flow with cattaneo-christov heat flux model in a channel with double slip effect [11]. The present study dealt with the analysis of heat transfer of the unsteady Maxwell nanofluid flow in a squeezed rotating channel of a porous extensile surface subject to the velocity and thermal slip effects incorporating the theory of heat flow intensity of Cattaneo-Christov model for the expression of the energy distribution in preference to the classical Fourier's law. A set of transformations were occupied to renovate the current model in a system of nonlinear ordinary differential equations that are numerically decoded with the help of MATLAB integrated function bvp4c. The effects of various flow control parameters were investigated for the momentum, temperature and diffusion profiles, as well as for the wall shearing stress and the heat and mass transfer. The results are finally described from the material point of view [11]. A comparison of heat flux models of Cattaneo-Christov and Fourier was also performed. An important result from the present work is that the squeezing parameter is strong enough in the middle of the channel to retard the fluid flow.

### 3. Formulation of the Problem

The movement of a conducting electrical, incompressible fluid in a laminar, two-dimensional, asymmetric flow in a non-uniform artery was taken into consideration. Let's define  $(x, y)$  as the coordinates in the system where the artery's axis served as the  $x$ -axis and the circumferential and radial directions served as the  $y$ -axis, respectively. In order to assess the flow's axial direction, field for electrical conductivity of blood plasma, there was applied a homogeneous magnetic field of strength  $B$ . The analysis of the transfer of heat phenomena took into account the constant temperature at the artery wall. **Figure 1** is a mathematical formulation illustrating the effect of magnetic field on the flow of blood plasma.



**Figure 1.** Physical configuration of the model problem.

### 3.1. Governing Equations

The governing equations were presented based on the following assumptions on blood plasma

- Plasma is considered to be Newtonian,
- Blood plasma is incompressible,
- Plasma is homogeneous and a viscous fluid.

The above assumptions explain the unsteady flow of blood in the presence of magnetic field governed by two-dimensional boundary layer equations where  $u$  and  $v$  are the velocity components in the direction of  $x$  and  $y$ , respectively at time  $t$  in the flow field.

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.1)$$

Momentum Equation

$$x\text{- Direction} - \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x \quad (1.2)$$

$$y\text{- Direction} - \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y \quad (1.3)$$

Energy Equation

$$\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \phi \quad (1.4)$$

where

$$\phi = \mu \left\{ 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 - \frac{2}{3} (\text{div}u)^2 \right] \right\} \quad (1.5)$$

### 3.2. Methodology

#### Numerical Solution of PDE

The finite difference method involves solving a partial differential equation numerically by transforming the continuous components of the PDE into discrete components. The discretization for these partial derivatives were replaced by forward difference approximation and by central difference approximation This involves the use of algebraic finite difference approximations where the latter is substituted in the partial differential equation and hence obtaining algebraic finite difference equations. After this, the resulting algebraic finite difference equations are solved. The method of finite differences hence solves a PDE by transforming the continuous problem into an algebraic problem by:

1) Transforming the continuous components of the PDE into discrete components.

2 Use of finite difference approximations to approximate the partial derivatives of the PDE.

- 3) Obtaining algebraic finite difference equations by substituting the FDA'S into the PDE.
- 4) Solving the algebraic finite difference equations.

### 3.3. Discretization of Energy Equation

An Energy Equation was considered.

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left[ \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right] + \phi \theta \tag{1.6}$$

It was used it to investigate the blood plasma temperature distribution. For the central difference scheme (CDS), the values,  $\theta_x$ ,  $\theta_y$ ,  $\theta_{xx}$  and  $\theta_{yy}$  were replaced by central difference approximation. When these values are substituted into the energy Equation (1.6), you get;

$$\begin{aligned} &u \left( \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta x} \right) + v \left( \frac{\theta_{i,j+1} - \theta_{i,j-1}}{2\Delta y} \right) \\ &= \left( \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta x)^2} + \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta y)^2} \right) + \phi \theta_{i,j} \end{aligned} \tag{1.7}$$

Take  $u = v = N_c = \theta = 1$ , multiply both sides by  $2(\Delta x)$  and let  $Pr = 0.71$  and take  $\Delta x = \Delta y = 0.1$  on a square mesh into Equation (1.7) you get the scheme;

$$-19\theta_{i+1,j} + (80 - 0.2\phi)\theta_{i,j} - 21\theta_{i-1,j} = 19\theta_{i,j+1} + 21\theta_{i,j-1} \tag{1.8}$$

Taking and  $i = 1, 2, 3, 4, 5$  and  $j = 1$  we form the following systems of linear algebraic equations;

$$\begin{aligned} &-19\theta_{2,1} + (80 - 0.2\phi)\theta_{1,1} - 121\theta_{0,1} = 19\theta_{1,2} + 21\theta_{1,0} \\ &-19\theta_{3,1} + (80 - 0.2\phi)\theta_{2,1} - 21\theta_{1,1} = 19\theta_{2,2} + 21\theta_{2,0} \\ &-19\theta_{4,1} + (80 - 0.2\phi)\theta_{3,1} - 21\theta_{2,1} = 19\theta_{3,2} + 21\theta_{3,0} \\ &-19\theta_{5,1} + (80 - 0.2\phi)\theta_{4,1} - 21\theta_{3,1} = 19\theta_{4,2} + 21\theta_{4,0} \\ &-19\theta_{6,1} + (80 - 0.2\phi)\theta_{5,1} - 21\theta_{4,1} = 19\theta_{5,2} + 21\theta_{5,0} \end{aligned} \tag{1.9}$$

With initial and boundary conditions  $\theta_{i,0} = \theta_{0,j} = 10$  and  $\theta_{i,2} = 0$  respectively, the algebraic Equation (1.9) can be written in matrix form as;

$$\begin{bmatrix} (8 - 0.2\phi) & 0.86 & 0 & 0 & 0 \\ -1.4 & (8 - 0.2\phi) & 0.86 & 0 & 0 \\ 0 & -1.4 & (8 - 0.2\phi) & 0.86 & 0 \\ 0 & 0 & -1.4 & (8 - 0.2\phi) & 0.86 \\ 0 & 0 & 0 & -1.4 & (8 - 0.2\phi) \end{bmatrix} \begin{bmatrix} \theta_{1,1} \\ \theta_{2,1} \\ \theta_{3,1} \\ \theta_{4,1} \\ \theta_{5,1} \end{bmatrix} = \begin{bmatrix} 1420 \\ 210 \\ 210 \\ 210 \\ 210 \end{bmatrix} \tag{1.10}$$

Equation (1.10) was solved using MATLAB to investigate the effects of  $\phi$ , on the blood plasma temperature distribution.

## 4. Results

### Effects of Viscous Dissipation on Blood Temperature Distribution

We hold constant the values of  $v = u = 1.0$  m/s, and varying  $\phi$  in Equation

(1.10). Solving Equation (1.10) for varying values of  $\phi$ , solutions were obtained as presented in **Table 1**.

The results in **Table 1** were represented graphically as shown in **Figure 2**.

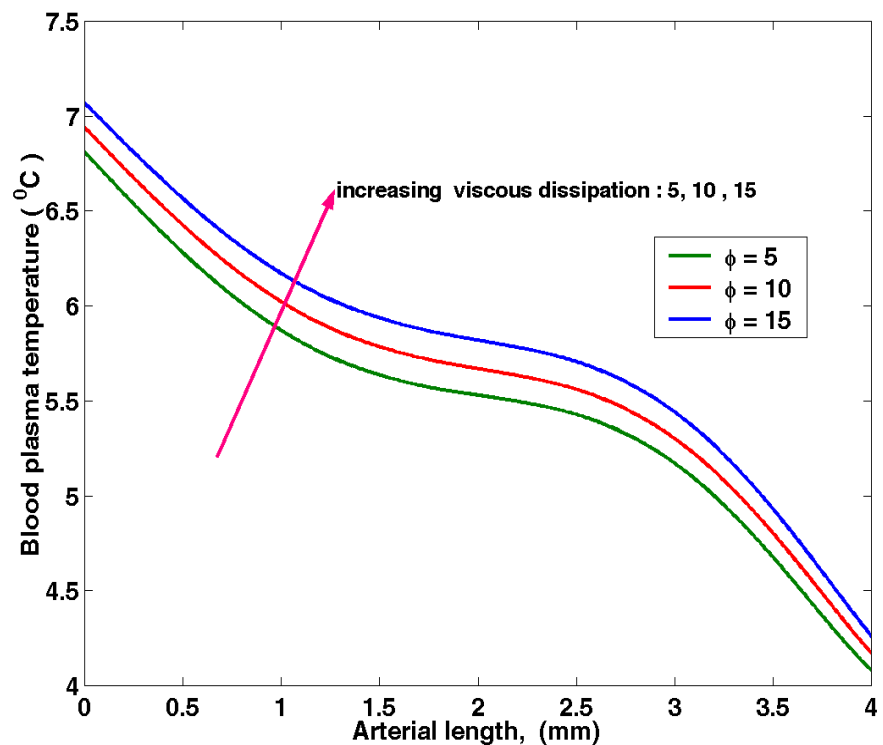
**Figure 2** showed several values of viscous dissipation, numerical results showed that for given value of these parameters which are listed in figure, the blood plasma surface heat transfer tends to decrease by increasing in  $\phi$ .

The convection current near the channel is enhanced by the viscous dissipation which in return affects the temperature of the fluid, thus causes decrease in fluid density, hence increases the fluid flow. It is also observed that the inter-facial temperature decreases on the heated surface while it increases on the cold surface which is due to increase in viscous dissipation.

The decrease in heat distribution reduces the flow rate of blood, which may be used to reduce bleeding during surgeries.

**Table 1.** Blood plasma temperature distribution for varying viscous dissipation.

Viscous Dissipation	Arterial length, $x$				
	0	1	2	3	4
$\phi = 5$	6.813856	5.872194	5.5395637	5.1702643	4.0835628
$\phi = 10$	6.941472	6.021216	5.6722278	5.3011632	4.172579
$\phi = 15$	7.069552	6.170012	5.823442	5.4426483	4.263325



**Figure 2.** Temperature against arterial length with varying viscous dissipation.



## 5. Discussion

The purpose of this study was to investigate the effect of heat transfer on plasma through an asymmetric non-uniform channel. The objectives of the study were to determine the effect of viscous dissipation ( $\phi$ ) on temperature distribution of blood plasma. With regards to this, we came up with the governing equations *i.e.* the Energy equation, momentum equation and concentration equation. For the equations to be solved in a simpler manner there was non-dimensionalization of the governing equations. After that, the equations were solved numerically using a finite difference scheme where the partial differential equations were converted into algebraic finite difference equations. After this, the equations were then solved using MATLAB. The results from this were illustrated in the form of graphs and the conclusions from this were as follows:

- 1) The blood plasma surface heat transfer tends to decrease by increasing in  $\phi$ .
- 2) Increasing the speed will increase the temperature of the blood, resulting in a decrease in the viscosity and shear stress of the blood plasma.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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### Nomenclature

$v$ , Velocity along  $y$ -axis ( $m^2$ );  $u$ , Velocity along  $x$ -axis ( $m^2$ );  $P$ , Pressure ( $N\ m^{-2}$ );  
 $\rho$ , Density ( $kg/m^3$ );  $\mu$ , Viscosity ( $N\ s/m^2$ );  $Q_0$ , Heat absorption coefficient;  
 $u_x = \frac{\partial u}{\partial x}$ ;  $u_t = \frac{\partial u}{\partial t}$ ;  $u_{xx} = \frac{\partial^2 u}{\partial x^2}$ ;  $u_{yy} = \frac{\partial^2 u}{\partial y^2}$ ;  $T$ , Temperature (K);  $C_p$ , Specific heat  
 at constant pressure;  $\rho g_x$ , External force in  $x$  direction;  $\rho g_y$ , External force in  
 $y$  direction;  $\phi$  Viscous dissipation.