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Contribution to the Mathematical Modeling of COVID-19 in Niger

Badjo Kimba Abdoul Wahid[1], Djibo Moustapha[1], Hassane Ganda Rabi[1], Saley Bisso[2]

[1]Department of Exact and Applied Sciences, Dosso Unversity, Dosso, Niger
[2]Department of Mathematics and Computer Sciences, Abdou Moumouni Unversity, Niamey, Niger
Email: abadjokimba@yahoo.fr, rabiganda@yahoo.fr, moustaphad530@gmail.com, bsaley@yahoo.fr

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Abstract

We are interested in this paper in the modeling and analysis of the disease of COVID-19 applied to the capital of Niger: Niamey. The model we are presenting takes into account the strategy that the country has adopted to fight this pandemic. The spread of the infectious agent within the population is a dynamic phenomenon: the number of the healthy and sick individuals changes over time, depending on the contacts during which the pathogen passes from an infected individual to a healthy individual. We model this propagation phenomenon by a set of differential systems equations and determine its behavior through a numerical resolution.

Keywords

COVID-19, Epidemiology, Differential Equations, Operators, Numerical Analysis, Matlab

1. Introduction

Of all the pandemics that have desolated the human race that of the Corona Virus Disease (COVID-19) is, without a doubt, the most dreaded; this plague left to itself can mow entire nations. Because despite the significant advances in medical and technological knowledge, which have led us to the era of telemedicine, humanity remains powerless in the face of this evil.

This infectious disease is caused by a new virus that has never before been identified in humans. It appears on November 17, 2019 in the city of Wuhan, in China [1], then spreads throughout the world. On March 11, 2020, the COVID-19 epidemic was qualified as a pandemic [2] by the WHO, which asked that essential protection measures be put in place in all countries to prevent this disease. At the date of this declaration, there was no cure or vaccine against this pandemic. As of April 4, 2020, 1,118,921 confirmed cases [3] have been recorded worldwide, including 58,937 deaths. To eradicate this pandemic, several strategies are adopted in the world aiming to interrupt the chain of contamination. These policies have the same objective, to reduce the contact rate of populations because humans are the vector of this disease. This is how certain countries have opted for confinement, among which we have France, Italy, etc. Sweden confines the elderly and the sick who may develop a severe form of this disease while leaving others in the hope that they will manage to develop immunity to COVID-19, so the disease will go away without any form of external interventions.

In Niger, the first confirmed case in the country was reported on March 19, 2020, and within 15 days, a total of 120 patients were diagnosed with COVID-19 [3]. A global pandemic, COVID-19 disrupts the daily life of affected populations every day. There is an urgent need to coordinate efforts to study and better understand the COVID-19 virus, to minimize the threat in the affected countries and to reduce the risk of further spread. It is in this context that our original research work falls, as there is so far no previous work, which is based on modeling and analysis of the evolution of COVID's disease-19 applied to the capital of Niger: Niamey. We count with the results obtained, helping our decision-makers to make good decisions.

2. Parameters of the Model and Mathematical Formulation

The model we are proposing takes into account the strategy that Niger has adopted to eradicate the coronavirus pandemic in the country. Thus, the population (N) is subdivided into eight compartments according to the epidemiological status of the individuals: susceptible (S), exposed (E), declared exposed (E_i), undeclared exposed (E_n), infected (I), undeclared infected (I_u), declared infected (I_r), finally the cured compartment (R).

The parameters used are:

- γ : the contact rate;
- b: the birth rate;
- μ_1 : the additional mortality rate of the individuals declared;
- μ_2 : the additional mortality rate of unreported individuals;
- μ : the natural mortality rate;
- α_1 : the reporting coefficient for suspect individuals;
- α_2 : the coefficient of undeclared suspects;
- λ_1 : the coefficient of infectivity of the individuals declared;
- λ_2 : the coefficient of false alarm of the individuals declared;
- θ_1 : the coefficient of infectivity of undeclared individuals;
- θ_2 : the false alarm coefficient for unreported individuals;
- k_1 : the proportion of undeclared infected individuals;
- k_2 : the proportion of infected individuals reported;
- η_1 : the cure rate of reported infected;
- η_2 : the cure rate of undeclared infected.

The propagation dynamics are described by the following Diagram 1.

The dynamics of disease spread are governed by the following system of differential equations:

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = bN(t) + rR(t) + \theta_2 E_n(t) + \lambda_2 E_i(t) - \dfrac{\gamma(t)}{N} S(t)\left(I(t) + I_u(t)\right) - \mu S(t) \\ \dfrac{\mathrm{d}E(t)}{\mathrm{d}t} = \dfrac{\gamma(t)}{N} S(t)\left(I(t) + I_u(t)\right) - (\alpha + \mu) E(t) \\ \dfrac{\mathrm{d}E_i(t)}{\mathrm{d}t} = \alpha_1 E(t) - (\lambda + \mu) E_i(t) \\ \dfrac{\mathrm{d}E_n(t)}{\mathrm{d}t} = \alpha_2 E(t) - (\theta + \mu) E_n(t) \\ \dfrac{\mathrm{d}I_r(t)}{\mathrm{d}t} = \lambda_1 E_i(t) + k_2 I(t) - (\mu_1 + \mu + \eta_1) I_r(t) \\ \dfrac{\mathrm{d}I(t)}{\mathrm{d}t} = \theta_1 E_n(t) - (\mu + k) I(t) \\ \dfrac{\mathrm{d}I_u(t)}{dt} = k_1 I(t) - (\mu_2 + \mu + \eta_2) I_u(t) \\ \dfrac{\mathrm{d}R(t)}{\mathrm{d}t} = \eta_1 I_r(t) + \eta_2 I_u(t) - (\mu + r) R(t) \end{cases} \quad (1)$$

3. Disease Free Equlibrium

To determine the equilibrium point without disease, it is assumed on the one hand that the functions $S, E, E_i, E_n, I_r, I, R$ and I_u are constant and on the other hand that we have no infected. We get the point of equilibrium

$$Q_0 = \left(0, 0, 0, 0, 0, 0, \frac{b}{\mu}, 0\right) \quad (2)$$

3.1. Reproductive Number

The infection compartments are E, E_i, E_n, I_r, I and I_u. To determine the number of reproduction, we consider the following vectors:

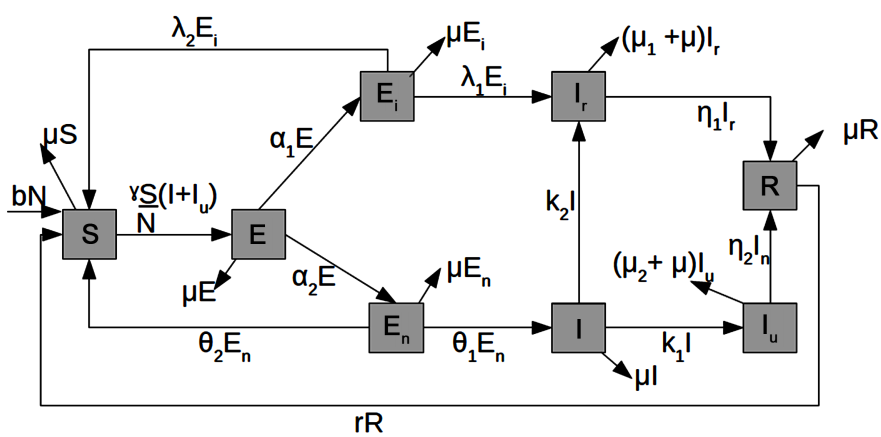

Diagram 1. The propagation of COVID-19 in Niamey (Niger).

$$f=\left(\frac{\gamma}{N}S\left(I+I_u\right),0,0,0,0,0\right) \text{ and } v=\begin{pmatrix} (\alpha+\mu)E \\ -\alpha_1 E+(\lambda+\mu)E_i \\ -\alpha_2 E+(\theta+\mu)E_n \\ -\lambda_1 E_i-k_2 I+(\mu_i+\mu+\eta_1)I_r \\ -\theta_1 E_n+(k+\mu)I \\ -k_1 I+(\eta_2+\mu_2+\mu)I_n \end{pmatrix}$$

By deriving these vectors with respect to E, E_i, E_n, I_r, I and I_u, we obtain the following respective Jacobian matrices taken at the equilibrium point Q_0:

$$F=\begin{pmatrix} 0 & 0 & 0 & 0 & \gamma & \gamma \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$V=\begin{pmatrix} \alpha+\mu & 0 & 0 & 0 & 0 & 0 \\ -\alpha_1 & \alpha+\mu & 0 & 0 & 0 & 0 \\ -\alpha_2 & 0 & \theta+\mu & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & \mu_1+\mu+\eta_1 & -k_2 & 0 \\ 0 & 0 & -\theta_1 & 0 & k+\mu & 0 \\ 0 & 0 & 0 & 0 & -k_1 & \eta_2+\mu_2+\mu \end{pmatrix}$$

The number of reproductions corresponds to the spectral radius of FV^{-1}, *i.e.* $\Re_0=\rho\left(FV^{-1}\right)$ [4] [5] [6]

$$\Re_0=\frac{\theta_1\alpha_2\gamma(\lambda+\mu)(k_1+\eta_2+\mu_2+\mu)}{(k+\mu)(\alpha+\mu)(\theta+\mu)(\eta_2+\mu_2+\mu)} \tag{3}$$

3.2. Signification of the Reproduction Number $\Re_0$

In epidemiology, we define $\Re_0$ as follows: $\Re_0$ is the basic reproduction rate, in other words, the average number of secondary cases produced by an infectious individual during the average time that he is infectious when he is introduced into a population made up entirely of susceptible individuals.

Intuitively any (infectious) disease, to spread in a community necessarily needs a minimum number (threshold) of patients below which it will disappear on its own. All the strategies for eradicating a disease aim to reduce the number of reproduction $\Re_0$ strictly less than 1. If this number remains strictly greater than 1, then the disease will persist in society.

4. Numerical Simulation and Interpretations

All the strategies adopted in the world aim to reduce the contact rate, which corresponds to the parameter that we noted γ in our model. Because currently, this is the only parameter on which we have the means to act to curb the spread of the disease.

In the analysis of our model, we considered the two possible cases:

A. when the number of declared infected individuals is greater than the number of unreported infected individuals;

B. when the number of infected individuals declared is less than the number of unreported infected individuals;

We present below the results obtained by simulation of the different cases A. and B.

- **Situation A.**

In this situation, we give the different figures for different values of $\Re_0$ and the contact rate γ.

Figure 1 and Figure 2 reflect the current situation of the evolution of the COVID-19 in the city of Niamey from 6h to 19h. During this period the majority of residents go about their business and therefore a high contact rate. It can be seen from Figure 1 that if no action is taken to reduce this contact rate, it will be difficult to overcome this disease. Figure 2 confirms that a reduction in this contact rate would allow us to achieve a reduction in the disease.

By drastically reducing the contact rate, we get Figure 3 and Figure 4.

It appears from the interpretation of Figure 3 that if $\Re_0 = 1$ and $\gamma = 0.15948$ in 95 days from 04/04/2020, the disease will disappear without any form of external interventions (without any form of treatment). The results of Figure 3 can be obtained by imposing the wearing of masks on the whole population.

As for Figure 4 if $\Re_0 = 0.0188$ and $\gamma = 0.003$, in 55 days (from 04/04/2020) we will be able to eradicate this scourge. This result could only be obtained in a context of total containment.

- **Situation B**

Figure 5 and Figure 6 give the same results as those of Figure 1 and Figure 2 with high numbers of infected individuals. This could cause a huge loss in terms of lives.

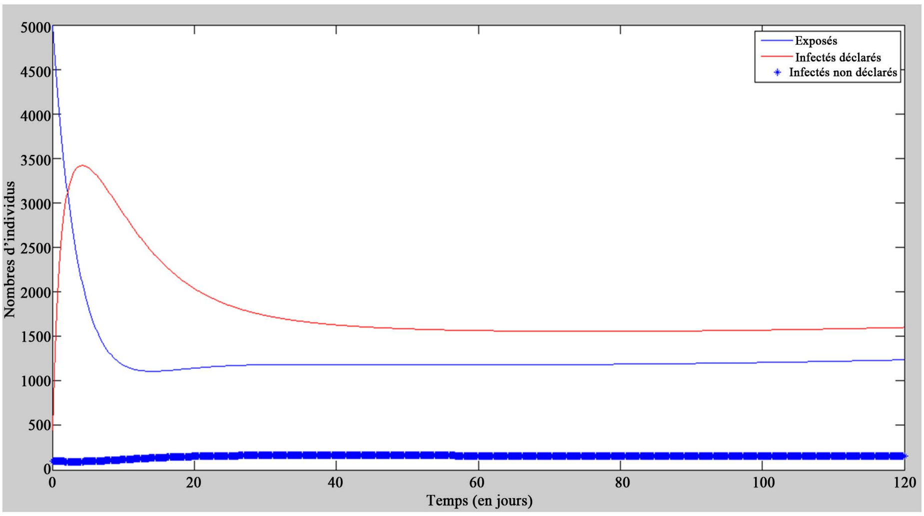

Figure 1. Evolution of the disease in Niamey with $\Re_0 = 5.6434$ and $\gamma = 0.9$.

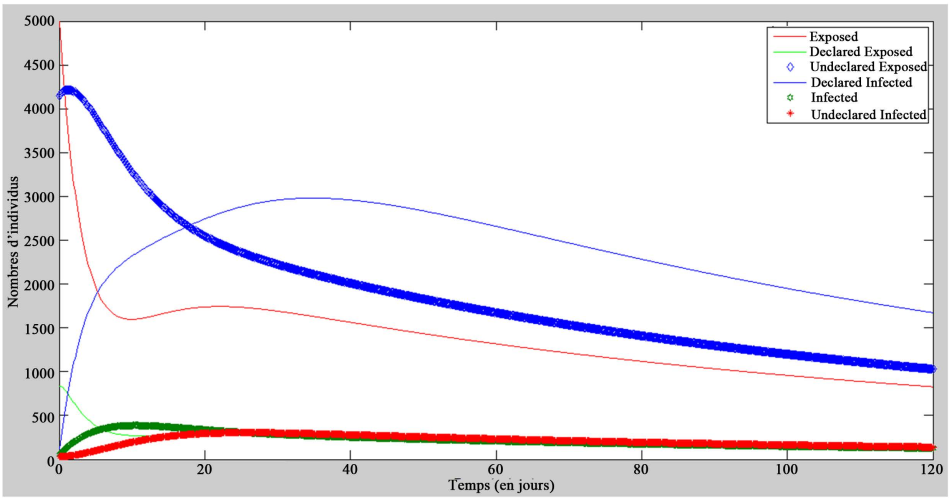

Figure 2. Evolution of the disease in Niamey with $\Re_0 = 4.3895$ and $\gamma = 0.7$.

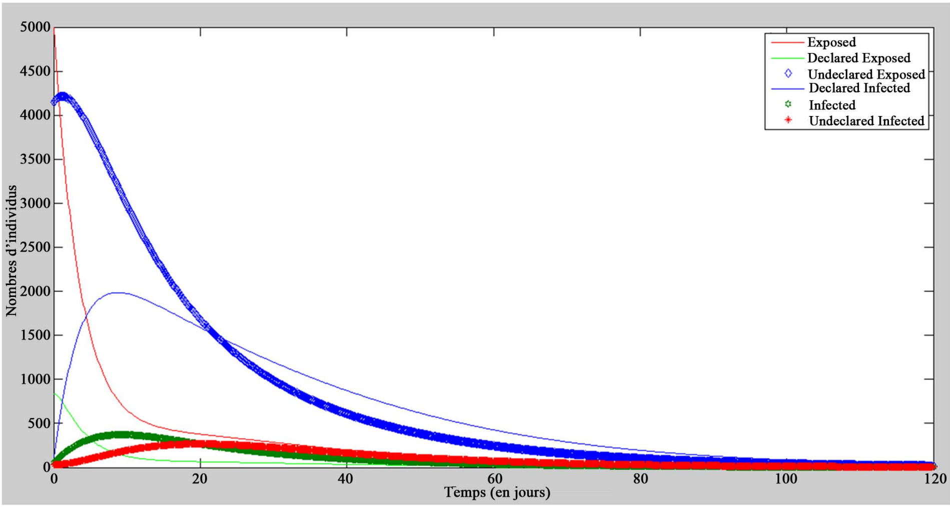

Figure 3. Evolution of the disease in Niamey with $\Re_0 = 1$ and $\gamma = 0.15948$.

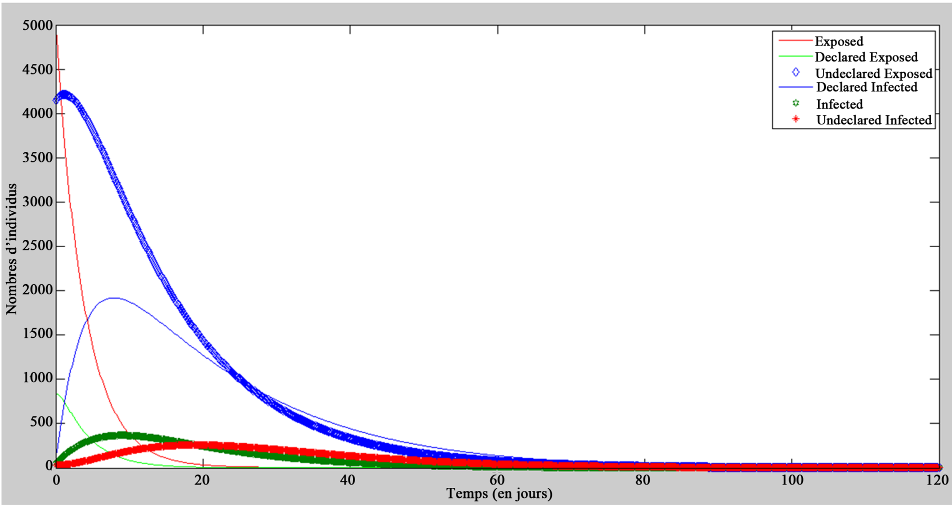

Figure 4. Evolution of the disease in Niamey with $\Re_0 = 0.0188$ and $\gamma = 0.003$.

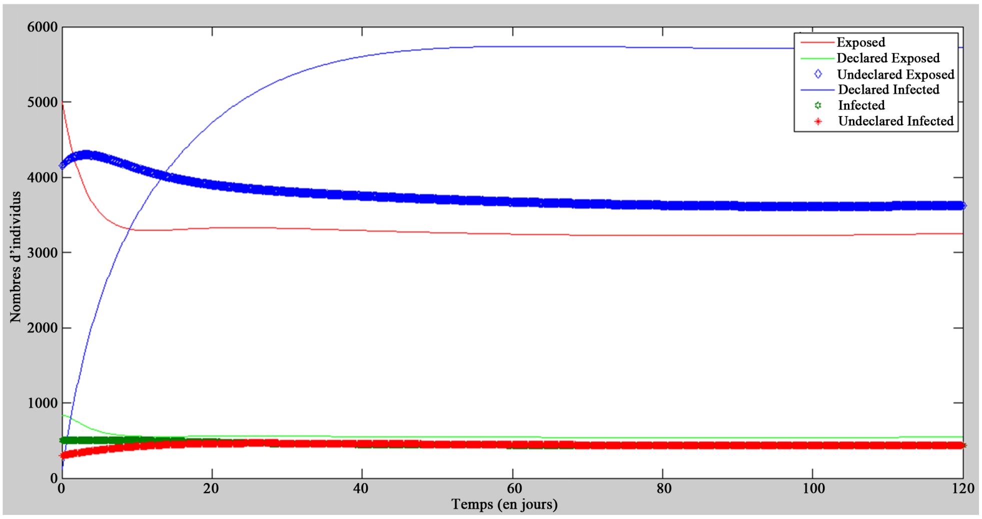

Figure 5. Evolution of the disease in Niamey with $\Re_0 = 5.6434$ and $\gamma = 0.9$.

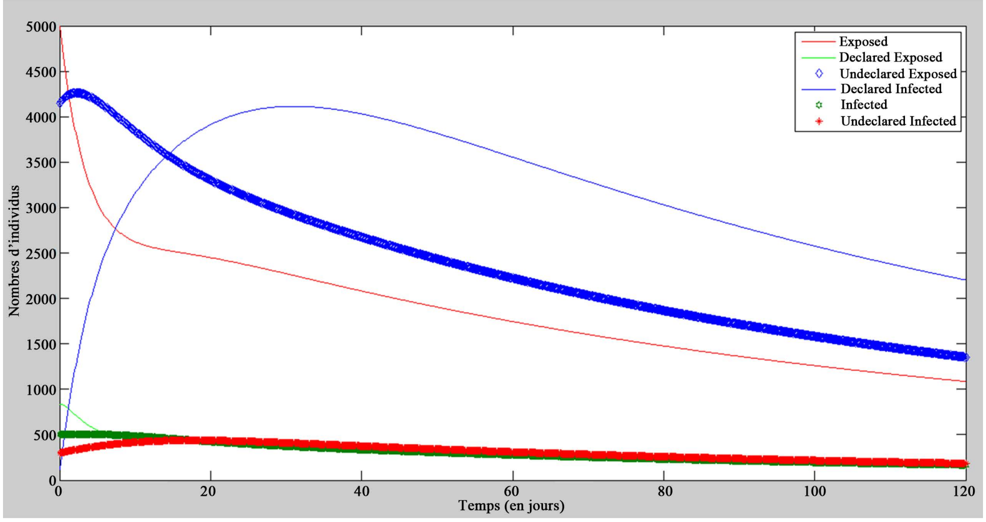

Figure 6. Evolution of the disease in Niamey with $\Re_0 = 4.3893$ and $\gamma = 0.7$.

Despite very low contact rates, if the number of unreported infected individuals exceeds that of declared individuals, we will take more than two months to eradicate this disease. All this is illustrated in Figure 7 and Figure 8.

In other words, in this context even in confinement we will take more than two months from 04/04/2020 to reach the end of the disease.

5. Conclusion and Perspectives

In this paper, we have proposed a mathematical model which describes the evolution of COVID-19 applied to the city of Niamey. This model takes into account most of the parameters describing the dynamics of the spread of this disease. In our study, we focused on the contact parameter (γ) which most influences the number of reproductions ($\Re_0$). The higher the contact parameter, the more the disease spreads within the population and vice versa.

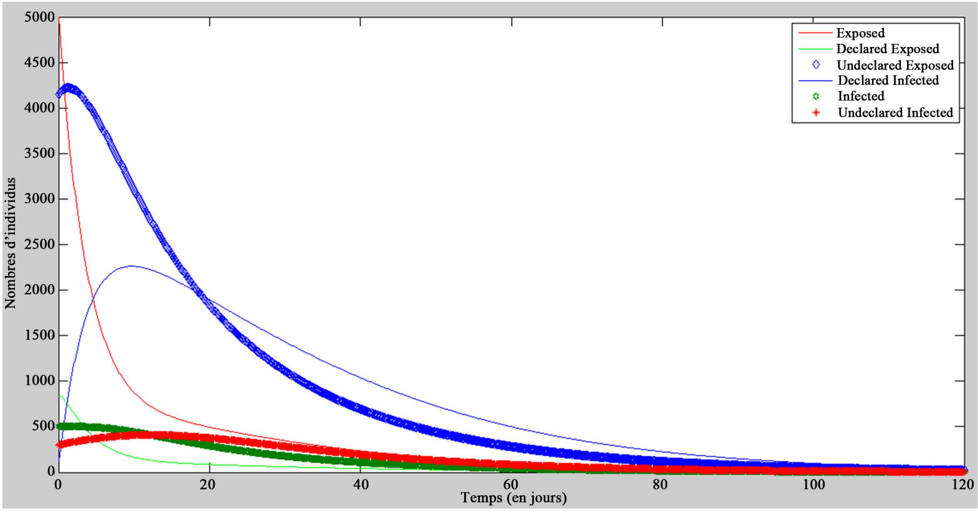

Figure 7. Evolution of the disease in Niamey with $\Re_0 = 1$ and $\gamma = 0.15948$.

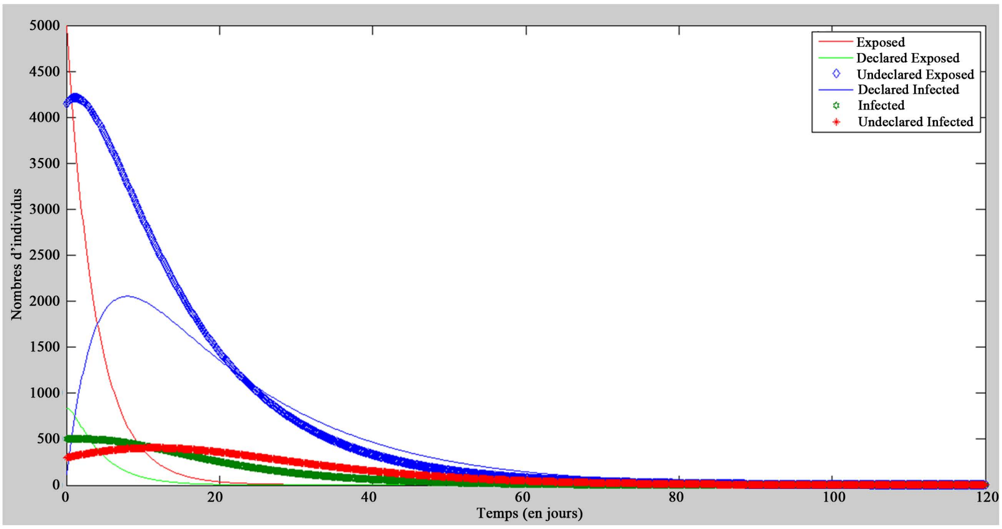

Figure 8. Evolution of the disease in Niamey with $\Re_0 = 0.0188$ and $\gamma = 0.003$.

At the end of the various simulations, it appears that if we reduce our contacts, we can eradicate this disease in 2 months at most. Otherwise, the situation will remain out of control.

We suggest to our decision makers:

1) To improve communication with the population, so that the latter can make its contribution by declaring suspect individuals.

2) In the absence of total containment, require the wearing of masks for the entire population of Niger.

3) Create a permanent unit of (scientific) experts for better management of any form of crisis.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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