

Global Dynamics of a New Huanglongbing Transmission Model with Quarantine Measures

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Abstract

An epidemic model which describes Huanglongbing transmission is proposed with the goal of investigating the effect of quarantine measures on the spread of diseases. First of all, the analytical formula for the basic reproduction number \mathcal{R}_0 is obtained by the means of next generation matrix, and the existence of disease-free equilibrium and endemic equilibrium is discussed. Then, the local stability and the global stability of equilibria are investigated by using Routh-Hurwitz criterion and Lyapunov function, respectively. Numerical simulations indicate that comprehensive quarantine measures can effectively control the spread of Huanglongbing. It provides a reliable tactic basis for preventing the epidemic outbreak.

Keywords

Huanglongbing, Basic Reproduction Number, Sensitivity Analysis, PRCC, Global Stability

1. Introduction

The citrus Huanglongbing (HLB), allied with the bacterium *Candidatus Liberibacter asiaticus* (CLAs), poses a great threat to the industry of citrus worldwide [1] [2]. The most vital method of disease breadth occurs by the citrus psyllids that serve as vectors [1]. The citrus psyllids consume sap from an infected host through their stylets, the viruses in the sap enter the salivary glands, circulate within the psyllid, and then cause infection between citrus trees and psyllids [3] [4] [5] [6] [7]. The citrus tree infected HLB shows yellow shoots, leaves with blotchy mottle, small lopsided fruits, overall stunting of the trees, in server cases, and ultimately the death of the entire tree [8] [9] [10]. In 2020, citrus Huanglongbing was included in the List of Crop pests and diseases by the Ministry of Agri-

culture and Rural Affairs [11].

There are no economically viable curative methods for citrus trees infected HLB due to absence of resistant or tolerant commercial citrus varieties [12]. However, a large number of researchers have studied this disease biologically and come to describe the dynamics of disease mathematically [1] [13]. Recent works have provided different insights into the dynamic mechanism of HLB propagation through differential equation modeling, statistical analysis and individual-based modeling [10]. Jacobsen *et al.* established a 6-dimensional dynamic model within a citrus orchard and analyzed how the number of each class changes over time due to bacterial transmission between citrus trees and psyllids [7]. Parry *et al.* fitted a spatially explicit disease model, using specialized statistical methods and software, and then obtained the effect of tree age on transmission parameters, which could be used to predict the spread of early epidemics [14]. Lee *et al.* [2] conducted a mathematical model that combined experiments and individual-based. It emphasized the need to keep the psyllid populations as low as possible due to the whole grove to be infected before the first symptoms appear on any tree. In fact, removing infected trees and quarantining them are also important measures to prevent as many trees from becoming infected. Specially, if HLB is not yet present, quarantine measures should be enforced to keep it out [15]. The quarantine measures, which began on June 17, 2010, were designed to prevent the transportation of citrus in Florida [7]. Therefore, motivated by these researches, we propose a dynamic model of HLB with quarantine measures for exposed and infected citrus trees, and then perform some theoretical analysis of its properties.

The organization of this paper is as follows: the model is developed and the feasible region, the basic reproduction number and the equilibria of the model are obtained in Section 2. By the Routh-Hurwitz criterion, the local stability of disease-free equilibrium and endemic equilibrium are proved in Section 3. By constructing a suitable Lyapunov function, the global stability of the equilibria is studied in Section 4. In addition, numerical simulations are illustrated in our theoretical results in Section 5 and a brief conclusion is given in the last section.

2. Model and Preliminaries

The dynamics of the model with a fraction of quarantined citrus trees involve citrus-psyllid interactions, therefore, the citrus tree population given as $N_h(t)$ is divided into susceptible host $S(t)$, exposed host $E(t)$, infected host $I(t)$, quarantined host $Q(t)$ and removed host $R(t)$, at any time t . Thus,

$N_h(t) = S(t) + E(t) + I(t) + Q(t) + R(t)$. The citrus psyllid population denoted by $N_v(t)$, is divided into susceptible class $X(t)$ and infected class $Y(t)$ at any time t . Thus, $N_v(t) = X(t) + Y(t)$. We assume that the citrus population grows at a recruitment rate Λ_h , moreover, q_1 and q_2 represent the rate of the infectious convert to the quarantined host and the rate of exposed to the quarantined host, respectively. According to the principle of the compartmental model, the model with first order nonlinear ordinary differential equations is

constructed as follows:

$$\begin{cases} \frac{dS(t)}{dt} = \Lambda_h - \beta_1SY - \mu_hS_h, \\ \frac{dE(t)}{dt} = \beta_1SY - (q_1 + \sigma + \gamma_1 + \mu_h)E, \\ \frac{dI(t)}{dt} = \sigma E - (q_2 + \gamma_2 + \mu_h + d)I, \\ \frac{dQ(t)}{dt} = q_1E + q_2I - (\gamma_3 + \mu_h)Q, \\ \frac{dR(t)}{dt} = \gamma_1E + \gamma_2I + \gamma_3Q - \mu_hR, \\ \frac{dX(t)}{dt} = \Lambda_v - \beta_2(\theta E + I)X - \mu_vX, \\ \frac{dY(t)}{dt} = \beta_2(\theta E + I)X - \mu_vY, \end{cases} \quad (2.1)$$

where the parameters involved in the model (2.1) and their values are showed in **Table 1**. The schematic diagram for the flow of the dynamics is depicted in **Figure 1**. In the subsequent section, we will explore the basic qualitative properties of the model (2.1).

2.1. Feasible Region Denote

$$\Omega_0 = \left\{ (S, E, I, Q, R, X, Y) \in \mathbb{R}_+^7 \mid S + E + I + Q + R \leq \frac{\Lambda_h}{\mu_h}, X + Y \leq \frac{\Lambda_v}{\mu_v} \right\}. \quad (2.2)$$

Lemma 1. *The feasible region is given by Ω_0 is a positively invariant and globally attractive set for system (2.1).*

Proof. Clearly, the solution set $(S(t), E(t), I(t), Q(t), R(t), X(t), Y(t))$ of the system (2.1) is positive.

Since $I(t)$ is non-negative, by adding the first to fifth equations and the last two equations of the system (2.1), respectively, the total number of citrus tree $N_h(t)$ and psyllid vector $N_v(t)$ satisfy

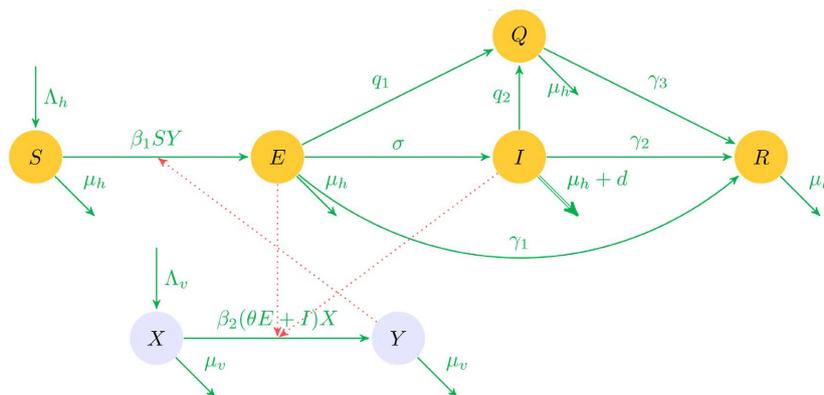


Figure 1. The schematic diagram of the model showing transitions to different categories for trees and psyllids.

Table 1. Parameters in the model (2.1).

Parameters	Description	Baseline (range)	unit	References
Λ_h	Citrus recruitment rate	6.6 (1 - 8)	month ⁻¹	[16]
μ_h	Natural death rate of citrus population	0.0033 (0.002 - 0.01)	month ⁻¹	[17]
β_1	Virus transmission probability from psyllid to plant	0.003125 (0.001 - 0.005)	(vector·month) ⁻¹	[10]
σ	Rate at which exposed become infectious	0.25 (0 - 0.5)	month ⁻¹	Estimation
q_1	Rate of the exposed to the quarantined	0.3 (0 - 1)	month ⁻¹	Estimation
q_2	Rate of the infectious convert to the quarantined	0.3 (0 - 1)	month ⁻¹	Estimation
d	Death rate of citrus tree due to the infection	0.025 (0.01 - 0.1)	month ⁻¹	[18]
γ_1	Rate at which exposed become recovered	0.1 (0 - 0.4)	month ⁻¹	Estimation
γ_2	Rate at which infected become recovered	0.05 (0 - 0.2)	month ⁻¹	Estimation
γ_3	Rate at which quarantined become recovered	0.2 (0 - 0.6)	month ⁻¹	Estimation
Λ_v	Psyllid recruitment rate	2000 (100 - 3000)	month ⁻¹	Estimation
μ_v	Natural death rate of psyllid population	5 (4 - 8)	month ⁻¹	[10]
β_2	Virus transmission probability from plant to psyllid	0.0025 (0.001 - 0.004)	(tree·month) ⁻¹	[10]
θ	Infection intensity relative to infected	0.1 (0 - 0.2)	-	Estimation

$$\dot{N}_h(t) \leq \Lambda_h - \mu_h N_h(t), \quad \dot{N}_v(t) = \Lambda_v - \mu_v N_v(t) \tag{2.3}$$

From Equation (2.3), we get

$$\begin{aligned} N_h(t) &\leq \frac{\Lambda_h}{\mu_h} - \left(\frac{\Lambda_h}{\mu_h} - N_h(0) \right) \exp\{-\mu_h t\}, \\ N_v(t) &= \frac{\Lambda_v}{\mu_v} - \left(\frac{\Lambda_v}{\mu_v} - N_v(0) \right) \exp\{-\mu_v t\}. \end{aligned} \tag{2.4}$$

This implies that

$$\limsup_{t \rightarrow +\infty} N_h \leq \frac{\Lambda_h}{\mu_h}, \quad \limsup_{t \rightarrow +\infty} N_v = \frac{\Lambda_v}{\mu_v}. \tag{2.5}$$

Thus, all solutions in Ω are uniformly bounded. This completes our proof.

2.2. The Basic Reproduction Number

Since the fourth and fifth equation is decoupled from the rest in system (2.1) and to better organize the analysis, we denote $\mu_1 = q_1 + \sigma + \gamma_1 + \mu_h$, $\mu_2 = q_2 + \gamma_2 + \mu_h + d$, and consider the following equivalent system of (2.1):

$$\begin{cases} \frac{dS(t)}{dt} = \Lambda_h - \beta_1 SY - \mu_h S_h, \\ \frac{dE(t)}{dt} = \beta_1 SY - \mu_1 E, \\ \frac{dI(t)}{dt} = \sigma E - \mu_2 I, \\ \frac{dX(t)}{dt} = \Lambda_v - \beta_2 (\theta E + I) X - \mu_v X, \\ \frac{dY(t)}{dt} = \beta_2 (\theta E + I) X - \mu_v Y. \end{cases} \tag{2.6}$$

It is easy to see

$$\Omega_1 = \left\{ (S, E, I, X, Y) \in \mathbb{R}_+^5 \mid S + E + I \leq \frac{\Lambda_h}{\mu_h}, X + Y \leq \frac{\Lambda_v}{\mu_v} \right\} \tag{2.7}$$

is a positively invariant and globally attractive set for system (2.6). From now on, we restrict the analysis of system (2.6) to the positive invariant set Ω_1 .

Clearly, the disease free equilibrium (DFE) of system (2.6) exists and is given by $P_0 = \left(\frac{\Lambda_h}{\mu_h}, 0, 0, \frac{\Lambda_v}{\mu_v}, 0 \right)$.

Now, we calculate the basic reproductive number R_0 of system (2.6) by using the next generation operator approach [19]. Considering the infectious compartments $x = (E, I, Y)^T$, let

$$F(x) = (F_1(x), F_2(x), F_3(x))^T = \begin{pmatrix} \beta_1 SY \\ 0 \\ \beta_2 (\theta E + I) X \end{pmatrix},$$

and

$$V(x) = (V_1(x), V_2(x), V_3(x))^T = \begin{pmatrix} \mu_1 E \\ \mu_2 I - \sigma E \\ \mu_v Y \end{pmatrix}$$

represent the rate of appearance of new infection and the rates of transfer into and out of the class of infected states, respectively. Then the Jacobian matrices $\mathcal{F}(x)$ and $\mathcal{V}(x)$ of system (2.6) with respect to P_0 are given below:

$$\mathcal{F}(x) = \begin{pmatrix} 0 & 0 & \frac{\Lambda_h \beta_1}{\mu_h} \\ 0 & 0 & 0 \\ \frac{\Lambda_v \beta_2 \theta}{\mu_v h} & \frac{\Lambda_v \beta_2}{\mu_v h} & 0 \end{pmatrix}, \quad \mathcal{V}(x) = \begin{pmatrix} \mu_1 & 0 & 0 \\ -\sigma & \mu_2 & 0 \\ 0 & 0 & \mu_v \end{pmatrix} \tag{2.8}$$

Using the next generation matrix theory, the expression of the basic reproduction number \mathcal{R}_0 can be obtained as

$$\mathcal{R}_0 = \sqrt{\mathcal{R}_0^1 + \mathcal{R}_0^2}, \quad \mathcal{R}_0^1 = \frac{\beta_1 \beta_2 \Lambda_h \Lambda_v \theta}{\mu_h \mu_1 \mu_v^2}, \quad \mathcal{R}_0^2 = \frac{\beta_1 \beta_2 \Lambda_h \Lambda_v \sigma}{\mu_h \mu_1 \mu_2 \mu_v^2}, \quad (2.9)$$

Remark 1: \mathcal{R}_0^1 represents the number of secondary infections by the circular transmission path from E to Y and back to E , and \mathcal{R}_0^2 indicates the number of secondary infections by the other circular transmission path from I to Y and back to I .

2.3. The Existence of Endemic Equilibrium

Next, we consider the existence of endemic equilibrium. Let the right side of equations of system (2.6) be equal to 0, then we obtain algebraic equations as follows:

$$\begin{aligned} \Lambda_h - \beta_1 S^* Y^* - \mu_h S_h^* &= 0, \\ \beta_1 S^* Y^* - \mu_1 E^* &= 0, \\ \sigma E^* - \mu_2 I^* &= 0, \\ \Lambda_v - \beta_2 (\theta E^* + I^*) X^* - \mu_v X^* &= 0, \\ \beta_2 (\theta E^* + I^*) X^* - \mu_v Y^* &= 0. \end{aligned} \quad (2.10)$$

From the third equation of system (2.10), we get

$$I^* = \frac{\sigma E^*}{\mu_2}. \quad (2.11)$$

By adding the first second equations and the last two equations of system (2.10) respectively, we find

$$S^* = \frac{\Lambda_h - \mu_1 E^*}{\mu_h}, \quad X^* = \frac{\Lambda_v}{\mu_v} - Y^* \quad (2.12)$$

Substituting Equations (2.11) and (2.12) into the second and fifth equation of (2.10), we have

$$E^* = \frac{\mu_h \mu_2 \mu_v^2 (\mathcal{R}_0^2 - 1)}{\beta_2 (\mu_2 \theta + \sigma) (\Lambda_v \beta_1 + \mu_h \mu_v)}, \quad Y^* = \frac{\mu_h \mu_1 \mu_2 \mu_v (\mathcal{R}_0^2 - 1)}{\beta_1 (\Lambda_h \beta_2 (\mu_2 \theta + \sigma) + \mu_1 \mu_2 \mu_v)}. \quad (2.13)$$

If $\mathcal{R}_0 > 1$, for system (2.6) there exists a unique endemic equilibrium P^* in the interior of Ω_1 . Therefore, from the above calculation, we get the following theorem.

Theorem 2. *There is always a disease-free equilibrium $P_0 \left(\frac{\Lambda_h}{\mu_h}, 0, 0, \frac{\Lambda_v}{\mu_v}, 0 \right)$, and the unique endemic equilibrium $P^* (S^*, E^*, I^*, X^*, Y^*)$ of system (2.6) emerges in Ω_1 when $\mathcal{R}_0 > 1$ and no endemic equilibrium when $\mathcal{R}_0 \leq 1$.*

3. Local Stability of Equilibria

In this section, we will investigate the local stability of the disease-free equilibrium

P_0 and the endemic equilibrium P^* .

Theorem 3. For system (2.6), the disease-free equilibrium P_0 is locally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.

Proof. The Jacobian matrix of system (2.6) at disease-free equilibrium

$$P_0 \left(\frac{\Lambda_h}{\mu_h}, 0, 0, \frac{\Lambda_v}{\mu_v}, 0 \right) \text{ is}$$

$$\mathcal{J}_0 = \begin{pmatrix} -\mu_h & 0 & 0 & 0 & -\frac{\beta_1 \Lambda_h \beta_1}{\mu_h} \\ 0 & -\mu_1 & 0 & 0 & \frac{\beta_1 \Lambda_h \beta_1}{\mu_h} \\ 0 & \sigma & -\mu_2 & 0 & 0 \\ 0 & -\frac{\Lambda_v \beta_2 \theta}{\mu_v h} & -\frac{\Lambda_v \beta_2}{\mu_v h} & -\mu_v & 0 \\ 0 & \frac{\Lambda_v \beta_2 \theta}{\mu_v h} & \frac{\Lambda_v \beta_2}{\mu_v h} & 0 & -\mu_v \end{pmatrix}. \tag{3.1}$$

The characteristic equation of matrix \mathcal{J}_0 is the following polynomial equation

$$(\lambda + \mu_h)(\lambda + \mu_v) \left(\frac{\beta_1 \beta_2 \Lambda_h \Lambda_v ((\lambda + \mu_2)\theta + \sigma)}{\mu_h \mu_v} - (\lambda + \mu_1)(\lambda + \mu_2)(\lambda + \mu_v) \right) = 0. \tag{3.2}$$

It is obviously that two eigenvalues of \mathcal{J}_0 are $\lambda_1 = -\mu_h$, $\lambda_2 = -\mu_v$, which are negative. The other three roots are determined by the following cubic equation

$$\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0, \tag{3.3}$$

where

$$\begin{aligned} a_1 &= \mu_1 + \mu_2 + \mu_v, \\ a_2 &= \mu_2(\mu_1 + \mu_v) + \mu_1 \mu_v \left(1 - \frac{\mu_2 \theta}{\mu_2 \theta + \sigma} \mathcal{R}_0^2 \right), \\ a_3 &= \mu_1 \mu_2 \mu_v (1 - \mathcal{R}_0^2). \end{aligned}$$

It is obviously that if $\mathcal{R}_0 < 1$, we have $a_1 > 0, a_2 > 0, a_3 > 0$, and

$$a_1 a_2 - a_3 = (\mu_2 + \mu_v) a_2 + \mu_1^2 \mu_2 + \mu_1^2 \mu_v \left(1 - \frac{\mu_2 \theta}{\mu_2 \theta + \sigma} \mathcal{R}_0^2 \right) + \mu_1 \mu_2 \mu_v \mathcal{R}_0^2 > 0,$$

however $a_3 < 0$ if $\mathcal{R}_0 > 1$. According to the Routh-Hurwitz criterion, we declare that all eigenvalues of \mathcal{J}_0 have negative real parts if and only if $\mathcal{R}_0 < 1$. This proof is complete.

Theorem 4. If $\mathcal{R}_0 > 1$, the endemic equilibrium P^* of the system (2.6) is locally asymptotically stable.

Proof. The Jacobian matrix of system (2.6) at endemic equilibrium

$$P^*(S^*, E^*, I^*, X^*, Y^*) \text{ is}$$

$$\mathcal{J}^* = \begin{pmatrix} -\mu_{h1} & 0 & 0 & 0 & -\beta_1 S^* \\ \beta_1 Y^* & -\mu_1 & 0 & 0 & \beta_1 S^* \\ 0 & \sigma & -\mu_2 & 0 & 0 \\ 0 & -\beta_2 \theta X^* & -\beta_2 X^* & -\mu_{v1} & 0 \\ 0 & \beta_2 \theta X^* & \beta_2 X^* & \beta_2 \theta E^* + \beta_2 I^* & -\mu_v \end{pmatrix}, \tag{3.4}$$

where $\mu_{h1} = \beta_1 Y^* + \mu_h$ and $\mu_{v1} = -\beta_2 \theta E^* + \beta_2 I^* + \mu_v$. The characteristic equation of matrix \mathcal{J}^* is given by

$$(\lambda + \mu_v) \left((\lambda + \mu_{h1})(\lambda + \mu_1)(\lambda + \mu_2)(\lambda + \mu_{v1}) - \frac{\mu_1 \mu_2 \mu_v (\lambda + \mu_h)(\theta \lambda + \mu_2 \theta + \sigma)}{\mu_2 \theta + \sigma} \right) = 0. \tag{3.5}$$

Clearly, there is a negative eigenvalue $\lambda_1 = -\mu_h$. By $\beta_1 S^* = \frac{\mu_1 E^*}{Y^*}$, $\beta_2 X^* = \frac{\mu_2 \mu_v Y^*}{(\mu_2 \theta + \sigma) E^*}$, the other four eigenvalues satisfy the following quartic polynomial

$$\lambda^4 + b_1 \lambda^3 + b_2 \lambda^2 + b_3 \lambda + b_4 = 0, \tag{3.6}$$

where

$$\begin{aligned} b_1 &= \mu_{h1} + \mu_1 + \mu_2 + \mu_{v1}, \\ b_2 &= \mu_{h1}(\mu_1 + \mu_2 + \mu_{v1}) + \mu_2(\mu_1 + \mu_{v1}) + \mu_1 \left(\frac{\mu_v \sigma}{\mu_2 \theta + \sigma} + \beta_2 \theta E^* + \beta_2 I^* \right), \\ b_3 &= \mu_{h1} \mu_2 (\mu_1 + \mu_{v1}) + \mu_1 \mu_2 (\beta_2 \theta E^* + \beta_2 I^*) + \mu_1 \mu_v \left(\frac{\mu_h \sigma}{\mu_2 \theta} + \beta_1 Y^* \right), \\ b_4 &= \mu_1 \mu_2 (\mu_{h1} \mu_{v1} - \mu_h \mu_v). \end{aligned}$$

It is obviously that if $\mathcal{R}_0 > 1$, we have $b_1 > 0, b_2 > 0, b_3 > 0, b_4 > 0$, and we can verify that $b_3(b_1 b_2 - b_3) - b_1^2 b_4 > 0$. According to the Routh-Hurwitz criterion, we obtain that all eigenvalues of \mathcal{J}^* have negative real parts if and only if $\mathcal{R}_0 > 1$. This proof is complete.

4. Global Stability of Equilibria

In this section, we will discuss the global stability of the DFE P_0 and the endemic equilibrium P^* .

Theorem 5. *The disease-free equilibrium P_0 of system (2.6) is globally asymptotically stable (GAS) in the interior of the set Ω_1 provided that $\mathcal{R}_0 \leq 1$.*

Proof. Constructing Lyapunov function

$$L(E, I, Y) = E + \frac{\mu_1}{\mu_2 \theta + \sigma} I + \frac{\mu_1 \mu_2 \mu_v}{\beta_2 \Lambda_v (\mu_2 \theta + \sigma)} Y, \tag{4.1}$$

then the derivative of (4.1) with respect to t along the solution curves of (2.6) is given by

$$\begin{aligned}
 \frac{dL(E, I, Y)}{dt} &= \frac{dE}{dt} + \frac{\mu_1}{\mu_2\theta + \sigma} \frac{dI}{dt} + \frac{\mu_1\mu_2\mu_v}{\beta_2\Lambda_v(\mu_2\theta + \sigma)} \frac{dY}{dt} \\
 &= \beta_1SY - \mu_1E + \frac{\mu_1}{\mu_2\theta + \sigma}(\sigma E - \mu_2I) \\
 &\quad + \frac{\mu_1\mu_2\mu_v}{\beta_2\Lambda_v(\mu_2\theta + \sigma)}(\beta_2(\theta E + I)X - \mu_vY) \\
 &\leq \frac{\beta_1\Lambda_h Y}{\mu_h} - \mu_1E + \frac{\mu_1}{\mu_2\theta + \sigma}(\sigma E - \mu_2I) \\
 &\quad + \frac{\mu_1\mu_2\mu_v}{\beta_2\Lambda_v(\mu_2\theta + \sigma)}\left(\frac{\beta_2(\theta E + I)\Lambda_v}{\mu_v} - \mu_vY\right) \\
 &= \frac{\mu_1\mu_2\mu_v^2(\mathcal{R}_0^2 - 1)}{\beta_2\Lambda_v(\mu_2\theta + \sigma)}Y.
 \end{aligned} \tag{4.2}$$

Thus, $\mathcal{R}_0 \leq 1$ ensures that $L(E, I, Y)' \leq 0$ for all $E, I, Y > 0$. It is easy to verify that the disease-free equilibrium $P_0\left(\frac{\Lambda_h}{\mu_h}, 0, 0, \frac{\Lambda_v}{\mu_v}, 0\right)$ is the largest invariant set in $\{(S, E, I, X, Y) \in \Omega_1 : L'(E, I, Y) = 0\}$, and hence by the LaSalle's invariance principle [20], we conclude that all trajectories starting in Ω_1 approach P_0 for $\mathcal{R}_0 \leq 1$. That is to say, P_0 is globally asymptotically stable in Ω_1 if $\mathcal{R}_0 \leq 1$.

Next, we will investigate the global stability of the endemic equilibrium P^* in the positively invariant set Ω_1 by constructing Lyapunov function.

Theorem 6. *The endemic equilibrium P^* of system (2.6) is globally asymptotically stable in the interior of the set Ω_1 if $\mathcal{R}_0 > 1$.*

Proof. From (2.10), we know that S^*, E^*, I^*, X^*, Y^* satisfy

$$\begin{cases} \mu_h = \frac{\Lambda_h - \beta_1 S^* Y^*}{S^*}, \\ \mu_1 = \frac{\beta_1 S^* Y^*}{E^*}, \\ \mu_2 = \frac{\sigma E^*}{I^*}, \\ \mu_v = \frac{\beta_2(\theta E^* + I^*) X^*}{Y^*}, \end{cases} \tag{4.3}$$

then the system (2.6) can be rewritten as the following form

$$\begin{cases} \frac{dS(t)}{dt} = \Lambda_h \left(1 - \frac{S}{S^*}\right) + \beta_1 S^* Y^* \left(1 - \frac{SY}{S^* Y^*}\right), \\ \frac{dE(t)}{dt} = \beta_1 S^* Y^* \left(\frac{SY}{S^* Y^*} - \frac{E}{E^*}\right), \\ \frac{dI(t)}{dt} = \sigma E^* \left(\frac{E}{E^*} - \frac{I}{I^*}\right), \\ \frac{dX}{dt} = \Lambda_v \left(1 - \frac{X}{X^*}\right) + \beta_2 \theta E^* X^* \left(1 - \frac{EX}{E^* X^*}\right) + \beta_2 I^* X^* \left(1 - \frac{IX}{I^* X^*}\right), \\ \frac{dY}{dt} = \beta_2 \theta E^* X^* \left(\frac{EX}{E^* X^*} - \frac{Y}{Y^*}\right) + \beta_2 I^* X^* \left(\frac{IX}{I^* X^*} - \frac{Y}{Y^*}\right). \end{cases} \tag{4.4}$$

To illustrate the uniqueness and global stability of P^* , we set

$$V_1(t) = \frac{1}{\beta_1 S^* Y^*} \left(S - S^* - S^* \ln \frac{S}{S^*} \right), \quad V_2(t) = \frac{1}{\beta_1 S^* Y^*} \left(E - E^* - E^* \ln \frac{E}{E^*} \right),$$

$$V_3(t) = \frac{1}{\sigma E^*} \left(I - I^* - I^* \ln \frac{I}{I^*} \right), \quad V_4(t) = X - X^* - X^* \ln \frac{X}{X^*} \quad \text{and}$$

$$V_5(t) = Y - Y^* - Y^* \ln \frac{Y}{Y^*}. \quad \text{Then the derivative of functions}$$

$V_1(t), V_2(t), V_3(t), V_4(t), V_5(t)$ along solutions of system (2.6) yield

$$\begin{aligned} V_1'(t)|_{(2.6)} &= \frac{1}{\beta_1 S^* Y^*} \left(1 - \frac{S^*}{S} \right) \left(\Lambda_h \left(1 - \frac{S}{S^*} \right) + \beta_1 S^* Y^* \left(1 - \frac{SY}{S^* Y^*} \right) \right) \\ &\leq \left(1 - \frac{S^*}{S} \right) \left(1 - \frac{SY}{S^* Y^*} \right) \\ &= 1 - \frac{S^*}{S} - \frac{SY}{S^* Y^*} + \frac{Y}{Y^*}, \end{aligned} \tag{4.5}$$

$$\begin{aligned} V_2'(t)|_{(2.6)} &= \frac{1}{\beta_1 S^* Y^*} \left(1 - \frac{E^*}{E} \right) \left(\beta_1 S^* Y^* \left(\frac{SY}{S^* Y^*} - \frac{E}{E^*} \right) \right) \\ &= 1 - \frac{E}{E^*} - \frac{SYE^*}{S^* Y^* E} + \frac{SY}{S^* Y^*}, \end{aligned} \tag{4.6}$$

$$\begin{aligned} V_3'(t)|_{(2.6)} &= \frac{1}{\sigma E^*} \left(1 - \frac{I^*}{I} \right) \left(\sigma E^* \left(\frac{E}{E^*} - \frac{I}{I^*} \right) \right) \\ &= 1 - \frac{I^*}{I} - \frac{EI^*}{E^* I} + \frac{E}{E^*}, \end{aligned} \tag{4.7}$$

$$\begin{aligned} V_4'(t)|_{(2.6)} &= \left(1 - \frac{X^*}{X} \right) \left(\Lambda_v \left(1 - \frac{X}{X^*} \right) + \beta_2 \theta E^* X^* \left(1 - \frac{EX}{E^* X^*} \right) \right. \\ &\quad \left. + \beta_2 I^* X^* \left(1 - \frac{IX}{I^* X^*} \right) \right) \\ &\leq \left(1 - \frac{X^*}{X} \right) \left(\beta_2 \theta E^* X^* \left(1 - \frac{EX}{E^* X^*} \right) + \beta_2 I^* X^* \left(1 - \frac{IX}{I^* X^*} \right) \right) \\ &= \beta_2 \theta E^* X^* \left(1 - \frac{X^*}{X} - \frac{EX}{E^* X^*} + \frac{E}{E^*} \right) \\ &\quad + \beta_2 I^* X^* \left(1 - \frac{X^*}{X} - \frac{IX}{I^* X^*} + \frac{I}{I^*} \right), \end{aligned} \tag{4.8}$$

and

$$\begin{aligned} V_5'(t)|_{(2.6)} &= \left(1 - \frac{Y^*}{Y} \right) \left(\beta_2 \theta E^* X^* \left(\frac{EX}{E^* X^*} - \frac{Y}{Y^*} \right) + \beta_2 I^* X^* \left(\frac{IX}{I^* X^*} - \frac{Y}{Y^*} \right) \right) \\ &= \beta_2 \theta E^* X^* \left(1 - \frac{Y}{Y^*} - \frac{EXY^*}{E^* X^* Y} + \frac{EX}{E^* X^*} \right) \\ &\quad + \beta_2 I^* X^* \left(1 - \frac{Y}{Y^*} - \frac{IXY^*}{I^* X^* Y} + \frac{IX}{I^* X^*} \right). \end{aligned} \tag{4.9}$$

Define Lyapunov function as follows:

$$V(t) = c_1V_1(t) + c_2V_2(t) + c_3V_3(t) + c_4V_4(t) + V_5(t), \tag{4.10}$$

where $c_i > 0 (i = 1, 2, 3, 4)$ are left unspecified. Then the derivative of function $V(t)$ with respect to t along solutions of system (2.6) is given by

$$\begin{aligned} V'(t)|_{(2.6)} &= c_1V_1'(t)|_{(2.6)} + c_2V_2'(t)|_{(2.6)} + c_3V_3'(t)|_{(2.6)} + c_4V_4'(t)|_{(2.6)} + V_5'(t)|_{(2.6)} \\ &\leq c_1 \left(1 - \frac{S^*}{S} - \frac{SY}{S^*Y^*} + \frac{Y}{Y^*} \right) + c_2 \left(1 - \frac{E}{E^*} - \frac{SYE^*}{S^*Y^*E} + \frac{SY}{S^*Y^*} \right) \\ &\quad + c_3 \left(1 - \frac{I^*}{I} - \frac{EI^*}{E^*I} + \frac{E}{E^*} \right) \\ &\quad + \beta_2 \theta E^* X^* \left(1 - \frac{Y}{Y^*} - \frac{EXY^*}{E^*X^*Y} + \frac{EX}{E^*X^*} \right) \\ &\quad + c_4 \left(\beta_2 \theta E^* X^* \left(1 - \frac{X^*}{X} - \frac{EX}{E^*X^*} + \frac{E}{E^*} \right) \right) \\ &\quad + c_4 \left(\beta_2 I^* X^* \left(1 - \frac{X^*}{X} - \frac{IX}{I^*X^*} + \frac{I}{I^*} \right) \right) \\ &\quad + \beta_2 I^* X^* \left(1 - \frac{Y}{Y^*} - \frac{IXY^*}{I^*X^*Y} + \frac{IX}{I^*X^*} \right) \\ &= c_1 + c_2 + c_3 + (c_4 + 1)(\beta_2 \theta E^* X^* + \beta_2 I^* X^*) - c_1 \frac{S^*}{S} - c_1 \frac{SY}{S^*Y^*} \\ &\quad - c_2 \frac{SYE^*}{S^*Y^*E} - c_3 \frac{EI^*}{E^*I} - c_4 (\beta_2 \theta E^* X^* + \beta_2 I^* X^*) \frac{X^*}{X} \\ &\quad - \beta_2 \theta E^* X^* \frac{EXY^*}{E^*X^*Y} - \beta_2 I^* X^* \frac{IXY^*}{I^*X^*Y} \\ &\quad + (c_1 - \beta_2 (\theta E^* + I^*) X^*) \frac{Y}{Y^*} + (c_2 - c_1) \frac{SY}{S^*Y^*} \\ &\quad + (c_3 - c_2 + c_4 \beta_2 \theta E^* X^*) \frac{E}{E^*} + (c_4 \beta_2 I^* X^* - c_3) \frac{I}{I^*} \\ &\quad + \beta_2 \theta E^* X^* (1 - c_4) \frac{EX}{E^*X^*} + \beta_2 I^* X^* (1 - c_4) \frac{IX}{I^*X^*}. \end{aligned} \tag{4.11}$$

To choose the suitable constants $c_i > 0 (i = 1, 2, 3, 4)$ such that $V'(t)|_{(2.6)}$ is negative definite or semidefinite, we eliminate the terms Y, SY, E, I, EX, IX by taking

$$c_1 = \beta_2 (\theta E^* + I^*) X^*, c_2 = \beta_2 (\theta E^* + I^*) X^*, c_3 = \beta_2 X^* I^*, c_4 = 1. \tag{4.12}$$

Consequently, the function (4.10) is specified. Then (4.11) is in turn given as

$$\begin{aligned} V'(t)|_{(2.6)} &\leq c_1 + c_2 + c_3 + (c_4 + 1)(\beta_2 \theta E^* X^* + \beta_2 I^* X^*) - c_1 \frac{S^*}{S} - c_1 \frac{SY}{S^*Y^*} \\ &\quad - c_2 \frac{SYE^*}{S^*Y^*E} - c_3 \frac{EI^*}{E^*I} - c_4 (\beta_2 \theta E^* X^* + \beta_2 I^* X^*) \frac{X^*}{X} \\ &\quad - \beta_2 \theta E^* X^* \frac{EXY^*}{E^*X^*Y} - \beta_2 I^* X^* \frac{IXY^*}{I^*X^*Y} \\ &= \beta_2 \theta E^* X^* \left(4 - \frac{S^*}{S} - \frac{SYE^*}{S^*Y^*E} - \frac{X^*}{X} - \frac{EXY^*}{E^*X^*Y} \right) \\ &\quad + \beta_2 I^* X^* \left(5 - \frac{S^*}{S} - \frac{SYE^*}{S^*Y^*E} - \frac{X^*}{X} - \frac{IXY^*}{I^*X^*Y} - \frac{EI^*}{E^*I} \right) \end{aligned} \tag{4.13}$$

By the relationship between the arithmetic and the associated geometric means, we have

$$\begin{aligned}
 4 - \frac{S^*}{S} - \frac{S Y E^*}{S^* Y^* E} - \frac{X^*}{X} - \frac{E X Y^*}{E^* X^* Y} &\leq 0, \\
 5 - \frac{S^*}{S} - \frac{S Y E^*}{S^* Y^* E} - \frac{X^*}{X} - \frac{I X Y^*}{I^* X^* Y} - \frac{E I^*}{E^* I} &\leq 0.
 \end{aligned}
 \tag{4.14}$$

That is, $V'(t)|_{(2.6)} \leq 0$ and the equality holds if and only if $S = S^*$, $X = X^*$, $\frac{E}{E^*} = \frac{Y}{Y^*} = \frac{I}{I^*}$. It can be easily verified that the largest invariant set of system (2.6) on the set

$$\left\{ (S, E, I, X, Y) : S = S^*, X = X^*, \frac{E}{E^*} = \frac{Y}{Y^*} = \frac{I}{I^*} \right\}$$

is the singleton $\{P^*\}$. Therefore, by the LaSalle's invariance principle [20], it follows that the endemic equilibrium $P^*(S^*, E^*, I^*, X^*, Y^*)$ is globally asymptotically stable in Ω_1 when $\mathcal{R}_0 > 1$. This completes the proof.

Remark 2: The basic reproduction number \mathcal{R}_0 gives a sharp threshold that completely determines their global dynamics [21].

5. Numerical Simulation

In this section, we first provide results from numerical simulations of model (2.1) that illustrate and support our theoretical results. In the model (2.1), all parameters are in months and their values are shown in Table 1. According to the above parameters of the model (2.1), we conduct a global sensitivity analysis on the basic reproductive number \mathcal{R}_0 by employing Latin Hypercube Sampling (LHS) and partial rank correlation coefficients (PRCCs) [22] [23]. Figure 2 and Figure 3 depict our sensitivity and uncertainty analysis, which involved computing the PRCCs of \mathcal{R}_0 using the LHS method [22]. From Figure 2, we

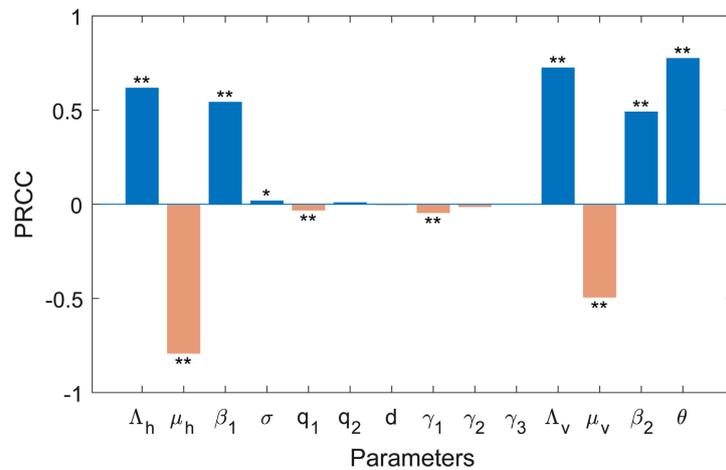


Figure 2. PRCC performed on the system (1). Parameters are listed in Table 1. * indicates that PRCCs are very significantly different from zero ($p < 0.05$), and ** denotes that PRCCs are very significantly different from zero ($p < 0.01$).

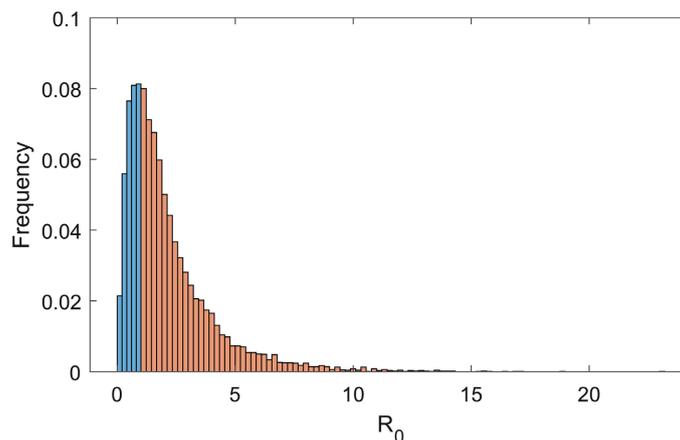


Figure 3. The distribution of \mathcal{R}_0 under LHS parameters.

can easily observe that \mathcal{R}_0 is sensitive to the natural death rate of citrus population (μ_h), infection intensity relative to infected (θ), psyllid recruitment rate (Λ_v), citrus recruitment rate (Λ_h), virus transmission probability from psyllid to plant (β_1), virus transmission probability from plant to psyllid (β_2), natural death rate of psyllid population (μ_v), where \mathcal{R}_0 increases with the increase of θ , Λ_v , Λ_h , β_1 and β_2 , but decreases with the increase of μ_h and μ_v . This shows that reducing θ , Λ_v , Λ_h , β_1 , β_2 , or increasing μ_h , μ_v will help control the spread of the disease. For uncertainty analysis, it can be seen from **Figure 3** that the probability of $\mathcal{R}_0 > 1$ is significantly higher than that of $\mathcal{R}_0 < 1$ (about 2:1), which indicates that disease control should be strengthened for preventing the occurrence of endemic diseases in the current situation.

To better understand the spread of the disease, we further do numerical simulations. Select parameter values as: $\Lambda_h = 6.6$, $\mu_h = 0.0033$, $\beta_1 = 0.003125$, $\sigma = 0.25$, $d = 0.025$, $\gamma_1 = 0.1$, $\gamma_2 = 0.05$, $\Lambda_v = 2000$, $\mu_v = 5$, $\beta_2 = 0.0025$, $\theta = 0.1$, and $q_1 = 0$, $q_2 = 0$, $\gamma_3 = 0$ (*i.e.*, do not take quarantine measures). We get $\mathcal{R}_0 = 3.4133$, $I^* = 54.3824$, the disease is persistent (**Figure 4**). Take certain isolation measures $q_1 = 0.3$, $q_2 = 0.3$, $\gamma_3 = 0.2$, we have $\mathcal{R}_0 = 1.2066$, $I^* = 2.0847$, there are still endemic disease (**Figure 5**). One interesting thing is that for $\mathcal{R}_0 > 1$, the number of the infected populations has a distinct wavy pattern, with multiple peaks and troughs, which means that at some time the infected individual goes to zero and that doesn't mean that the epidemic is under control, it's going to erupt in the near future. Further, if $q_1 = 0.45$, $q_2 = 0.45$, $\gamma_3 = 0.2$, we have $\mathcal{R}_0 = 0.9444$, the endemic disease disappears and the disease is well controlled (**Figure 6**). This shows that the quarantine measure is effective in controlling the spread of HLB.

In addition, the rate of the exposed to the quarantined (q_1) and rate of the infectious convert to the quarantined (q_2) also have a certain influence on \mathcal{R}_0 . Therefore, in order to further analyze the influence of q_1 and q_2 on \mathcal{R}_0 , the sensitivity of parameters q_1, q_2 for \mathcal{R}_0 is shown in **Figure 7**. It can be seen that controlling q_2 alone cannot make \mathcal{R}_0 drop below 1, and controlling q_1

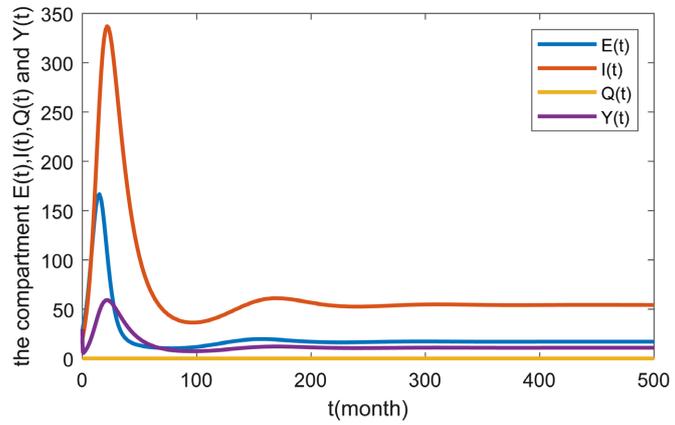


Figure 4. Time series plot of infectious citrus tree without quarantine ($q_1 = 0, q_2 = 0, \gamma_3 = 0$).

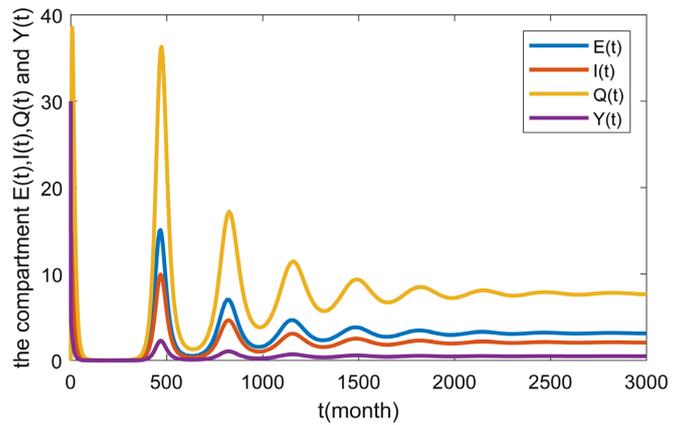


Figure 5. Time series plot of infectious citrus tree with quarantine ($q_1 = 0.3, q_2 = 0.3, \gamma_3 = 0.2$).

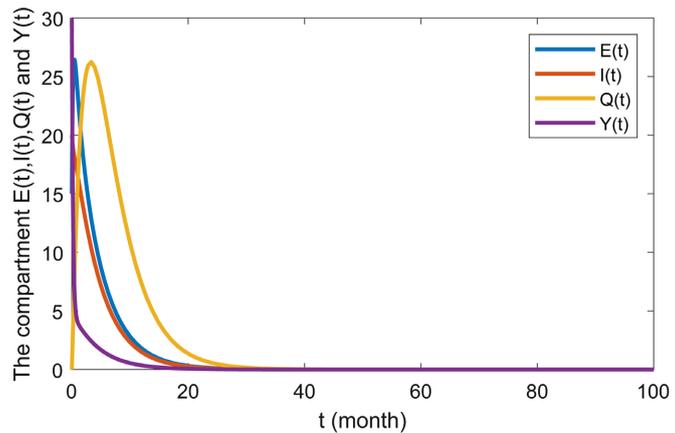


Figure 6. Time series plot of infectious citrus tree with quarantine ($q_1 = 0.45, q_2 = 0.45, \gamma_3 = 0.2$).

alone can only make \mathcal{R}_0 close to 1, but the joint control of q_1, q_2 can make $\mathcal{R}_0 < 1$.

According to PRCC analysis (see **Figure 2**), virus transmission probability from psyllid to plant (β_1), virus transmission probability from plant to psyllid (β_2), natural death rate of citrus population (μ_h) and natural death rate of psyllid population (μ_v) have great influence on \mathcal{R}_0 . We conduct sensitivity analysis of these four parameters to \mathcal{R}_0 , as shown in **Figure 8** and **Figure 9**. It implies that disease control can be achieved by inhibiting β_1 and β_2 or increasing μ_h and μ_v simultaneously.

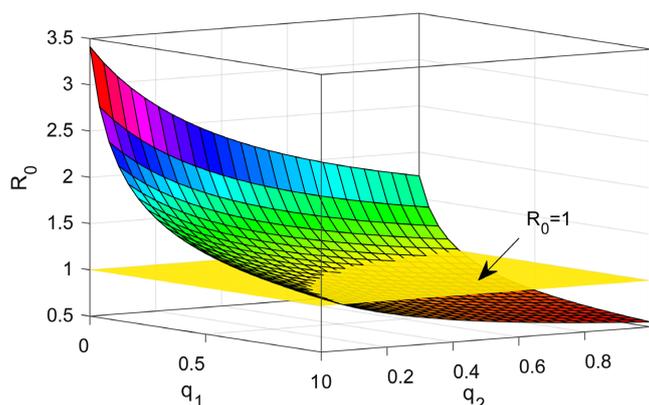


Figure 7. The sensitivity analysis of for \mathcal{R}_0 with the parameters q_1, q_2 .

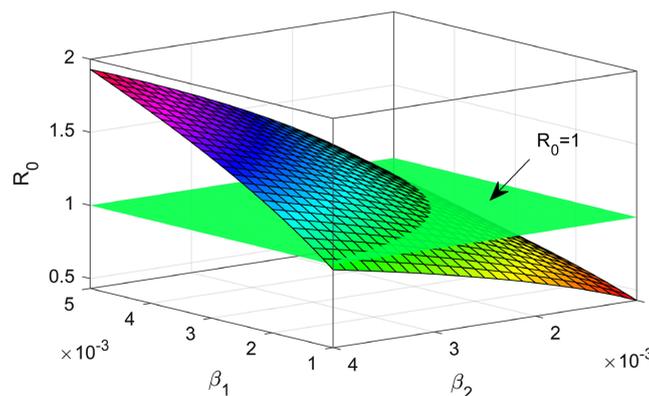


Figure 8. The sensitivity analysis of for \mathcal{R}_0 with the parameters β_1, β_2 .

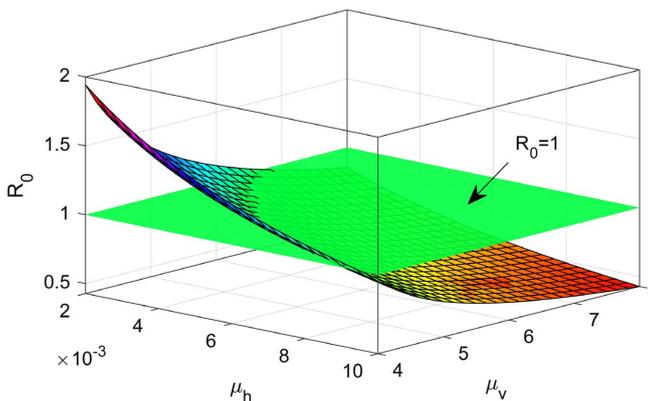


Figure 9. The sensitivity analysis of for \mathcal{R}_0 with the parameters μ_h, μ_v .

6. Conclusions

In this work, a citrus-psyllid dynamic model with quarantine measure is formulated. Based on the method of next-generation matrix, we obtain the expression of basic reproductive number \mathcal{R}_0 . The global stability of disease-free equilibrium and endemic equilibrium are demonstrated by constructing Lyapunov functions.

Numerical simulation shows that when $\mathcal{R}_0 > 1$, the number of infected hosts has obvious wavy, with multiple peaks and troughs, which implies that at a time infected individuals tending to zero does not mean that the disease is under control, instead, it may break out soon. Additionally, our investigations show that a certain degree quarantine measure is effective, and it is more effective to isolate the exposed than the infected host. Moreover, comprehensive quarantine can more effectively control the outbreak of disease. These results can provide a reference for fruit industry to conduct comprehensive management of HLB.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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