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Iterating with Fuzzy Parameters to Produce Exact Results

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Abstract

Iteration problems such as compound interest calculations have well-specified parameters and aim to derive an exact value. Not all problems offer well-specified parameters, even for well-defined dynamic equations; the linear "weak field approximation" of general relativity is iteratively equivalent to Einstein's non-linear field equation, but the exact parameters involved in some applications are unknown. This paper develops a theory based on "fuzzy" parameters that must produce exact results. The problem is analyzed and example calculations are produced.

Keywords

Iteration, Energy of Motion, Gravitomagnetism, Dimensional Analysis, Quantum Potential, Kinetic Energy

1. Introduction

Fundamental concepts in physics may lack rigor. For example, Newton's force equation $f = ma$ defines each term in terms of the other two: $a = f/m$, $m = f/a$. Feynman noted that while this definition of force is somewhat circular, it is valuable in that, when objects are observed to accelerate, it tells us to search for an appropriate force. Yet fundamental forces are not unambiguously understood; of the four forces—gravitational, electromagnetic, weak and strong—detailed understanding of at least the strong force is missing. And general relativity effectively replaces the concept of force with geodesic paths.

Similarly ill-defined is *kinetic energy*, defined as the "energy of motion". Introduced in this way to high school students, it is still understood in this way by Ph.D. physicists. The other key energy, *potential energy*, is typically understood as "energy of position"; however in 2015 Holland [1] proposed "*quantum poten-*

tial energy as concealed motion": "…*since what is termed* '*kinetic energy*' *in quantum mechanics may be regarded literally as* ***energy due to motion***".

The anomaly is that most forms of energy, gravitational, electromagnetic, chemical, nuclear are associated with real physical fields, whereas energy of motion does not define any physical entity or medium to store energy as it accumulates. The vagueness of the mechanics does not typically detract from its utility in dynamic equations, but it may underlie Feynman's [2] view of the continuity of movement, conservation of linear momentum, as an unexplained mystery:

"*The reason why things coast forever has never been found out. The law of inertia has no known origin*".

A hypothesis that kinetic energy at the particle level is stored in a field faces several obstacles—identification of the field; coupling of the field to inertial mass; calculation of the energy stored in the field; explanation of the dynamics of the field. These issues are treated in this paper.

2. Relevant Background

Kinetic energy typically shows up in Lagrangian physics: $L = T - V$ where T is kinetic energy and V is potential energy; the Euler-Lagrange equation leads to the equations of motion. Holland addressed the issue of *quantum potential* as "concealed" energy associated with kinetic energy of particle interactions—hidden degrees of freedom. To this end he breaks kinetic energy T into T_q and T_Q where coordinates q_i are observable and coordinates Q_i are "concealed" in typical quantum treatments. Via the standard definition of canonical momentum, he defines

$$P_i = \frac{\partial L}{\partial \dot{Q}_i} = A_{ij}(q)\dot{Q}_j \tag{1}$$

In these terms the modified Lagrangian function is

$$L'(q,\dot{q},P) = L - P_i\dot{Q}_i = B_{ij}(q)\dot{q}_i\dot{q}_j - A_{ij}^{-1}(q)P_iP_j \tag{2}$$

The Euler-Lagrange equations for q_i derive from L':

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L'}{\partial \dot{q}_i}\right) = \frac{\partial L'}{\partial q_i} \tag{3}$$

such that the motion proceeds *as if* this *q*-component of the system possesses potential energy

$$V_q = T_Q, \quad L' = T_q - V_q. \tag{4}$$

That is, in the physics of the *concealed motion*, the *kinetic energy* of the *Q*-system now shows up as *potential energy* of the *q*-system. The above was essentially shown by Routh [3]. Holland applies these results to quantum potential via a series of complex choices leading to a Lagrangian in which the effects due to the concealed motion are attributed to the quantum potential energy $\left(\hbar^2/8m\right)\rho_0 u_i u_j$. He obtains the quantum mechanical formula for what is termed *kinetic energy* in quantum mechanics, regarded literally as *energy due to mo-*

tion, u_i. The resulting Hamiltonian is:

$$H = (1/2)\int m\rho_0(x) g_{\mu\nu}(x)\dot{\xi}_\mu(x)\dot{\xi}_\nu(x)\mathrm{d}^3x \tag{5}$$

Holland's model has the status "proof of concept"; no explanation exists for the *density-dependent* factor in the concealed kinetic energy. It appears in the continuity equation where it differs from the quantum [probability] expression $\rho_0 \sim |\psi|^2$. His specific choices allow other ways to formulate this problem; in fact, an alternate model would let the concealed variables represent internal freedoms, such as *rotation*. We focus on an interpretation of "*energy due to motion*".

3. Field Energy

Unlike discrete mass m_0 at a point, physical fields *distribute energy over three dimensions*, plus time. A field $\boldsymbol{F}(\boldsymbol{r},t)$ has energy *density* proportional to $|\boldsymbol{F}\cdot\boldsymbol{F}|$. Field equations are expressed in terms of the source of the field, *s*, and are typically solved via

$$\nabla f = s \quad \Rightarrow \quad f = \nabla^{-1}s \tag{6}$$

where ∇^{-1} is a Green's function-based anti-derivative [4] of the form: $f = \nabla^{-1}s \Rightarrow \int_M s' + \int_{\partial M} s$.

The *gravitomagnetic field*, denoted by $\boldsymbol{C}$, is described by the following equations [5]:

$$\nabla\times\boldsymbol{C} = -\kappa\rho\boldsymbol{v} + \partial_t\boldsymbol{G} \tag{7a}$$

$$\boldsymbol{C} = \kappa\rho\boldsymbol{v}\times\boldsymbol{r} \tag{7b}$$

$$\rho_C = \kappa^{-1}|\boldsymbol{C}\cdot\boldsymbol{C}|. \tag{7c}$$

Consider the C-field circulation induced by rest mass m_0 in motion. At distance $\boldsymbol{r}$, the circulation is $\sim\rho\boldsymbol{v}$ where $\rho = m_0/r^3$ is local mass density. At rest m_0 is not accompanied by C-field circulation. The minus sign in Equation (7a) is associated with the direction of circulation, not its magnitude. Gravitomagnetic *torsion* supports only left-handed circulation; for momentum pointing in the direction of the left thumb the C-field curls in the directions of the fingers of the left-hand. Transition from rest $\boldsymbol{v}=0$ to velocity $\boldsymbol{v}=\Delta\boldsymbol{v}$ at time *t* is analogous to transition from velocity $\boldsymbol{v}$ to velocity $\boldsymbol{v}+\Delta\boldsymbol{v}$. Every increase in velocity $\mathrm{d}v/\mathrm{d}t$ is accompanied by positive definite change in magnitude of C-field circulation, an iterative condition.

4. Iteration Rate

A defining parameter is *speed of sound* in a field viewed as a perfect fluid. The "speed of sound" in gravity is now known to be the speed of light [6]. The recognition that every increase in local velocity $\boldsymbol{v}\to\boldsymbol{v}+\Delta\boldsymbol{v}$ induces a corresponding increase in C-field circulation means that acceleration is an iterative procedure. A field cannot self-interact over a region of interest faster than the

speed of light. For a region with radius r this implies a "cycle time" $\sim r/c$. For an electron, if $r \approx 10^{-8}$ m and $c \approx 10^{8}$ m/sec then the iteration period is $\sim 10^{-16}$ sec or 100 *attosec*; the iteration process can repeat approximately 10^{16} times every second. The limiting factor becomes the "interest rate" which will be used to compound $\sim 10^{16}$ times. Every step of acceleration produces an induced circulation, which, in turn, implies increased energy of the circulating field, hence increased induction.

The initial density of the rest mass is ρ. After acceleration to velocity $\boldsymbol{v}$ C-field circulation is proportional to ρ while C-field energy-density is proportional to $\kappa'\rho^2$ where κ' subsumes velocity and radius parameters and any other necessary dimensional factors. This naïve interpretation leads to the relevant density at this step $\rho' = \rho + \kappa'\rho^2$: $\rho' = \rho(1+\kappa'\rho)$. The next acceleration step is based on added density ρ'; the corresponding increased density is represented $\rho'' = \rho'(1+\kappa''\rho') = \rho(1+\kappa'\rho)\big(1+\kappa''(1+\kappa'\rho)\big)$. We simplify this formalism by unrealistically fixing $\kappa = \kappa' = \kappa'' = \cdots$ such that the iterated form becomes $\rho^{n+1} = \rho^n\left(1+\kappa\rho^n\right)$,

$$\begin{gathered}\rho \\ \rho(1+\kappa\rho) \\ \rho(1+\kappa\rho)\big(1+\kappa\rho(1+\kappa\rho)\big) \\ \rho(1+\kappa\rho)\big(1+\kappa\rho(1+\kappa\rho)\big)\big(1+\kappa\rho(1+\kappa\rho)\big(1+\kappa\rho(1+\kappa\rho)\big)\big)\end{gathered} \tag{8}$$

Physically the local mass density m_0/r^3, when put in motion, induces a local circulation $\nabla\times\boldsymbol{C}$, that itself induces more local circulation, etc. In this way, a macroscopically tiny force of gravity, can deliver surprising energy at realistic density, say that of an electron, experimentally claimed to have a radius less than 10^{-18} m and theoretically claimed to be greater than Planck's length, 10^{-33} m. Our immediate goal is to show that $\rho^{n+1} = \rho^n\left(1+\kappa\rho^n\right)$ "*can keep up with*" reality [*i.e.*, finite acceleration] and that the key to the "compound interest" formula is *periodicity* at which the "principle" [C-field energy] is compounded. The *compound interest* can grow at any natural growth as long as the compounding period is sufficiently short. The goal is to show that the C-field iteration implied by equations

$$\boldsymbol{C} = \kappa\rho\boldsymbol{v}\times\boldsymbol{r},\ \nabla\times\boldsymbol{C} = -\kappa\rho\boldsymbol{v} + \partial_t\boldsymbol{G}, \text{ and } \rho_C = \kappa^{-1}\left|\boldsymbol{C}\cdot\boldsymbol{C}\right|,$$

can "keep up with" any quadratic force applied to m_0. If change in circulation can "keep up with" change in velocity of source density, then every erg of energy injected via accelerating force $\boldsymbol{F} = m\boldsymbol{a} = m\,\mathrm{d}\boldsymbol{v}/\mathrm{d}t$ is stored in the circulating C-field induced thereby. The plot shown in **Figure 1** is "capped" at 10 on the vertical axis; the slope becomes nearly vertical; density grows from 10 to the greatest number supported by the system, $\sim 10^{247}$, at which point the calculation stops; implying that density can *keep up with* the energy being applied to accelerate the particle for variable ranges: $0 < \kappa < 0.3$ and $0 < \rho < 0.3$. If these "fuzzy" parameters are chosen to be significantly smaller, then many more iterations are required to achieve the same result. In physical reality, these occur on the order of 10^{16} per second, but computer calculations take far longer.

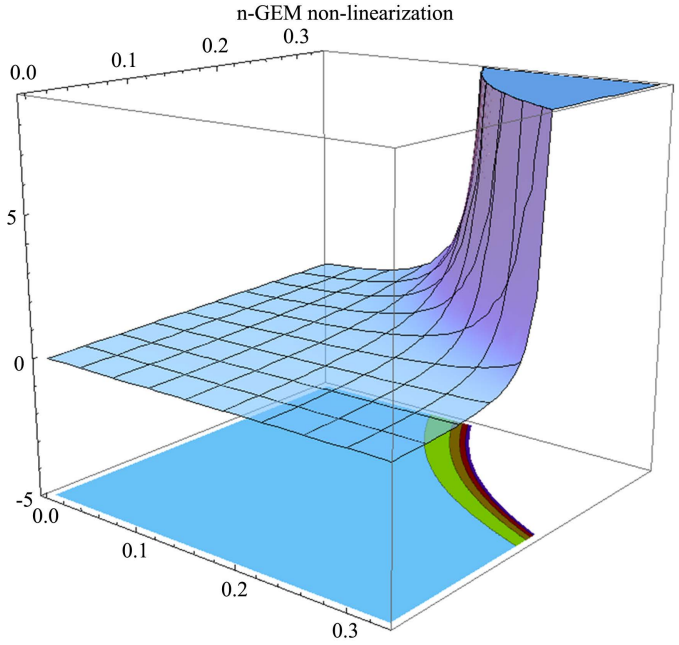

Figure 1. Plot of $\rho^{n+1} = \rho^n\left(1+\kappa\rho^n\right)$.

The variables are chosen to demonstrate iterative behavior rather than exact physical simulation. The ground plane shows a density contour plot with colors chosen by the system; the small initial blue density is smoothly varying. As the *corner is turned* the larger density shows yellow, then orange, then red. The white region is where calculated density exceeds 10^{247}, the largest number that the system supports. Iterated density rises from 10 to $>10^{247}$ almost vertically. Iteration in this region should "keep up with" any acceleration from physically real forces.

5. Calculations

In order to discuss physical reality, we derive approximate values for real particles and perform dimensional analysis to check the integrity of our expressions. Begin with $\kappa = g/c^2$ where g is Newton's gravitational constant, 6.67×10^{-11} $m^3 \cdot kg^{-1} \cdot sec^{-2}$. If length = l, mass = m, and time = t, the dimensional representation of g is l^3/mt^2. For the speed of light, $c = 3\times10^8$ m/sec, then $\kappa = g/c^2 \sim 10^{-27}$ m/kg, with dimension l/m, *i.e.*, length over mass. Therefore, for mass density $\rho = mass/vol \sim m/l^3$, velocity $\boldsymbol{v}$ m/sec $\sim l/t$, and distance $r \sim l$ we have:

$$\boldsymbol{C} = \kappa\rho\boldsymbol{v}\times\boldsymbol{r} \quad \rightarrow \quad \frac{l}{m}\frac{m}{l^3}\frac{l}{t}\frac{l}{1} = \frac{1}{t}. \text{ The C-field has dimensions of frequency.} \tag{9}$$

A field defined by circulation is reasonably characterized by circular frequency. Real physical fields have energy density proportional to the field strength squared, so we write the definition of C-field energy density ρ_C as

$$\rho_C = \kappa^{-1}\left|\boldsymbol{C}\cdot\boldsymbol{C}\right| = \kappa^{-1}\kappa\kappa\rho^2 v^2 r^2 \tag{10}$$

The dimensional check:

$$\rho_C \sim \frac{m}{l}\frac{l}{m}\frac{l}{m}\frac{m^2}{l^6}\frac{l^2}{t^2}\frac{l^2}{1} = \frac{m}{l^3}\frac{l^2}{t^2} \rightarrow \quad \sim 10^{-27}\frac{mv^2}{vol} \tag{11}$$

C-field energy density has dimensions of kinetic energy distributed over volume: $\rho_C \sim \rho v^2$. For an electron with rest mass $m_0 \sim 9.1\times10^{-31}$ kg and measured radius $\leq 10^{-18}$ m the density of the electron is large $\sim 10^{24}$ kg/m^3, but the density we are most concerned with is relative to the volume associated with the field induced by a moving electron with kinetic energy $m_0v^2/2$. Assume the C-field induced by a moving electron is distributed over several atomic radii, typically 10^{-10} m. If we assume a radius of 10 angstroms, 10^{-9} m, the relevant C-field volume becomes $\sim 10^{-27}$ m^3. According to the above equations the C-field energy density over the relevant volume is on the order of the kinetic energy of the electron:

$$\rho_C \sim \frac{10^{-27}}{10^{-27}}mv^2 \quad \rightarrow \quad mv^2 \tag{12}$$

with inertial mass $m = \gamma m_0$ where inertial factor $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$, Einstein's $E = mc^2$ yields:

$$E \cong m_0c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) \quad \Rightarrow \quad m_0c^2 + m_0v^2/2 \tag{13}$$

The m_0c^2 term represents the rest energy of the particle and the $m_0v^2/2$ term represents the C-field energy induced by the accelerating force. In these calculations based on "fuzzy" variables the symbol ~ means *approximately* or "*on the order of*". It may, for example, indicate that scalar constants such as 4π have been suppressed in favor of focusing on the physical variable of interest.

6. Iterations

Thus, depending upon volume occupied by C-field circulation, the initial induced energy density $\sim m_0v^2$ may be quite small, say $0.001m_0v^2$. This energy density represents *new* mass density, which itself induces C-field circulation; the increased circulation induces still more circulation in iterative fashion. The initial energy density is $\rho = m_0c^2/r^3$. A more realistic physical iteration is:

$$\rho = \rho$$

$$\rho' = \rho + \alpha\rho v_1^2 = \rho\left(1 + \alpha v_1^2\right), \quad \alpha < 1.0$$

$$\rho'' = \rho'\left(1 + \alpha v_2^2\right) = \rho\left(1 + \alpha v_1^2\right)\left(1 + \alpha v_2^2\right)$$

$$\rho''' = \rho''\left(1 + \alpha v_3^2\right) = \rho\left(1 + \alpha v_1^2\right)\left(1 + \alpha v_2^2\right)\left(1 + \alpha v_3^2\right)$$

Actually, $\boldsymbol{v}_i$ is increasing $\boldsymbol{v}_i = \boldsymbol{v}_{i-1} + \mathrm{d}\boldsymbol{v}_i$, $\mathrm{d}\boldsymbol{v}_i = \dfrac{q\boldsymbol{E}}{m_i}\mathrm{d}t$.

Note that increment $\mathrm{d}\boldsymbol{v}_i$ decreases with mass.

In **Figure 2**, the velocity $\boldsymbol{v}$ is linearly increased from zero to 30 and $\alpha = 0.05$. Compare this to compound interest formula $A = p\left(1 + r/n\right)^{nt}$ or

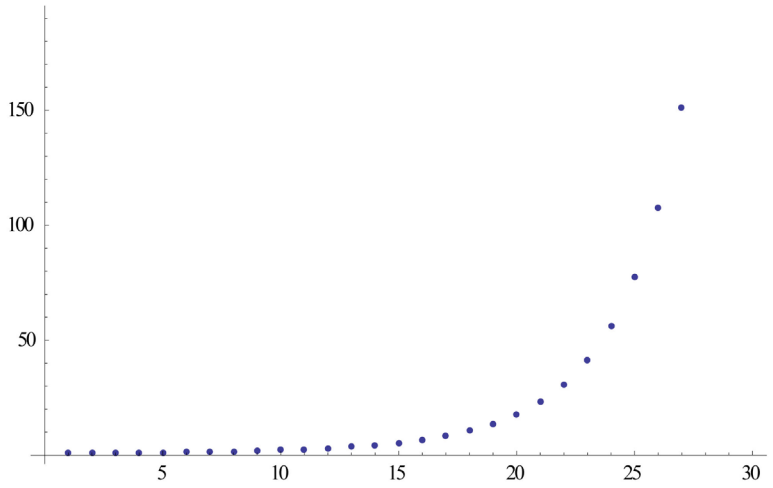

Figure 2. Plot of $\rho_{i+1} = \rho_i\left(1+\alpha v_{i+1}^2\right)$.

Wikipedia's $(1+\alpha)^n$. Here α is not an *annual* rate, but a "per period" rate where the period is the cycle time, ~10^{-16} sec for electrons. The ρ_C we have calculated is the energy density of the C-field. In the iteration above it is added to mass density ρ of the electron, therefore ρ_C must be converted to mass density, *i.e.*, the scalar α must contain a factor $1/c^2$ if we choose units other than $c=1$.

7. Kinetic Energy Has Value $mv^2/2$

Kinetic energy depends only on mass and velocity. One might wonder how an iterated formula can produce exactly this result, which is required if kinetic energy actually *is* he C-field. In the above calculations, we found approximately $m_0v^2/2$, based on several approximations. Clearly, C-field energy induced by the accelerating force cannot *exceed* the value of the kinetic energy, but our results do not preclude the C-field energy immediately induced being much *less than* the kinetic energy. In the following we consider the immediate C-field circulation induced by mv^2 to be on the order of $0.001mv^2$, for purposes of discussion.

According to our electron example, the exact density of the electron is unknown as the electron radius has been measured only down to ~10^{-18} m. It could be much smaller, with corresponding higher density. Nor do we know the exact volume that C-field energy is distributed over during the acceleration process. Nevertheless, reasonable guesses lead to a "per cycle" rate ~0.001. This "interest rate", compounded every 100 attosec-onds, yields an increase in energy density as large as we desire (limited, of course, by the available driving energy). If this assumption is valid, as can be shown for typical cases, then we are provided with a limiting process that yields $mv^2/2$ regardless of the unknown parameters. We know the accelerating force is inputting energy to accelerate the rest particle to velocity v, and the energy transferred to the particle is $mv^2/2$. This energy is assumed to be stored in the C-field induced by the acceleration.

A process that involves the transfer of energy from the work $\boldsymbol{F}\cdot\boldsymbol{x}$ done by accelerating force, $\boldsymbol{F}$ accelerates the particle to velocity $\boldsymbol{v}$. The total energy transferred is $mv^2/2$. There is no more available energy to be stored in the C-field at this point, therefore the self-interacting process cannot continue to increase the local energy $> mv^2/2$. That is, energy compounding every 100 attosec can keep up with the energy available to the process but it *cannot get ahead of it*. When all available energy has been extracted from the driving force, the self-interactive process stops.

8. Simplifying the Problem

The physics involved is quite sophisticated and actual numbers hard to come by. For this reason, it may be useful to remove the physics and focus on the calculations. Compound interest is an analogy that everyone understands. For example (Wiki) the periodic compounding formula $P' = P\left(1+r/n\right)^{nt}$, if the principle $P=$ 1500 is compounded *quarterly* at 4.3% for six years, yields:

$$P' = 1500\times\left(1+\frac{0.043}{4}\right)^{4\times 6} \sim 1938.84 \tag{14}$$

An operational analogy: Most businessmen learn that bankers loan money only if you don't need it, *i.e.*, loans are 100% secured. Assume that I have 1 million cash and I borrow $10,000 at a rate of 0.001 compounded hourly for three years. The question is when I will run out of money. As soon as the payments exhaust my $1 million assets, I will be unable to pay back the bank (assuming I have not put the borrowed cash to productive use.) Also assume that the bank is broke and cannot loan me more. *The process halts*, unless I can acquire more money, which immediately goes to the bank.

In place of assets guaranteeing a loan, the physics analogy begins by pouring energy E_0 into an object by accelerating it to velocity $\boldsymbol{v}$ at which point the object has $\left(mv^2/2\right)=E_0$ kinetic energy; with no more energy left to accelerate the object further; the process halts. If new energy becomes available, compounding will continue until that energy is used up also. This continues as long as the self-interaction (compounding) can *keep up with* the transfer of available energy.

9. To Summarize

Self-interaction of the C-field, modeled by an iterative procedure, occurs over an infinitesimal (~100 attosec) period and compounds every such period. We hypothesize that this process, in reality, can "keep up with" *any* accelerating force. That is, the C-field can absorb energy from the driving force as fast as it is delivered; that is all that is required! When the force is accelerated to particle velocity $\boldsymbol{v}$, the energy stored in the C-field is $mv^2/2$.

Infinite energy, force, or acceleration does not exist in the physical world. Schiller [7] has shown that the hypothesis of maximum force $\le c^4/g$ leads to Einstein's field equations. This places an upper bound on acceleration, thus giv-

ing the self-interactive "compounding" a maximum target with which it has to keep up. The iteration process that *keeps up with* but *does not get ahead of* the transfer of energy from the accelerating force is not sensitive to local parameters such as the exact radius of the particle or the exact volume occupied by the C-field. Iteration is assumed to *keep up* based on *any realistic parameters*. Iteration is thus *not* used to calculate C-field energy. It is used to prove that this process can keep up with the physics. The final result, $mv^2/2$, is *not* calculated from the iteration process; it is typically based on conservation of energy-momentum.

One might ask why one would wish to calculate via iteration a result that one derives from quite another physical formulation, e.g. Newton's equations. The answer is that self-interaction of the gravitational field explains a number of issues that have not heretofore been explained.

10. Discussion of Results

Gravitomagnetic field equations were first derived by Heaviside [8] in 1893 and later derived from Einstein's field equations in the *weak field approximation*. Existence of the C-field was established circa 2006 by Tajmar [9] and 2011 by *Gravity Probe B* [10]. The utility of the C-field was recently demonstrated by C. Will [11] who calculated the C-field contribution of the other planets to the advance of the perihelion of Mercury. The iterated C-field equations are identical to Einstein's full nonlinear field equations, per Feynman [12] and Ohanian and Ruffini [13].

Energy analysis of a C-field induced by an accelerating object describes a formulation of C-field energy yielding kinetic energy, $mv^2/2$. This is of physical interest since the mechanism by which kinetic energy is actually stored has never been explained. The key assumption is that C-field circulation induced by accelerating mass density possesses its own mass density, which increases the mass being accelerated and hence induces even more C-field energy. Where does this iterative process end? The process does not drive itself. When the driving force terminates, the increase in C-field energy terminates. Nor can the C-field process extract energy from the power source at a rate faster than the power couples to the mass. The self-interacting C-field extracts all available local energy, then circulation ceases to increase. We know from Newton's laws that the moving object will have kinetic energy $mv^2/2$. Thus the interesting situation where a self-interacting process with unknown parameters must iterate an unknown number of times and yet must produce a precise result $mv^2/2$. The iteration process must produce sufficient field energy to match the kinetic energy of an object accelerated to v, but cannot overshoot this well-defined value.

Note that the force is *not* equal to the change in C-field circulation; only a fraction κ of the force causes a change in circulation. But this change in the field induces further change in the field and this extracts more energy from the driving force; continuing until field energy equals $mv^2/2$. The self-interaction frequency limit is set by the speed of light over the region containing the field

and thus occurs much faster than typical mechanical motions. Iterative self-interaction extracts power from source, but cannot exceed the power available from the source.

Our analysis explains how a *fuzzy* iteration process whose local parameters are not known with precision can yield the well-specified kinetic energy value. We show by iterated examples that no matter how small the *interest rate*, if compounded sufficiently frequently, iteration eventually reaches a point where it *turns a corner*, quickly yielding almost unlimited growth.

Next, recall that $\rho = m/vol$ and $m\boldsymbol{v}$ is momentum, $\boldsymbol{p}$; we simplify by choosing $vol = 1$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\nabla \times \boldsymbol{C} = -\frac{\kappa \boldsymbol{p}}{vol}\right] \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(\nabla \times \boldsymbol{C}\right) = -\kappa \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \tag{15}$$

where $\mathrm{d}\boldsymbol{p}/\mathrm{d}t$ is the accelerating force. This equation is symmetrical: if force $\mathrm{d}\boldsymbol{p}/\mathrm{d}t$ is non-zero, the field circulation changes (in left-handed direction) proportional to the force. But let the external force $\mathrm{d}\boldsymbol{p}/\mathrm{d}t$ be zero, and assume that circulation changes. There is no reason to believe that the circulation can increase with no external force applied, so any change must represent a decrease in field circulation. This effectively induces an internal force that resists the change. In electrodynamics, this force is the *emf* or *electromotive force*, known as the Lenz-law effect, upon which automobile "spark plug" operation was based for over a century. The Lenz-law-like phenomenon sustains the velocity; a decrease in velocity is opposed by a corresponding change in circulation. This action conserves linear momentum, which Feynman declared to be a mystery! Observe that Lenz-law operation also suggests relevance to quantum mechanical tunneling.

We have defined a problem that addresses several uncertainties in physics, from the nature of *energy of motion* to conservation of linear momentum. We have presented the relevant equations, invented by Heaviside and derived from Einstein's field equations and have discussed an iterative procedure that produces an exact value, $mv^2/2$, based on approximate but unknown fundamental parameters, including density and volume of the relevant objects; particles and induced fields.

By analyzing an electron in this framework, we have opened the way to apply this to quantum mechanics, based on Holland's interpretation of "concealed motion" as quantum potential, perhaps most relevant to de Broglie-Bohm's interpretation of quantum mechanics. Holland's unexplained term, $m\rho_0 = m|\psi_0|^2$, is compatible with field energy density ρ.

The significance of our results in a nutshell: the gravitational field is self-interactive; as reflected in Einstein's non-linear field equations, which, as noted above, are identically equal to the linear field Equations (7) iterated appropriately. Nonlinearity generally presents unsolvable problems; the few ingenious and lucky guesses—Schwarzschild, Kerr, Kasner—yield *exact* mathematical solutions but do not resolve all physical questions. Over a century of effort

has failed to find an expression for the gravitational self-energy tensor in general relativity. It is proposed herein that actions of nonlinear gravity yield the world we live in while providing explanations for heretofore unexplained aspects of physics. If, as proposed, iteration based on "fuzzy" physical parameters can extract the available energy and this energy is described by the linear equations of Newton, then we are justified in calling upon nonlinear physics for explanation, while continuing to use linear physics based on conservation principles for computation. The iterative solution of linear equations is equivalent to the nonlinear solution, if such exists.

Finally, we establish reasonable bounds on maximum force, maximum compounding rate, and minimum densities involved in the problem. It is hoped that these bounds will enable a rigorous proof that this mechanism works for all realistic ranges of physical parameters.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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