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Distributed Synchronization of Coupled Time-Delay Neural Networks Based on Randomly Occurring Control

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Abstract

In this paper, the distributed synchronization of stochastic coupled neural networks with time-varying delay is concerned via randomly occurring control. We use two Bernoulli stochastic variables to describe the occurrence of distributed adaptive control and updating law according to certain probabilities. The distributed adaptive control and updating law for each vertex in the network depend on the state information on each vertex's neighborhood. Based on Lyapunov stability theory, Itô differential equations, etc., by constructing the appropriate Lyapunov functional, we study and obtain sufficient conditions for the distributed synchronization of such networks in mean square.

Keywords

Complex Dynamical Networks, Time-Varying Delay Coupling, Randomly Occurring Control, Synchronization

1. Introduction

With the rapid development of the information age, complex networks have received extensive attention as a frontier of interdisciplinary and challenging research fields. In real life, complex networks are everywhere, from the huge Internet to WWW [1], from large power networks to transportation networks, from neural networks to the metabolism of living things and social networks. It is no exaggeration to say that we live in a world full of various complex networks.

As one of the more significant dynamic behaviors of complex dynamical networks, synchronization widely exists in real life, for example, the chorus of crickets, the applause of the audience at a concert, the spread of computer viruses, the epidemic of infectious diseases in the crowd, etc. All of these are well-known phenomena of synchronization, because it can describe many natural phenomena and has many potential applications, such as image processing, neuronal synchronization, secure communication, etc. [2] [3] [4] [5] [6]. The synchronization of large-scale complex networks has been extensively studied in various fields of science and engineering.

As we all know, there are random uncertainties in any actual system and its external environment, for example, stochastic errors in system input and output, environmental noise, etc. With the deepening of research, people gradually realized that stochastic models can sometimes better reflect the inherent essential properties of natural and engineering systems. Various random emergencies or environmental noise will have a significant impact on natural systems, engineering systems, stock prices and other socio-economic systems, and some of them may even be subversive. For example, due to the existence of random interference factors, the modeling and analysis of the system become complicated, and the control and design of the system become difficult. Conventional deterministic algebraic equations and differential equations are no longer sufficient to accurately describe such network systems. In fact, since scientists discovered the Brownian motion (1827, Brown), proposed the Markov chain (1907, Markov), presented Langevin stochastic differential equation (1908, Langevin), gave an accurate mathematical description of Brownian motion (1923, Wiener), proposed Ornstein-Uhlenbeck noise (1930), laid the mathematical theoretical foundation of stochastic process (1931, Kolmogorov), discussed the mathematical theory of stochastic process (1951, Doob), and established Itô calculus theory, published a famous paper on stochastic differential equations (1951, K.Itô), laid the theoretical foundation of stochastic differential equations. In the past two hundred years, scientists have studied and analyzed stochastic phenomena, explored stochastic laws, and established a complete theoretical system of stochastic systems. Since then, the theory of stochastic systems based on stochastic analysis and stochastic differential equations has received full attention, and a large number of research results have been obtained which have permeated into many research fields and been widely applied. Therefore, the stochastic system control theory has become a hot research topic today and has important research value.

In the past ten years, many scholars have obtained a large number of constructive results on the study of synchronization of complex networks, and have proposed many effective methods. In [7], Barahona and Pecora proposed master stability functions to characterize the synchronization of complex networks. In [8], an improved simulated annealing method was used to detect optimal synchronization networks. Researchers have further studied the synchronization of complex networks with time-delay coupling [9] [10], weighted coupling matrices [11], linearly coupled form [12], adaptive updating weights [13], randomly occurring nonlinearities [14], stochastic coupling [15]. In [16], an adaptive strategy is used to study the synchronization of general complex networks when network topology is slowly time-varying. However, synchronization of complex networks may not be guaranteed if control is not introduced [17]. Researchers have developed many control-based methods to synchronize complex dynamical networks. For example, in [18], the problem of pinning control of complex dynamical networks is studied. When synchronizing and controlling networks, pinning control has been shown to be a simple but effective technique for stabilization and synchronization [17] [19] [20] [21] [22]. In [23] [24] [25] [26], some studies have discovered how to effectively control complex networks with a small number of nodes. The distributed synchronization of networks has attracted more and more attention from researchers in various fields. Vertex i in the network synchronizes the system state based not only on the state of vertex i, but also on the states of its neighboring vertices according to the given complex network topology [27]. In [28], an adaptive technique was proposed to synchronize Complex networks, where only neighborhood information was used to design updating law. Literature [29] has designed a distributed adaptive controller, the main feature of which is that nodes can more effectively use the local information on its neighbors without the need for global information on the entire networks. The actual implementation of the controller is always disturbed by various uncertainties caused by internal and external environments [30] [31]. Such disturbances widely exist in control implementation and system design, and are due to random abrupt changes [14] [30]. The distributed controller designed in [29] takes stochastic disturbances into consideration and proposes a distributed controller that occurs randomly. The first reason is that the signals in the network system cannot be completely transmitted or cannot be controlled, just as in the cases of packet dropouts, random failures and repairs of actuators [30] [31]. Another reason is positive. With consideration of economic or system life, control will be suspended from time to time [31]. Therefore, control activation and network systems may occur in a probabilistic or switching manner, and vary randomly in terms of their types and/or intensity [14] [30] [31] [32]. However, the synchronization research of complex networks mainly focuses on fixed controllers [15] [23] [33] [34]. Although it is of practical significance to study the randomly occurring control of complex networks, it has received little attention. In particular, the distributed synchronization of neural networks under randomly occurring control has received little attention from the academic community.

Time-delay is a universal objective phenomenon in nature and human society, which is usually caused by limited signal transmission and memory effects. For example, in the field of automation and information technology, information acquisition, processing and transmission of sensors all require time. In fact, communication systems, transmission systems, and power systems are all typical time-delay systems. Therefore, the phenomenon of time delay is ubiquitous. In the topological structure of complex dynamical networks, there are inevitably coupled time-delays. Moreover, the coupled time-delay of the network will have a certain impact on the synchronization capability and control effect of the network. Therefore, the analysis and research on time-delay systems is very meaningful.

In summary, randomness and time-delay are the most common phenomena in real systems, such as power and biological systems in actual engineering systems are typical time-delayed stochastic systems. In the past, the research on synchronization of complex networks mainly focused on the fixed controller. In reference [29], considering that the controlled system is interfered by random mutations, a method of randomly occurring control is proposed, and the synchronization behavior of complex networks is studied, but it does not take into account the impact of time-delay factors, so its model is somewhat conservative.

Based on the above viewpoints, a more general coupled time-varying delay complex dynamical network model under randomly occurring control is proposed in this paper. This model has the following characteristics: both controller activation and updating law of control gain occur in a probabilistic way, and the synchronization of stochastic complex networks is studied by considering randomly occurring control and updating law. The complex network model proposed in reference [29] is extended to the complex dynamical network model with coupled time-varying delay. Based on the Lyapunov stability theory, by constructing appropriate Lyapunov functional, the sufficient conditions for the distributed synchronization of such networks in mean square are obtained.

2. Model Description and Preliminaries

Notations: Let $\mathbb{R}\left(\mathbb{R}^{+}\right)$ denote the set of real (positive) numbers. $\mathbb{R}^{n}$ and $\mathbb{R}^{n\times m}$ denote , respectively, the n-dimensional Euclidean space and the set of all $n\times m$ -dimensional real matrices. $A>0$ denotes that the matrix A is a symmetric and positive definite matrix. The notation A^{T} is the transpose of a vector or matrix A. I represents the identity matrix with appropriate dimensions. $\|A\|$ denotes the Euclidean norm of a matrix A and $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) denotes the maximum (respectively, minimum) eigen value of matrix A. The symbol $trace(A)$ represents the trace of square matrix $A=\left(a_{ij}\right)_{n\times n}$, *i.e.* $trace(A)=\sum_{i=1}^{n}a_{ii}$. $\otimes$ stands for the Kronecker product. And $E\{X\}$ represents the expectation of the random variable X. Define a graph by $\mathcal{G}=[\mathcal{V},\mathcal{E}]$, where $\mathcal{V}=\{1,\cdots,N\}$ denotes the vertex set and $\mathcal{E}=\{e(i,j)\}$ denotes the edge set. $N(i)$ denotes the neighborhood of vertex i in the sense $N(i)=\{j\in\mathcal{V}:e(i,j)\in\mathcal{E}\}$. In this paper, graph $\mathcal{G}$ is supposed to be undirected [$e(i,j)\in\mathcal{E}$ implies $e(j,i)\in\mathcal{E}$] and simple (without self-loops and multiple edges). Let $L=\left[l_{ij}\right]_{i,j=1}^{N}$ be the Laplacian matrix of graph $\mathcal{G}$, which is defined as follows: for any pair $i\neq j$, $l_{ij}=l_{ji}=-1$ if $e(i,j)\in\mathcal{E}$ and $l_{ij}=l_{ji}=0$ otherwise . $l_{ii}=-\sum_{j=1,j\neq i}^{N}l_{ij}$ stands for the degree of vertex $i(i=1,2,\cdots,N)$. Let $(\Omega,\mathcal{F},\mathcal{P})$ be a complete probability space, where Ω represents a sample space, $\mathcal{F}$ is called an σ-algebra, and $\mathcal{P}$ is a probability measure.

In this paper, we consider the following model of a complex network stochastic system with coupled time-varying delays, which can be expressed as:

$$\begin{aligned} \mathrm{d}x_i(t) = & \left[f(x_i,t) + C \sum_{j\in N(i)} \Gamma\left(x_j(t-\tau(t)) - x_i(t-\tau(t))\right)\right]\mathrm{d}t \\ & + \sigma\left(x_i(t),t\right)\mathrm{d}w(t), \quad i=1,2,\cdots,N \end{aligned} \tag{1}$$

where $x_i(t) = \left[x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t)\right]^{\mathrm{T}} \in \mathbb{R}^n \ (i=1,2,\cdots,N)$ is the state vector of the ith vertex, $f(x_i,t) = \left[f_1(x_i,t), f_2(x_i,t), \cdots, f_n(x_i,t)\right]^{\mathrm{T}}$ is a continuous vector function, C is the coupling strength of the network. The inner coupling matrix Γ is a constant diagonal matrix, and $\Gamma > 0$. $\tau(t)$ is a time-varying coupling delay. Furthermore, $\sigma(x,t): \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is the noise intensity function vector, and $w(t)$ is a scalar Brownian motion defined on $(\Omega, \mathcal{F}, \mathcal{P})$ satisfying $E\{\mathrm{d}w(t)\} = 0$ and $E\left\{\left[\mathrm{d}w(t)\right]^2\right\} = \mathrm{d}t$. Additionally, from the Gershgorin disk theorem, all the eigenvalues of the Laplacian matrix L corresponding to graph $\mathcal{G}$ satisfy $0 = \lambda_1(L) \le \lambda_2(L) \le \cdots \le \lambda_N(L)$. Furthermore, $\mathcal{G}$ is connected if and only if $\lambda_2(L) > 0$: *i.e.*, L is irreducible.

In order to achieve the synchronization of the stochastic complex network in (1), controllers are added to each vertex.

$$\begin{aligned} \mathrm{d}x_i(t) = & \left[f(x_i,t) + C \sum_{j\in N(i)} \Gamma\left(x_j(t-\tau(t)) - x_i(t-\tau(t))\right) + u_i(t)\right]\mathrm{d}t \\ & + \sigma\left(x_i(t),t\right)\mathrm{d}w(t), \quad i=1,2,\cdots,N \end{aligned} \tag{2}$$

where $u_i(t)$ is a distributed adaptive controller.

For the ith vertex, $u_i(t)$ is designed as

$$u_i(t) = \rho(t) \sum_{j\in N(i)} \varepsilon_i(t)\Gamma\left(x_j(t) - x_i(t)\right), \quad i=1,2,\cdots,N \tag{3}$$

where $\varepsilon_i(t)$ is the control strength of vertex i.

In (3), $\rho(t)$ is a Bernoulli stochastic variable that describes the following random events for (2):

$$\begin{cases} \text{Event 1:} & \text{(2) experiences (3)} \\ \text{Event 2:} & \text{(2) does not experience (3)} \end{cases} \tag{4}$$

Let $\rho(t)$ be defined by

$$\rho(t) = \begin{cases} 1, & \text{if Event 1 occurs} \\ 0, & \text{if Event 2 occurs} \end{cases} \tag{5}$$

where the probability of event $\{\rho(t)=1\}$ is $\Pr\{\rho(t)=1\} = \rho \in [0,1]$, so the expectation of random variable $\rho(t)$ is $E\{\rho(t)\} = \rho$.

The distributed controller (3) in this paper takes stochastic disturbances into account. The distributed controller $u_i(t)$ occurs in a probabilistic manner and uses feedback information from neighboring points. Different from the conventional adaptive controller, the distributed controller $u_i(t)$ is not always implemented and it can model control failure in a stochastic way. In short, randomly occurring distributed control can effectively use the information of neighboring points to simulate real-world disturbances.

$\varepsilon_i(t)$ in (3) is updated according to the following randomly occurring distributed updating law:

$$\mathrm{d}\varepsilon_i(t)=\xi(t)\alpha\left[\sum_{j\in N(i)}\left(x_j(t)-x_i(t)\right)\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)}\left(x_j(t)-x_i(t)\right)\right]\mathrm{d}t, \tag{6}$$
$$i=1,2,\cdots,N$$

where $\alpha>0$ and $\xi(t)$ is a Bernoulli stochastic variable representing the following random events for (6):

$$\begin{cases}\text{Event 3}: & \varepsilon_i(t)\,\text{experiences (6)}\\ \text{Event 4}: & \varepsilon_i(t)\,\text{does not experience (6)}\end{cases} \tag{7}$$

Similarly, Let $\xi(t)$ be defined by

$$\xi(t)=\begin{cases}1, & \text{if Event 3 occurs}\\ 0, & \text{if Event 4 occurs}\end{cases} \tag{8}$$

where the probability of event $\{\xi(t)=1\}$ is $\Pr\{\xi(t)=1\}=\xi\in[0,1]$, so the expectation of stochastic variable $\xi(t)$ is $E\{\xi(t)\}=\xi$.

Remark 1: If $\rho=1$ and $\xi=1$, the control and updating rule will be simplified to normal control and updating law, as shown in [25] [28]. If $\rho=0$ and $\xi=0$, the problem considered in this article will be simplified to the synchronization of complex networks without controllers.

It can be seen from the above model that the network studied in this paper has the following characteristics:

1) The activation of the controller and the updating law of control gain both occur in a probabilistic manner, and the distributed synchronization of stochastic complex networks is studied by considering randomly occurring control and updating law.

2) Considering the effect of coupled time-delays, the model is more general.

In order to get the main results, the following definitions, assumptions and lemmas are needed.

Definition 1: Let $x_i(t)(1\le i\le N)$ be the solution of the stochastic complex network with coupled time-varying delay in (1) or (2), if they satisfy the following condition:

$$\lim_{t\to\infty}E\sum_{i=1}^{N}\left\|x_i(t)-x_j(t)\right\|^2=0,\quad i,j=1,2,\cdots,N \tag{9}$$

then the stochastic complex network is said to achieve synchronization in mean square.

Lemma 1 (Itô formula) [35]: Let $x(t)$ be an Itô process on $t\ge0$ with the stochastic differential

$$\mathrm{d}x(t)=f(t)\mathrm{d}t+g(t)\mathrm{d}w(t)$$

Let $V(x,t)$ be a real-valued function, which is continuously twice differentiable in x and once differentiable in t. Then $V(x(t),t)$ is again an Itô process with the stochastic differential given by

$$\begin{aligned}&\mathrm{d}V(x(t),t)\\&=\left[V_t(x(t),t)+V_x(x(t),t)f(t)+\frac{1}{2}trace\left(g^{\mathrm{T}}(t)V_{xx}(x(t),t)g(t)\right)\right]\mathrm{d}t\\&\quad+V_x(x(t),t)g(t)\mathrm{d}w(t)\end{aligned}\tag{10}$$

Lemma 2 [36]: For any $x, y\in\mathbb{R}^n$ and any positive definite symmetric matrix $P\in\mathbb{R}^{n\times n}$, the following inequalities hold

$$\pm 2x^{\mathrm{T}}y\le x^{\mathrm{T}}Px+y^{\mathrm{T}}Py. \tag{11}$$

Lemma 3 [37]: Assuming that $P\in\mathbb{R}^{n\times n}$ is a positive definite matrix, for any $x\in\mathbb{R}^n$, the following inequalities hold

$$\lambda_{\min}(P)\|x\|^2\le x^{\mathrm{T}}Px\le\lambda_{\max}(P)\|x\|^2. \tag{12}$$

Assumption 1 [28] [38]: A vector-valued continuous function $f(x,t):\mathbb{R}^n\times\mathbb{R}^+\to\mathbb{R}^n$ is said to be uniformly decreasing for a matrix $\Gamma\in\mathbb{R}^{n\times n}$ if there exist $\theta>0$ and $\Delta>0$ such that

$$\begin{aligned}&(x-y)^{\mathrm{T}}\left[f(x,t)-f(y,t)-\theta\Gamma(x-y)\right]\\&\le-\Delta(x-y)^{\mathrm{T}}(x-y)\end{aligned}\tag{13}$$

holds for all $x, y\in\mathbb{R}^n$ and $t\ge 0$.

Assumption 2 [14]: $\sigma(x,t):\mathbb{R}^n\times\mathbb{R}^+\to\mathbb{R}^n$ is said to be locally uniformly Lipschitz continuous with respect to t if there exist a constant $k>0$ such that the following inequality holds for all $x, y\in\mathbb{R}^n$:

$$\|\sigma(x,t)-\sigma(y,t)\|\le\sqrt{k}\|x-y\| \tag{14}$$

Assumption 2 can describe many real-world systems very well. It has been widely employed or discussed in [14] [35].

Assumption 3: Time-varying delay $\tau(t)$ satisfies: $0\le\dot{\tau}(t)\le\overline{\tau}<1$, where $\dot{\tau}(t)$ represents the derivative of $\tau(t)$ to t.

3. Main Results

In this section, we study the distributed synchronization problem of the stochastic complex network in (2) coupled with time-varying delay via randomly occurring control and updating law and obtain sufficient conditions for the distributed synchronization of such networks in mean square.

Theorem 1: Suppose that Assumptions 1-3 hold, then the stochastic complex network in (2) under the distributed adaptive controller (3) and updating law (6) will be globally synchronized in mean square, if the following conditions are satisfied:

$$\begin{cases} \frac{k}{2} - \Delta + \frac{C}{2} \cdot \lambda_2(L)\left(\lambda_{\max}\left(\Gamma\Gamma^{\mathrm{T}}\right) + \frac{1}{1-\bar{\tau}}\right) < 0 \\ \theta - b\rho \cdot \lambda_2(L) < 0 \end{cases} \tag{15}$$

Proof: It should be noted that the stochastic dynamical network in (2) with $\rho(t)$ and $\xi(t)$ is a special stochastic system with Markovian switching. Therefore, the existence and uniqueness of solutions to (2) can be transformed into the existence and uniqueness of solutions to a stochastic system with Markovian switching. The proof of the existence and uniqueness of solutions to a stochastic system with Markovian switching can be found in [32]. Also, the proof of the existence and uniqueness of solutions to (2) is shown in Supporting Information.

Let $e_{ij}(t) = x_i(t) - x_j(t), \forall\, i, j = 1, 2, \cdots, N$ and $x = \left[x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \cdots, x_N^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{nN}$.

Consider the following Lyapunov function

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{16}$$

where

$$V_1(t) = \frac{1}{4}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}} e_{ij} \tag{17}$$

$$V_2(t) = \frac{\rho}{2\xi\alpha}\sum_{i=1}^{N}\left(\varepsilon_i(t) - b\right)^2 \tag{18}$$

$$V_3(t) = \frac{C}{2(1-\bar{\tau})}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t}\left[\sum_{j\in N(i)} e_{ij}(s)\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}(s)\right]\mathrm{d}s \tag{19}$$

where b is a positive constant. According to (2) and (3) we can easily obtain

$$\begin{aligned} \mathrm{d}e_{ij}(t) &= \mathrm{d}x_i(t) - \mathrm{d}x_j(t) \\ &= \Bigg\{ f(x_i, t) - f(x_j, t) + C\sum_{k\in N(i)}\Gamma\big(x_k(t-\tau(t)) - x_i(t-\tau(t))\big) \\ &\quad - C\sum_{m\in N(j)}\Gamma\big(x_m(t-\tau(t)) - x_j(t-\tau(t))\big) \\ &\quad + \rho(t)\varepsilon_i(t)\Gamma\sum_{k\in N(i)}\left[x_k(t) - x_i(t)\right] \\ &\quad - \rho(t)\varepsilon_j(t)\Gamma\sum_{m\in N(j)}\left[x_m(t) - x_j(t)\right]\Bigg\}\mathrm{d}t \\ &\quad + \left[\sigma\left(x_i(t), t\right) - \sigma\left(x_j(t), t\right)\right]\mathrm{d}w(t) \end{aligned} \tag{20}$$

By the Lemma 1 (Itô formula), the stochastic derivative of $V(t)$ can be obtained as

$$\mathrm{d}V(t) = \mathscr{L}V(t)\mathrm{d}t + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}\left[\sigma\left(x_i(t), t\right) - \sigma\left(x_j(t), t\right)\right]\mathrm{d}w(t) \tag{21}$$

and according to (20), the Itô differential operator $\mathscr{L}$ is given as

$$\begin{aligned}\mathscr{L}V(t) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}\Bigg\{ f\left(x_i,t\right)-f\left(x_j,t\right)+C\sum_{k\in N(i)}\Gamma\left(x_k\left(t-\tau(t)\right)-x_i\left(t-\tau(t)\right)\right) \\ &-C\sum_{m\in N(j)}\Gamma\left(x_m\left(t-\tau(t)\right)-x_j\left(t-\tau(t)\right)\right)+\rho(t)\varepsilon_i(t)\Gamma\sum_{k\in N(i)}\left[x_k(t)-x_i(t)\right] \\ &-\rho(t)\varepsilon_j(t)\Gamma\sum_{m\in N(j)}\left[x_m(t)-x_j(t)\right]\Bigg\}+\sum_{i=1}^{N}\frac{\rho}{\xi}\varepsilon_i(t)\cdot\xi(t)\left[\sum_{j\in N(i)}e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)}e_{ij}\right] \\ &-\sum_{i=1}^{N}\frac{\rho}{\xi}b\cdot\xi(t)\left[\sum_{j\in N(i)}e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)}e_{ij}\right]+\frac{C}{2(1-\overline{\tau})}\sum_{i=1}^{N}\left[\sum_{j\in N(i)}e_{ij}(t)\right]^{\mathrm{T}}\left[\sum_{j\in N(i)}e_{ij}(t)\right] \\ &-\frac{C}{2(1-\overline{\tau})}\sum_{i=1}^{N}\left[\sum_{j\in N(i)}e_{ij}\left(t-\tau(t)\right)\right]^{\mathrm{T}}\left[\sum_{j\in N(i)}e_{ij}\left(t-\tau(t)\right)\right]\cdot\left(1-\dot{\tau}(t)\right) \\ &+\sum_{i=1}^{N}\sum_{j\in N(i)}\frac{1}{4}\left[\sigma\left(x_i(t),t\right)-\sigma\left(x_j(t),t\right)\right]^{\mathrm{T}}\left[\sigma\left(x_i(t),t\right)-\sigma\left(x_j(t),t\right)\right]\end{aligned} \tag{22}$$

Note that

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}\Bigg\{\rho(t)\varepsilon_i(t)\Gamma\sum_{k\in N(i)}\left[x_k(t)-x_i(t)\right] \\ &-\rho(t)\varepsilon_j(t)\Gamma\sum_{m\in N(j)}\left[x_m(t)-x_j(t)\right]\Bigg\} \\ &=-\sum_{i=1}^{N}\left(\sum_{j\in N(i)}e_{ij}\right)^{\mathrm{T}}\rho(t)\varepsilon_i(t)\Gamma\left(\sum_{j\in N(i)}e_{ij}\right)\end{aligned} \tag{23}$$

and take expectations of $\rho(t)$ and $\xi(t)$, we obtain $E\left[\rho(t)\right]=\rho$ and $E\left[\frac{\rho}{\xi}\xi(t)\right]=\rho$, furthermore

$$\begin{aligned}&E\Bigg\{\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}\Bigg(\rho(t)\varepsilon_i(t)\Gamma\sum_{k\in N(i)}\left[x_k(t)-x_i(t)\right] \\ &-\rho(t)\varepsilon_j(t)\Gamma\sum_{m\in N(j)}\left[x_m(t)-x_j(t)\right]\Bigg) \\ &+\sum_{i=1}^{N}\frac{\rho}{\xi}\varepsilon_i(t)\cdot\xi(t)\left[\sum_{j\in N(i)}e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)}e_{ij}\right]\Bigg\} \\ &=E\Bigg\{-\sum_{i=1}^{N}\left(\sum_{j\in N(i)}e_{ij}\right)^{T}\rho(t)\varepsilon_i(t)\Gamma\left(\sum_{j\in N(i)}e_{ij}\right) \\ &\quad+\sum_{i=1}^{N}\frac{\rho}{\xi}\varepsilon_i(t)\cdot\xi(t)\left[\sum_{j\in N(i)}e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)}e_{ij}\right]\Bigg\} \\ &=0\end{aligned} \tag{24}$$

Then, using Assumptions 1 and 2, (22), and (24), we have

$$\begin{aligned}
&E\mathscr{L}V(t) \\
&= E\left\{\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}\left[f(x_i,t)-f(x_j,t)-\theta\Gamma e_{ij}\right]+\frac{\theta}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}\Gamma e_{ij}\right. \\
&\quad -C\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]-\rho b\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)} e_{ij}\right] \\
&\quad +\frac{C}{2(1-\bar{\tau})}\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}(t)\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}(t)\right] \\
&\quad -\frac{C}{2(1-\bar{\tau})}\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]\cdot(1-\dot{\tau}(t)) \\
&\quad \left.+\sum_{i=1}^{N}\sum_{j\in N(i)}\frac{1}{4}\left[\sigma(x_i(t),t)-\sigma(x_j(t),t)\right]^{\mathrm{T}}\left[\sigma(x_i(t),t)-\sigma(x_j(t),t)\right]\right\} \\
&\le E\left\{-\frac{\Delta}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}e_{ij}+\frac{\theta}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}\Gamma e_{ij}-C\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]\right. \\
&\quad -\rho b\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)} e_{ij}\right]+\frac{C}{2(1-\bar{\tau})}\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}(t)\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}(t)\right] \\
&\quad -\frac{C}{2(1-\bar{\tau})}\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]\cdot(1-\dot{\tau}(t)) \\
&\quad \left.+\frac{1}{4}\sum_{i=1}^{N}\sum_{j\in N(i)} k e_{ij}^{\mathrm{T}}e_{ij}\right\}
\end{aligned} \tag{25}$$

From the definitions of e_{ij}, x and the Laplacian matrix L, we get

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N(i)} e_{ij}^{\mathrm{T}}e_{ij}=x^{\mathrm{T}}(L\otimes I_n)x \tag{26}$$

$$\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)} e_{ij}\right]=x^{\mathrm{T}}\left(L^2\otimes\Gamma\right)x \tag{27}$$

Besides, by using Lemma 2 and Lemma 3, one obtains that

$$\begin{aligned}
&-C\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\Gamma\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right] \\
&\le\frac{C}{2}\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\Gamma\Gamma^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}\right] \\
&\quad +\frac{C}{2}\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right] \\
&\le\frac{C}{2}\lambda_{\max}\left(\Gamma\Gamma^{\mathrm{T}}\right)\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}\right] \\
&\quad +\frac{C}{2}\sum_{i=1}^{N}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]^{\mathrm{T}}\left[\sum_{j\in N(i)} e_{ij}(t-\tau(t))\right]
\end{aligned} \tag{28}$$

From Assumption 3 we can see that $1-\frac{1-\dot{\tau}(t)}{1-\bar{\tau}} \le 0$. And substitute (26), (27) and (28) into (25), one gets that

$$
\begin{aligned}
E\mathscr{L}V(t) \le{} & E\left\{\left(\frac{k}{2}-\Delta\right)\cdot x^{\mathrm{T}}\left(L\otimes I_n\right)x + x^{\mathrm{T}}\left(\theta L\otimes\Gamma\right)x + x^{\mathrm{T}}\left(-\rho bL^2\otimes\Gamma\right)x\right. \\
& \left. +\left(\frac{C}{2}\lambda_{\max}\left(\Gamma\Gamma^{\mathrm{T}}\right)+\frac{C}{2(1-\bar{\tau})}\right)\cdot x^{\mathrm{T}}\left(L^2\otimes I_n\right)x\right\} \\
={} & E\left\{x^{\mathrm{T}}\left[\left\{\left(\frac{k}{2}-\Delta\right)I_N+\left(\frac{C}{2}\lambda_{\max}\left(\Gamma\Gamma^{\mathrm{T}}\right)+\frac{C}{2(1-\bar{\tau})}\right)L\right\}L\otimes I_n\right]x\right. \\
& \left. + x^{\mathrm{T}}\left[\left(\theta I_N-\rho bL\right)L\otimes\Gamma\right]x\right\}
\end{aligned}
\tag{29}
$$

Note that condition (15) in Theorem 1 yields

$$
\begin{aligned}
& \frac{k}{2}-\Delta+\frac{C}{2}\cdot\lambda_2(L)\left(\lambda_{\max}\left(\Gamma\Gamma^{\mathrm{T}}\right)+\frac{1}{1-\bar{\tau}}\right)<0 \\
& \Rightarrow\left\{\left(\frac{k}{2}-\Delta\right)I_N+\left(\frac{C}{2}\lambda_{\max}\left(\Gamma\Gamma^{\mathrm{T}}\right)+\frac{C}{2(1-\bar{\tau})}\right)L\right\}L\le 0
\end{aligned}
\tag{30}
$$

$$
\theta-b\rho\cdot\lambda_2(L)<0\Rightarrow\left(\theta I_N-\rho bL\right)L\le 0 \tag{31}
$$

Therefore, from (26), (29), (30) and (31) we can derive

$$
E\mathscr{L}V(t)\le E\left\{-\mu\cdot x^{\mathrm{T}}\left(L\otimes I_n\right)x\right\}=E\left\{-\frac{\mu}{2}\cdot\sum_{i=1}^{N}\sum_{j\in N(i)}e_{ij}^{\mathrm{T}}e_{ij}\right\}\le 0 \tag{32}
$$

where μ is a positive constant. Thus, the distributed synchronization of the stochastic complex network with coupled time-varying delay in (2) via randomly occurring control and updating law can be achieved in mean square.

When the delay in the stochastic complex network in (2) is a constant delay, the following simpler conditions can be obtained.

Corollary 1: Suppose that Assumptions 1 and 2 hold, then the stochastic complex network in (2) under the distributed adaptive controller (3) and updating law (6) will be globally synchronized in mean square, if the following conditions are satisfied:

$$
\begin{cases}
\frac{k}{2}-\Delta+\frac{C}{2}\cdot\lambda_2(L)\left(\lambda_{\max}\left(\Gamma\Gamma^{\mathrm{T}}\right)+1\right)<0 \\
\theta-b\rho\cdot\lambda_2(L)<0
\end{cases}
\tag{33}
$$

Proof: The Lyapunov function constructed at this time becomes

$$
\begin{aligned}
V(t)={} & \frac{1}{4}\sum_{i=1}^{N}\sum_{j\in N(i)}e_{ij}^{\mathrm{T}}e_{ij}+\frac{\rho}{2\xi\alpha}\sum_{i=1}^{N}\left(\varepsilon_i(t)-b\right)^2 \\
& +\frac{C}{2}\sum_{i=1}^{N}\int_{t-\tau}^{t}\left[\sum_{j\in N(i)}e_{ij}(s)\right]^{\mathrm{T}}\left[\sum_{j\in N(i)}e_{ij}(s)\right]\mathrm{d}s
\end{aligned}
\tag{34}
$$

where b is a positive constant. The subsequent proof process is similar to the proof of Theorem 1, and will not be described in detail here.

4. Conclusion

In this paper, we have studied the distributed synchronization problem of stochastic complex networks with coupled time-varying delays via randomly occurring control and updating law. The activation of the controller and the updating law of control gain both occur in a probabilistic manner. According to the Lyapunov stability theory, Itô differential equations, etc., by constructing the appropriate Lyapunov functional, the sufficient conditions for the distributed synchronization of such networks in mean square are obtained.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Albert, R., Jeong, H. and Barabasi, A.L. (1999) Diameter of the World Wide Web. *Nature*, **410**, 130-131. https://doi.org/10.1038/43601

[2] Ott, E. (2002) Chaos in Dynamical Systems. 2nd Edition, Cambridge Univ. Press, Cambridge. https://doi.org/10.1017/CBO9780511803260

[3] Pikovsky, A., Rosenblum, M. and Kurths, J. (2001) Synchronization: A Universal Concept in Nonlinear Sciences. Cambridge Univ. Press, Cambridge. https://doi.org/10.1017/CBO9780511755743

[4] Tang, Y., Gao, H., Zou, W. and Kurths, J. (2012) Identifying Controlling Nodes in Neuronal Networks in Different Scales. *PLoS ONE*, **7**, e41375. https://doi.org/10.1371/journal.pone.0041375

[5] Zhang, W., Tang, Y., Fang, J. and Wu, X. (2012) Stability of Delayed Neural Networks with Time-Varying Impulses. *Neural Networks*, **36**, 59-63. https://doi.org/10.1016/j.neunet.2012.08.014

[6] Wu, L., Feng, Z. and Zheng, W.X. (2010) Exponential Stability Analysis for Delayed Neural Networks with Switching Parameters: Average Dwell Time Approach. *IEEE Transactions on Neural Networks*, **21**, 1396-1407. https://doi.org/10.1109/TNN.2010.2056383

[7] Barahona, M. and Pecora, L. (2002) Synchronization in Small-World Systems. *Physical Review Letters*, **89**, Article ID: 054101. https://doi.org/10.1103/PhysRevLett.89.054101

[8] Donetti, L., Hurtado, P. and Munoz, M. (2005) Entangled Networks, Synchronization, and Optimal Network Topology. *Physical Review Letters*, **95**, Article ID:

188701. https://doi.org/10.1103/PhysRevLett.95.188701

[9] Oguchi, T., Nijmeijer, H. and Yamamoto, T. (2008) Synchronization in Networks of Chaotic Systems with Time-Delay Coupling. *Chaos*, **18**, Article ID: 037108. https://doi.org/10.1063/1.2952450

[10] Hunt, D., Korniss, G. and Szymanski, B. (2010) Network Synchronization in a Noisy Environment with Time Delays: Fundamental Limits and Trade-Offs. *Physical Review Letters*, **105**, Article ID: 068701. https://doi.org/10.1103/PhysRevLett.105.068701

[11] Chavez, M., Hwang, D., Amann, A., Hentschel, H. and Boccaletti, S. (2005) Synchronization Is Enhanced in Weighted Complex Networks. *Physical Review Letters*, **94**, Article ID: 218701. https://doi.org/10.1103/PhysRevLett.94.218701

[12] Wu, C. and Chua, L. (1995) Synchronization in an Array of Linearly Coupled Dynamical Systems. *IEEE Transactions on Circuits and Systems—I: Fundamental Theory and Applications*, **42**, 430-447. https://doi.org/10.1109/81.404047

[13] Zhou, C. and Kurths, J. (2006) Dynamical Weights and Enhanced Synchronization in Adaptive Complex Networks. *Physical Review Letters*, **96**, Article ID: 164102. https://doi.org/10.1103/PhysRevLett.96.164102

[14] Wang, Z., Wang, Y. and Liu, Y. (2010) Global Synchronization for Discrete-Time Stochastic Complex Networks with Randomly Occurred Nonlinearities and Mixed Time-Delays. *IEEE Transactions on Neural Networks*, **21**, 11-25. https://doi.org/10.1109/TNN.2009.2033599

[15] Liang, J., Wang, Z., Liu, Y. and Liu, X. (2008) Robust Synchronization of an Array of Coupled Stochastic Discrete-Time Delayed Neural Networks. *IEEE Transactions on Neural Networks*, **19**, 1910-1921. https://doi.org/10.1109/TNN.2008.2003250

[16] Sorrentino, F. and Ott, E. (2008) Adaptive Synchronization of Dynamics on Evolving Complex Networks. *Physical Review Letters*, **100**, Article ID: 114101. https://doi.org/10.1103/PhysRevLett.100.114101

[17] Wang, X. and Chen, G. (2002) Pinning Control of Scale-Free Dynamical Networks. *Physica A*, **310**, 521-531. https://doi.org/10.1016/S0378-4371(02)00772-0

[18] Li, X., Wang, X. and Chen, G. (2004) Pinning a Complex Dynamical Network to Its Equilibrium. *IEEE Transactions on Circuits and Systems I: Regular Papers*, **51**, 2074-2087. https://doi.org/10.1109/TCSI.2004.835655

[19] Grigoriev, R., Cross, M. and Schuster, H. (1997) Pinning Control of Spatiotemporal Chaos. *Physical Review Letters*, **79**, 2795-2798. https://doi.org/10.1103/PhysRevLett.79.2795

[20] Lu, J., Kurths, J., Cao, J., Mahdavi, N. and Huang, C. (2012) Synchronization Control for Nonlinear Stochastic Dynamical Networks: Pinning Impulsive Strategy. *IEEE Transactions on Neural Networks and Learning Systems*, **23**, 285-292. https://doi.org/10.1109/TNNLS.2011.2179312

[21] Tang, Y., Gao, H., Kurths, J. and Fang, J. (2012) Evolutionary Pinning Control and Its Application in UAV Coordination. *IEEE Transactions on Industrial Informatics*, **8**, 828-838. https://doi.org/10.1109/TII.2012.2187911

[22] Tang, Y., Wang, Z., Gao, H., Swift, S. and Kurths, J. (2012) A Constrained Evolutionary Computation Method for Detecting Controlling Regions of Cortical Networks. *IEEE-ACM Transactions on Computational Biology and Bioinformatics*, **9**, 1569-1581. https://doi.org/10.1109/TCBB.2012.124

[23] Zhou, J., Lu, J. and Lu, J. (2008) Pinning Adaptive Synchronization of a General Complex Dynamical Network. *Automatica*, **44**, 996-1003.

https://doi.org/10.1016/j.automatica.2007.08.016

[24] Porfiri, M. and Bernardo, M. (2008) Criteria for Global Pinning-Controllability of Complex Networks. *Automatica*, **44**, 3100-3106. https://doi.org/10.1016/j.automatica.2008.05.006

[25] Yu, W., Chen, G. and Lu, J. (2009) On Pinning Synchronization of Complex Dynamical Networks. *Automatica*, **45**, 429-435. https://doi.org/10.1016/j.automatica.2008.07.016

[26] Lellis, P., Bernardo, M. and Russo, G. (2011) On QUAD, Lipschitz, and Contracting Vector Fields for Consensus and Synchronization of Networks. *IEEE Transactions on Circuits and Systems I*: *Regular Papers*, **58**, 576-583. https://doi.org/10.1109/TCSI.2010.2072270

[27] Saber, R., Fax, J. and Murray, R. (2007) Consensus and Cooperation in Networked Multi-Agent Systems. *Proceedings of the IEEE*, **95**, 215-233. https://doi.org/10.1109/JPROC.2006.887293

[28] Lu, W. (2007) Adaptive Dynamical Networks via Neighborhood Information: Synchronization and Pinning Control. *Chaos*, **17**, Article ID: 023122. https://doi.org/10.1063/1.2737829

[29] Tang, Y. and Wong, W.K. (2013) Distributed Synchronization of Coupled Neural Networks via Randomly Occurring Control. *IEEE Transactions on Neural Networks and Learning Systems*, **24**, 435-447. https://doi.org/10.1109/TNNLS.2012.2236355

[30] Hespanha, J., Naghshtabrizi, P. and Xu, Y. (2007) A Survey of Recent Results in Networked Control Systems. *Proceedings of the IEEE*, **95**, 138-162. https://doi.org/10.1109/JPROC.2006.887288

[31] Sun, X., Liu, G., Rees, D. and Wang, W. (2008) Stability of Systems with Controller Failure and Time-Varying Delay. *IEEE Transactions on Automatic Control*, **53**, 2391-2396. https://doi.org/10.1109/TAC.2008.2007528

[32] Mao, X. and Yuan, C. (2008) Stochastic Differential Equations with Markovian Switching. Imperial College Press, London. https://doi.org/10.1533/9780857099402

[33] Zhang, H., Xie, Y., Wang, Z. and Zheng, C. (2007) Adaptive Synchronization between Two Different Chaotic Neural Networks with Time Delay. *IEEE Transactions on Neural Networks*, **18**, 1841-1845. https://doi.org/10.1109/TNN.2007.902958

[34] Lu, J., Ho, D. and Wang, Z. (2009) Pinning Stabilization of Linearly Coupled Stochastic Neural Networks via Minimum Number of Controllers. *IEEE Transactions on Neural Networks*, **20**, 1617-1629. https://doi.org/10.1109/TNN.2009.2027810

[35] Mao, X. (2007) Stochastic Differential Equations and Applications. 2nd Edition, Horwood, Chichester.

[36] Dai, A.D. (2014) Synchronization Analysis and Control for Switching Complex Networks Based on M-Matrix Approach. PhD Thesis, Donghua University, Shanghai.

[37] Sun, D.W. (2017) Research on Quantitative Feedback and Event-Triggering Control of Networked Control Systems. Master Thesis, Dalian Maritime University, Dalian.

[38] Yu, W., Lellis, P., Chen, G., Bernardo, M. and Kurths, J. (2012) Distributed Adaptive Control of Synchronization in Complex Networks. *IEEE Transactions on Automatic Control*, **57**, 2153-2158. https://doi.org/10.1109/TAC.2012.2183190