American Journal of Operations Research, 2024, 14, 59-73
https://www.scirp.org/journal/ajor
ISSN Online: 2160-8849
ISSN Print: 2160-8830

A Note on an Order Level Inventory Model with Varying Two-Phased Demand and Time-Proportional Deterioration

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How to cite this paper: Mohanty, S., Singh, T., Routary, S.S. and Naik, C. (2024) A Note on an Order Level Inventory Model with Varying Two-Phased Demand and Time-Proportional Deterioration. *American Journal of Operations Research*, **14**, 59-73.
https://doi.org/10.4236/ajor.2024.141003

Received: December 25, 2023
Accepted: January 19, 2024
Published: January 22, 2024

Abstract

The main purpose of this paper is to generalize the effect of two-phased demand and variable deterioration within the EOQ (Economic Order Quantity) framework. The rate of deterioration is a linear function of time. The two-phased demand function states the constant function for a certain period and the quadratic function of time for the rest part of the cycle time. No shortages as well as partial backlogging are allowed to occur. The mathematical expressions are derived for determining the optimal cycle time, order quantity and total cost function. An easy-to-use working procedure is provided to calculate the above quantities. A couple of numerical examples are cited to explain the theoretical results and sensitivity analysis of some selected examples is carried out.

Keywords

Deteriorating Items, EOQ (Economic Order Quantity), Inventory, Time-Proportional Deterioration, Two-Phased Demand

1. Introduction

In recent decades, there has been a spate of interest in analyzing and formulating inventory models for deteriorating items from the point of view of practical applications. The classical EOQ (Economic Order Quantity) models of Harris [1] and Wilson [2] consider the exhaustion of inventory only due to the effect of unchanging demand. But, it has been observed that the depletion of the inventory is due to some demand patterns and some natural phenomena like decay, dryness, evaporation and spoilage. These natural phenomena are directly or indirectly called the process of "deterioration". Firstly, Ghare and Schrader [3] derived the mathematical expression of the EOQ inventory model with several factors such as inventory level during the cycle time, demand function and instantaneous deterioration rate. They represented these factors with the help of a differential equation

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t}+Z(t)I(t)+D(t)=0, 0\le t\le T,$$

where $I(t)$, $D(t)$, $Z(t)$ and T refer to the inventory level, the demand and the deterioration and time period, respectively. Donaldson [4] examined a complicated analytic solution procedure for the classical no-shortage inventory model for deteriorating items with linear trend in demand over a finite horizon cycle time. An order level no-shortage inventory system with constant demand as well as a constant rate of deterioration was developed by Aggarwal [5]. An inventory model for deteriorating items with linear trends demand, constant deterioration and no-shortages was studied by Dave and Patel [6]. Later, Sachan [7] extended Dave and Patel's [6] model considering a new factor as shortages. Singh and Pattnayak [8] studied an inventory model for a deteriorating item with varying time-dependent constant demand and Weibull distribution deterioration under permissible delay in payment. Some of the minimization problems on inventory models for deteriorating items under various conditions have been solved by [9]. Reviews of advances in deteriorating inventory policies were presented by Raafat [10], Goyal and Giri [11], Li *et al.* [12] and Janssen *et al.* [13].

In the detailed survey of review literature on inventory policies, it was observed that the assumption of a constant demand rate is not always suitable for many inventory items (cosmetics, fashion apparel, consumable items and electronics goods) because they experience fluctuations in demand during time of consideration. Thereafter, researches were developed on mainly two types of demand rates such as linear and exponential. The linear and exponential time-varying demands indicate a uniform change and rapid change in demand rates of the item per unit time, respectively those which seldom occur in the real market situation. The optimal replenishment policies for deteriorating items with linear trended demand shortages in all cycles were presented by Goswami and Chaudhuri [14] and Chakrabarti and Chaudhuri [15]. Chung and Ting [16] established an EOQ inventory model for deteriorating items with linear demand, no-shortages and time-proportional deterioration rate. Wee [17] presented an optimal replenishment policy with the assumptions of varying demand patterns and shortages in the declining situation of the market. Hollier and Mak [18] and Ouyang *et al.* [19] studied the optimal models for deteriorating items with constant deterioration rates and exponential demand rates. Srivastava and Gupta [20] formulated a no-shortage EOQ model for deteriorating items with a constant deterioration rate and both the constant and linearly increasing demand rate. Singh and Pattnayak [21] formulated an optimal two-warehouse inventory model with the incorporation of linear trended demand, constant deterioration and delay in payment conditions. Because of uniform and rapid changes in demand rate, some researchers have not considered the linear and exponential demand patterns for the development of their models. They included a time-dependent quadratic demand pattern in their models as it experiences either accelerated growth or retard growth in demand. The general form of the quadratic demand rate is $D(t)=a+bt+ct^2$. Here, $c=0$ and $b=c=0$ refer to the linear and constant demand patterns, respectively. An order level inventory model with no-shortages, time-dependent quadratic demand and constant deterioration was presented by Khanra and Chaudhuri [22]. Ghosh and Chaudhuri [23] established an optimal inventory model by incorporating the two-parameter Weibull distribution deterioration, time-dependent quadratic demand pattern and shortages. Khanra *et al.* [24] discussed an order-level inventory model for a deteriorating item with time-dependent quadratic demand rate and constant deterioration rate. Singh *et al.* (2017) formulated an ordering policy for deteriorating items focusing on the time-proportional deterioration rate and both constant and time-dependent linear demand rates and no-shortages.

Generally, items start deteriorating after a certain period of time on inclusion into the stock. Therefore, the constant deterioration rate is not always suitable for the consideration for formulation of the inventory model. This type of lacking is filled by Weibull distribution deterioration rates because it varies with the passage of time. In this context, Covert and Philip [25] modified Ghare and Schrader's [3] model by replacing the constant deterioration rate with the two-parameter Weibull distribution deterioration rate. Singh *et al.* [26] studied an optimal policy for deteriorating items with variable deterioration rates and constant and time-dependent demand rates. Singh *et al.* [27] presented a three-cased EOQ optimal policy for deteriorating items with the help of trapezoidal type demand, three-parameter Weibull distribution deterioration and shortages. A note on an inventory model for a deteriorating item with varying cubic demand and variable deterioration under permissible delay in payment was studied by Mohanty and Singh [28].

In practice, the assumptions of the constant deterioration rate and the constant demand pattern are debatable. A common practical situation is when the demand is constant for a certain period and then it varies with time for the rest part of the inventory cycle time. Recently, Srivastava and Gupta [20] studied a no-shortages inventory model for deteriorating items with a constant deterioration rate and both the constant and linear demand patterns. In this work, an optimal EOQ inventory policy is developed with the inclusion of a two-phased demand pattern, variable deterioration and no shortages. The two-phase demand describes the constant demand for the first part and time-dependent quadric demand for the rest part of the inventory cycle time. This pattern of demand rate shows the demand for items such as newly launched mobiles, automobiles, fashion apparels, cosmetics etc. as they experience constant demand in the first part and time-dependent quadric quadratic in the rest part of the cycle time. In addition, the deterioration rate is considered as a linear function of time for the second part of the inventory cycle. We think such type of two-phase demand pattern and time-proportional deterioration is quite realistic for developing no-shortage inventory model for deteriorating items. The objective of the proposed work is to optimize the average total cost and ordered quantity with respect to the cycle time. Finally, the model is illustrated with a couple of numerical examples and a sensitivity analysis of a selected example is presented.

The rest portion of the paper is arranged as follows: In Section 2, the notations and the fundamental assumptions involved in this problem are described. The mathematical formulation of the model and its working procedure are given in Section 3. Section 4 addresses a couple of numerical examples followed by sensitivity analysis of all system parameters in Section 5. Finally, a summary and some suggestions for future work are provided in Section 6.

2. Notations and Assumptions

The mathematical analysis of the EOQ inventory model is based on the following notations and assumptions:

2.1. Notations

The notations used in this paper are given below:

$\theta(t)$: The linear deterioration function, *i.e.*, $\theta(t)=\theta t$, where $0<\theta\ll 1$ and $t>0$. For $t=1$, the variable deterioration rate reduces to a constant deterioration rate.

$D(t)$: The demand rate, *i.e.*, $D(t)=\begin{cases} a, & 0\le t\le \eta \\ a+b(t-\eta)+c(t-\eta)^2, & \eta\le t\le T \end{cases}$.

During the first interval $[0,\eta]$, the demand is constant at the rate of a units per unit time, *i.e.*, it does not vary with time and during the second interval $[\eta,T]$, the demand rate is a quadratic function of time.

$I(t)$: The inventory level at any time t.

T: The length of the cycle.

I_s: The number of items received at the starting of the inventory system.

c_o: The ordering cost per order.

h_c: The inventory holding cost per unit per unit of time.

d_c: The unit cost of the item per unit per unit of time.

η: The time point at which the demand increases with time as well as the deterioration starts.

$TC(T)$: The average total cost per unit per unit time.

T^*: The optimal value of time T.

I_s^*: The optimal value of I_s.

$TC(T^*)$: The optimal average total cost per unit per unit time.

2.2. Assumptions

The following assumptions are considered in this paper:

1) The inventory system involves only one type of item.

2) There is no deterioration for the first part and the varying deterioration rate in the rest part of the cycle.

3) The demand is of two-phase pattern, *i.e.*, it is constant for the first part and quadratic function of time in the rest part of the cycle.

4) No shortages and backlogging are allowed to occur.

5) The occurrence of replenishment is instantaneous and the delivery lead time is zero.

6) Only a typical planning schedule of the length of the planning horizon is considered and all the remaining cycles are identical.

7) Deteriorated units are not replaced or repaired during the prescribed cycle period.

8) All types of costs such as ordering cost, holding cost and unit cost remain fixed over time.

3. Mathematical Formulation of the Model and Its Solution Procedure

The following model as well as its solution procedure is started below:

3.1. Model Development

Initially, the order quantity brings the level of the system up to I_s units. Because of the constant demand, the level of the inventory declines, say, *a* units per unit time in the interval $[0,\eta]$. Due to time-dependent quadratic demand and time-varying deterioration, the inventory level gradually depletes over time $t=T$. The nature of the inventory system is shown in **Figure 1**.

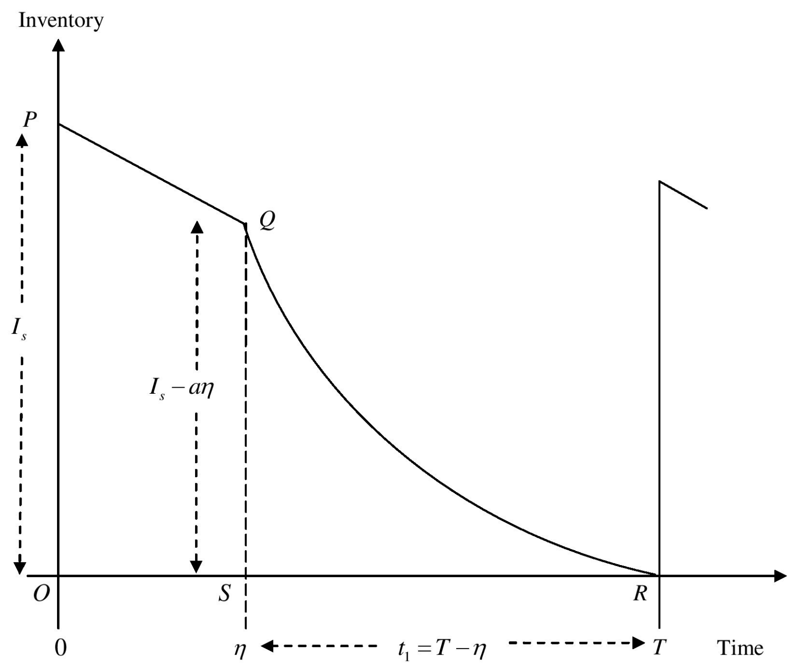

Figure 1. The behavior of inventory level with time.

The objective of the proposed problem is to optimize the cycle length T so that the present value of the average total cost $TC(T)$ is minimized.

Initially, during the interval $[0,\eta]$, the system demand rate (*SDR*) per unit time and the total demand (*TDR*) are given by

$$SDR = a\,, \tag{3.1}$$

and

$$TDR = a\eta\,, \tag{3.2}$$

respectively.

Therefore, the inventory level is decreased by the portion $a\eta$ and thus, the remaining portion of the inventory (*SRI*) during the interval $[\eta,T]$ is given by

$$SRI = I_s - a\eta\,. \tag{3.3}$$

In order to make the calculation easier, the interval $[\eta,T]$ can be taken as

$$t_1 = T - \eta\,. \tag{3.4}$$

Consequently, the instantaneous inventory level $I(t)$ at any time t during the time interval $0 \le t \le t_1$ satisfies the following differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta(t)I(t) = -\left[a + b(t-\eta) + c(t-\eta)^2\right],\quad 0 \le t \le t_1 \tag{3.5}$$

where $\theta(t) = \theta t, (0 < \theta \ll 1)$ and $I(0) = I_s - a\eta$.

Here the integrating factor (*IF*) and the solution of the differential equation are

$$IF = \mathrm{e}^{\frac{\theta t^2}{2}}\,, \tag{3.6}$$

and

$$I(t)\cdot \mathrm{e}^{\frac{\theta t^2}{2}} = -\int\left[a + b(t-\eta) + c(t-\eta)^2\right]\cdot \mathrm{e}^{\frac{\theta t^2}{2}}\,\mathrm{d}t + k\,, \tag{3.7}$$

where k is a constant of integration, respectively.

Solving the differential Equation (3.5), we have

$$\begin{aligned} I(t) = \Bigg[& I_s - a\eta - a\left(t + \frac{\theta t^3}{6}\right) - b\left(-2\eta + t^2 - \frac{\eta\theta t^3}{6} + \frac{\eta t^4}{8}\right) \\ & - c\left[\eta^2 t - \eta t^2 + \left(\frac{2+\theta\eta^2}{6}\right)t^3 - \frac{\eta\theta t^4}{4} + \frac{\theta t^5}{10}\right]\Bigg]\cdot \mathrm{e}^{-\frac{\theta t^2}{2}},\quad 0 \le t \le t_1 \end{aligned} \tag{3.8}$$

by ignoring the terms containing the powers like $\theta^2, \theta^3, \theta^4, \cdots \quad 0 < \theta \ll 1$.

The Equation (3.8) at $I(t_1) = 0$ is given by

$$\begin{aligned} I_s = {} & a\eta + a\left(t_1 + \frac{\theta t_1^3}{6}\right) + b\left(-\eta t_1 + \frac{t_1^2}{2} - \frac{\eta\theta t_1^3}{6} + \frac{\eta t_1^4}{8}\right) \\ & + c\left[\eta^2 t_1 - \eta t_1^2 + \left(\frac{2+\theta\eta^2}{6}\right)t_1^3 - \frac{\eta\theta t_1^4}{4} + \frac{\theta t_1^5}{10}\right] \end{aligned} \tag{3.9}$$

Using the relation $t_1 = T - \eta$, Equation (3.9) is equivalent to

$$I_s = a\left[T + \frac{\theta(T-\eta)^3}{6}\right] + b\left[-\eta(T-\eta) + \frac{(T-\eta)^2}{2} - \frac{\eta\theta(T-\eta)^3}{6} + \frac{\eta(T-\eta)^4}{8}\right]$$
$$+ c\left[\eta^2(T-\eta) - \eta(T-\eta)^2 + \left(\frac{2+\theta\eta^2}{6}\right)(T-\eta)^3 - \frac{\eta\theta(T-\eta)^4}{4} + \frac{\theta(T-\eta)^5}{10}\right] \tag{3.10}$$

In order to obtain the optimum total average cost, following cost components are needed:

1) Ordering cost (*SSC*):

$$SSC = c_o. \tag{3.11}$$

2) Holding cost (*SCC*) during the period $[0,T]$ is the sum of the holding cost during the period $[0,\eta]$ and the holding cost during the period $[\eta,T]$, *i.e.*,

$$SCC = h_c \times [\text{area of trapezium PQSO} + \text{area of triangle QRS}]$$
$$= h_c\left[\frac{1}{2}\cdot(I_s + (I_s - a\eta))\cdot\eta + \frac{1}{2}\cdot t_1\cdot(I_s - a\eta)\right] \tag{3.12}$$
$$= h_c\left[\frac{a\eta^2}{2} + (I_s - a\eta)\left(\eta + \frac{t_1}{2}\right)\right]$$

3) The deterioration cost (*SDC*) during the period $[0,T]$ is

$$SDC = d_c\left[I_s - a\eta - \int_0^{t_1}\left[a + b(t-\eta) + c(t-\eta)^2\right]\mathrm{d}t\right]$$
$$= d_c\left[I_s - a(t_1+\eta) - \frac{b(t_1-\eta)^2}{2} - \frac{c(t_1-\eta)^3}{3} - \frac{b\eta^2}{2} + \frac{c\eta^3}{3}\right] \tag{3.13}$$

Hence, using the previous results, the average total cost per unit time ($TC(T)$) of the system during the period $[0,T]$:

$$TC(T) = \frac{1}{T}[SSC + SCC + SDC]$$
$$= \frac{1}{T}\left[c_o + h_c\left[\frac{a\eta^2}{2} + (I_s - a\eta)\left(\eta + \frac{t_1}{2}\right)\right] + d_c\left[I_s - a(t_1+\eta) - \frac{b(t_1-\eta)^2}{2} - \frac{c(t_1-\eta)^3}{3} - \frac{b\eta^2}{2} + \frac{c\eta^3}{3}\right]\right]$$
$$= \frac{c_o}{T} + \frac{h_c a\eta^2}{2T} - \frac{d_c}{T}\left[a(T-\eta) + \frac{b(T-2\eta)^2}{2} + \frac{c(T-2\eta)^3}{3} + \frac{b\eta^2}{2} - \frac{c\eta^3}{3}\right]$$
$$+ \frac{T-\eta}{T}\left[a\left(1 + \frac{\theta(T-\eta)^2}{6}\right) + b\left(-\eta + \frac{T-\eta}{2} - \frac{\eta\theta(T-\eta)^2}{6} + \frac{\theta(T-\eta)^3}{8}\right)\right.$$
$$\left. + c\left(\eta^2 - \eta(T-\eta) + \left(\frac{2+\theta\eta^2}{6}\right)(T-\eta)^2 - \frac{\eta\theta(T-\eta)^3}{4} + \frac{\theta(T-\eta)^4}{10}\right)\right] \tag{3.14}$$
$$\cdot\left[h_c\left(\frac{T+\eta}{2}\right) + d_c\right]$$

by using the Equations (3.2) and (3.7).

In order to determine the optimum value of ($TC(T)$), the necessary condition is

$$\frac{\mathrm{d}\left(TC(T)\right)}{\mathrm{d}T}=0, \tag{3.15}$$

provided it satisfies the sufficient condition

$$\frac{\mathrm{d}^2\left(TC(T)\right)}{\mathrm{d}T^2}=0. \tag{3.16}$$

From the Equation (3.15), we have

$$\begin{aligned}\frac{\mathrm{d}\left(TC(T)\right)}{\mathrm{d}T}=&\frac{1}{T}\left[a\left(1+\frac{\theta(T-\eta)^2}{6}\right)+b\left(-\eta+\frac{T-\eta}{2}-\frac{\eta\theta(T-\eta)^2}{6}+\frac{\theta(T-\eta)^3}{8}\right)\right.\\&\left.+c\left(\eta^2-\eta(T-\eta)+\left(\frac{2+\theta\eta^2}{6}\right)(T-\eta)^2-\frac{\eta\theta(T-\eta)^3}{4}+\frac{\theta(T-\eta)^4}{10}\right)\right]\\&\cdot\left[h_cT+d_c\right]+\frac{T-\eta}{T}\left[\frac{a\theta(T-\eta)}{3}+b\left(\frac{1}{2}-\frac{\eta\theta(T-\eta)}{3}+\frac{3\theta(T-\eta)^2}{2}\right)\right.\\&\left.+c\left(-\eta+\left(\frac{2+\theta\eta^2}{3}\right)(T-\eta)-\frac{3\eta\theta(T-\eta)^2}{4}+\frac{2\theta(T-\eta)^3}{5}\right)\right]\\&\cdot\left[h_c\left(\frac{T+\eta}{2}\right)+d_c\right]-\frac{d_c}{T}\left[a+b(T-2\eta)+c(T-2\eta)^2\right]-\frac{TC(T)}{T}\\=&0\end{aligned} \tag{3.17}$$

provided the sufficient condition $\frac{\mathrm{d}^2\left(TC(T)\right)}{\mathrm{d}T^2}>0$ (see **Appendix**).

Special cases: 1) Putting $c=0$ in the demand pattern $D(t)=\begin{cases}a, & 0\le t\le\eta\\ a+b(t-\eta)+c(t-\eta)^2, & \eta\le t\le T\end{cases}$, the model reduces to that of Srivastava and Gupta [20].

2) Putting $t=1$ in the deterioration rate $\theta(t)=\theta t$, the model also reduces to that of Srivastava and Gupta [20].

A step by step solution procedure for above problem is given below.

3.2. Solution Procedure: Algorithms

To obtain the values of ($TC(T)$) and I_s, the following steps are required.

Step I. Input the appropriate value of the parameters.

Step II. Obtain the value of T from the Equation (3.15) by Numerical method.

Step III. Compare T with η.

1) If $T>\eta$, then T is a feasible solution, say T^*. Go to Step IV.

2) If $T<\eta$, then T is infeasible.

Step IV. Substitute T^* in the Equations (3.14) and (3.10) to get ($TC(T^*)$) and I_s^*, respectively.

4. Numerical Examples

To illustrate the results obtained from the inventory problem with two-phased demand and time-proportional deterioration, the following numerical examples are cited.

Example 1: Let us take the parametric values of the inventory model of deteriorating items in their units as follows: $a = 20$ units, $b = 0.2$, $c = 100$, $\theta = 0.02$, $\eta = 0.4$ days, $c_o = \$80$, $h_c = \$0.5/\text{unit}/\text{day}$ and $d_c = \$18/\text{unit}$.

Solving the Equation (3.15), the optimal cycle time is $T^* = 1.75651$ days which satisfies the sufficient condition, *i.e.*, $\frac{\mathrm{d}^2(TC(T))}{\mathrm{d}T^2} = 157.701 > 0$. Substituting the value of $T^* = 1.75651$ in the Equations (3.14) and (3.10), the optimal value of the average total cost and the optimal order quantity are $TC(T^*) = 113.074$ and $I_s^* = 67.0517$ units, respectively.

Example 2: Let us take the parametric values of the inventory model of deteriorating items in their units as follows: $a = 20$ units, $b = 0.2$, $c = 160$, $\theta = 0.02$, $\eta = 0.4$ days, $c_o = \$80$, $h_c = \$0.5/\text{unit}/\text{day}$ and $d_c = \$18/\text{unit}$.

Solving the Equation (3.15), the optimal cycle time is $T^* = 1.71125$ days which satisfies the sufficient condition, *i.e.*, $\frac{\mathrm{d}^2(TC(T))}{\mathrm{d}T^2} = 229.733 > 0$. Substituting the value of $T^* = 1.71125$ in the equations (3.14) and (3.10), the optimal value of the average total cost and the optimal order quantity are $TC(T^*) = 174.205$ and $I_s^* = 78.6997$ units, respectively.

5. Sensitivity Analysis

The effect of changes in the values of various parameters *a*, *b*, *c*, θ, η, c_o, h_c and d_c for the optimum cost and optimum order quantity is studied with the help of sensitivity analysis. It works by changing the each of the parameters by +50%, +20%, +10%, −10%, −20% and −50% at a time and keeping remaining parameters unchanged. The analysis is developed on the basis of **Example-2** and the results are displayed in Table 1. The following points are noted.

1) T^* decreases while $TC(T^*)$ and I_s^* increase with the increase in the value of the parameter *a*. Here T^*, $TC(T^*)$ and I_s^* are insensitive to changes in *a*.

2) T^*, $TC(T^*)$ and I_s^* decrease with the increase in the value of the parameter *b*. Here T^*, $TC(T^*)$ and I_s^* are insensitive to changes in *b*.

3) T^* decreases while $TC(T^*)$ and I_s^* increase with the increase in the value of the parameter *c*. Here T^*, $TC(T^*)$ and I_s^* are moderately sensitive to changes in *c*.

4) T^* and I_s^* decrease while $TC(T^*)$ increases with the increase in the value of the parameter θ and h_c. Here T^*, $TC(T^*)$ and I_s^* are moderately sensitive to changes in θ and h_c.

5) T^*, $TC(T^*)$ and I_s^* increase with the increase in the value of the parameter η. Here T^*, $TC(T^*)$ and I_s^* are highly sensitive to changes in η.

6) T^*, $TC(T^*)$ and I_s^* increase with the increase in the value of the parameter c_o and d_c. Here T^*, $TC(T^*)$ and I_s^* are moderately sensitive to changes in c_o and d_c.

Table 1. Sensitivity analysis.

Parameter	% change in parameter	T^*	% change in T^*	$TC(T^*)$	% change in $TC(T^*)$	I_s^*	% change in I_s^*
a	+50	1.69458	−0.97414	152.264	+3.43670	93.1643	+18.3795
	+20	1.70457	−0.39035	149.236	+1.37971	84.5144	+7.38847
	+10	1.70791	−0.19517	148.222	+0.69087	81.6121	+3.70065
	−10	1.71459	+0.19517	146.185	−0.69291	75.7770	−3.71374
	−20	1.71794	+0.39094	145.163	−1.38718	72.8456	−7.43853
	−50	1.72799	+0.97823	142.080	−2.48154	63.9901	−18.6908
b	+50	1.71065	−0.03506	147.052	−0.10393	78.6402	−0.07560
	+20	1.71101	−0.01402	147.144	−0.04143	78.6759	−0.03024
	+10	1.71113	−0.00701	147.174	−0.02105	78.6878	−0.01512
	−10	1.71137	+0.00701	147.236	+0.02105	78.7116	+0.01512
	−20	1.71149	+0.01402	147.266	+0.04143	78.7235	+0.03024
	−50	1.71184	+0.03447	147.358	+0.10393	78.7576	+0.07357
c	+50	1.68215	−1.70051	192.354	+30.6708	94.5202	+20.1024
	+20	1.69711	−0.82629	165.293	+12.2876	85.0034	+8.00981
	+10	1.70363	−0.44528	156.255	+6.14789	81.8457	+3.99747
	−10	1.72025	+0.52593	138.138	−6.15944	75.5645	−3.98375
	−20	1.73108	+1.15880	129.051	−12.3325	72.4471	−7.94488
	−50	1.78280	+4.18115	101.571	−31.0003	63.2716	−19.6038
θ	+50	1.66492	−2.70738	150.376	+2.15414	72.0672	−8.42751
	+20	1.69131	−1.16523	148.538	+0.90554	75.7724	−3.71958
	+10	1.70101	−0.59839	147.883	+0.45990	77.1824	−1.92796
	−10	1.72208	+0.63287	146.501	−0.47824	80.3366	+2.07993
	−20	1.73357	+1.30431	145.768	−0.97619	82.1101	+4.33343
	−50	1.77295	+3.60555	143.372	−2.60385	88.4816	+12.4294
η	+50	2.24873	+31.4086	266.410	+80.9789	119.598	+51.9675
	+20	1.92160	+12.7012	186.730	+26.8503	92.7249	+17.8212
	+10	1.81516	+6.07217	165.599	+12.4357	85.2893	+8.37309
	−10	1.61093	−5.86238	131.567	−10.6233	73.0312	−7.20270
	−20	1.51549	−11.4396	118.703	−19.3621	68.3794	−13.1135
	−50	1.27237	−25.6467	96.6536	−34.5446	61.7054	−21.5939
c_o	+50	1.76569	+3.18130	170.207	+15.6258	87.6296	+11.3468
	+20	1.73397	+1.32768	156.493	+6.30957	82.3149	+4.59366
	+10	1.72278	+0.67377	151.864	+3.16497	80.5147	+2.30623
	−10	1.69935	−0.69539	142.514	−3.18671	76.8684	−2.32695
	−20	1.68705	−1.41417	137.789	−6.39651	75.0197	−4.67600
	−50	1.64746	−3.74522	123.396	−16.1740	69.3665	−11.8593
h_c	+50	1.62771	−4.88181	157.400	+6.92572	66.7111	−15.2333
	+20	1.67479	−2.13061	151.495	+2.91430	73.2210	−6.96153
	+10	1.69244	−1.09920	149.390	+1.48432	75.8243	−3.65364
	−10	1.73135	+1.17458	144.932	−1.54411	81.8900	+4.05376
	−20	1.75290	+2.43389	142.561	−3.15478	85.4486	+8.57551
	−50	1.82823	+6.83594	134.731	−8.47390	99.0608	+25.8719
d_c	+50	1.74198	+1.79576	186.157	+26.4611	83.6274	+6.26140
	+20	1.72525	+0.81811	162.816	+10.6049	80.9088	+2.80700
	+10	1.71859	+0.42892	155.016	+5.30621	79.9073	+1.53444
	−10	1.70311	−0.47563	139.379	−5.31640	77.4424	−1.59759
	−20	1.69405	−1.00511	131.538	−10.6430	76.0663	−3.34614
	−50	1.65948	−3.02527	107.882	−26.7131	71.0356	−9.73841

Table 2. Effect of η on the optimal solution.

η	Change (%) in η	T^*	Comparison between η and T^*	$TC(T^*)$	I_s^*
0.500	+25.0000	1.97551 (+1544.25)	$\eta < T^*$	198.317 (+34.7216)	96.7430 (+22.9268)
1.000	+150.000	3.33619 (+94.9563)	$\eta < T^*$	708.653 (+381.406)	253.465 (+222.066)
2.000	+400.000	5.86408 (+242.678)	$\eta < T^*$	3075.06 (+1988.96)	935.278 (+1088.41)
5.000	+1150.00	12.6870 (+641.388)	$\eta < T^*$	22916.5 (+15467.7)	22916.5 (+15467.7)
6.000	+1400.00	14.8515 (+767.874)	$\eta < T^*$	34479.0 (+23322.4)	14334.9 (+18114.7)
6.100	+1425.00	15.0663 (+780.427)	$\eta < T^*$	35787.8 (+24211.5)	15017.4 (+18981.9)
6.200	+1450.00	15.2808 (+792.961)	$\eta < T^*$	37125.3 (+25120.1)	15722.6 (+19878.0)
6.300	+1475.00	15.4950 (+805.478)	$\eta < T^*$	38491.7 (+26048.4)	16450.8 (+20803.3)
6.400	+1500.00	15.7089 (+817.978)	$\eta < T^*$	39887.4 (+26996.5)	17202.4 (+21758.3)
6.500	+1525.00	15.9226 (+830.466)	$\eta < T^*$	41312.4 (+27964.5)	17978.2 (+22744.1)
6.510	+1527.50	15.9439 (+831.711)	$\eta < T^*$	41456.5 (+28062.4)	41456.5 (+28062.4)
6.550	+1537.50	16.0293 (+836.701)	$\eta < T^*$	42036.0 (+28456.1)	16596.0 (+20987.8)
6.560	+1540.00	16.0507 (+837.952)	$\eta < T^*$	42181.6 (+28555.0)	18455.4 (+23350.4)
6.570	+1542.50	16.0720 (+839.1960)	$\eta < T^*$	42327.5 (+28654.1)	18535.7 (+23452.4)
6.571	+1542.75	16.0740 (839.31300)	$\eta < T^*$	42342.2 (+28664.1)	18543.3 (+23462.1)
6.572	+1543.00	16.0763 (+839.4480)	$\eta < T^*$	42356.8 (+28674.0)	18551.8 (+23472.9)
6.573	+1543.25	16.0784 (+839.5700)	$\eta < T^*$	42371.4 (+28683.9)	18559.8 (+23483.1)
6.574	+1543.50	16.0806 (+839.6990)	$\eta < T^*$	42386.0 (+28693.9)	18568.0 (+23493.5)
6.575	+1543.75	0.843227 (−50.7245)	$\eta > T^*$	…	…
6.576	+1544.00	0.845722 (−50.5758)	$\eta > T^*$	…	…
7.000	+1650.00	1.67175 (−2.308250)	$\eta > T^*$	…	…

The effects of η on the basis of **Example-2** are also discussed in **Table 2**.

Here "…" denotes the infeasible solution.

It reveals that if the parameter η is increased by 1543.5%, then the value of the optimal cycle time is increased by 839.699%, the optimal average total cost is increased by 28693.9% and the optimal order quantity is increased by 23493.5%. Further, if the parameter η is increased by 1543.75%, then the value of the optimal cycle time is decreased by −50.7245%.

The notable point is that the increase in the value of the parameter η after 6.574 gives an infeasible solution as $\eta > T^*$.

6. Conclusions

This study develops an EOQ inventory model for deteriorating items with a variable deterioration rate and a two-phased demand rate. The two-phased demand rate experiences constant and quadratic demand functions in its first and second phases, respectively. The deterioration rate is a linear function of time.

The reason for assuming two-phased demand is due to the newly launched items like mobiles, automobiles, cosmetics, fashion apparels, etc. The demand for such items remains constant for some period of time and then varies with the quadratic function of time. Any types of shortages like partially backlogged and complete backlogged are not allowed to occur. The article concludes with a couple of numerical examples and a sensitivity analysis of various parameters to support the theoretical results. The objective of the proposed model is to generalize the demand rate involved in the model of Srivastava and Gupta (2007). The general form of the quadratic demand is considered as $D(t)=a+bt+ct^2$ where $a>0$, $b\neq 0$ and $c\neq 0$. Here $c=0$ and $b=c=0$ refer the time-dependent linear demand pattern and constant demand, respectively. The variable deterioration of the system is of the form $\theta(t)=\theta t,(0<\theta\ll 1)$. If $t=1$, then the variable deterioration rate becomes a constant deterioration rate. To be more precise, the proposed model has been studied for the computation of the cycle time, order quantity and total average cost under the factors of time-dependent two-phased demand and variable deterioration.

A future study will further include the proposed model into several realistic approaches like stochastic demand, generalized demand pattern, stock and price dependent demand. We could extend the work to several variable deteriorations like the two-parameter Weibull distribution deterioration and Gamma distribution deterioration. Finally, this work can be extended by incorporating the concept of shortages or partial backlogging.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

$$
\begin{aligned}
&\frac{\mathrm{d}^2\left(TC(T)\right)}{\mathrm{d}T^2}\\
&=\frac{h_c}{T}\left[a\left(1+\frac{\theta(T-\eta)^2}{6}\right)+b\left(-\eta+\frac{T-\eta}{2}-\frac{\eta\theta(T-\eta)^2}{6}+\frac{\theta(T-\eta)^3}{8}\right)\right.\\
&\quad\left.+c\left(\eta^2-\eta(T-\eta)+\left(\frac{2+\theta\eta^2}{6}\right)(T-\eta)^2-\frac{\eta\theta(T-\eta)^3}{4}+\frac{\theta(T-\eta)^4}{10}\right)\right]\\
&\quad+\frac{1}{T}\left[\frac{a\theta(T-\eta)}{3}+b\left(\frac{1}{2}-\frac{\eta\theta(T-\eta)}{3}+\frac{3\theta(T-\eta)^2}{8}\right)\right.\\
&\quad\left.+c\left(-\eta+\left(\frac{2+\theta\eta^2}{3}\right)(T-\eta)-\frac{3\eta\theta(T-\eta)^2}{4}+\frac{2\theta(T-\eta)^3}{5}\right)\right]\\
&\cdot\left[h_c\left(\frac{3T+\eta+1}{2}\right)+2d_c\right]+\frac{T-\eta}{T}\left[\frac{a\theta}{3}+b\left(-\frac{\eta\theta}{3}+\frac{3\theta(T-\eta)}{4}\right)\right.\\
&\left.+c\left(\frac{2+\theta\eta^2}{3}-\frac{3\eta\theta}{2}(T-\eta)+\frac{6\theta(T-\eta)^2}{5}\right)\right]\cdot\left[h_c\left(\frac{T+\eta}{2}\right)+d_c\right]-\frac{d_c}{T}\left[a+2b(T-2\eta)\right]\\
&-\frac{2}{T^2}\left[a\left(1+\frac{\theta(T-\eta)^2}{6}\right)+b\left(-\eta+\frac{T-\eta}{2}-\frac{\eta\theta(T-\eta)^2}{6}+\frac{\theta(T-\eta)^3}{8}\right)\right.\\
&\left.+c\left(\eta^2-\eta(T-\eta)+\left(\frac{2+\theta\eta^2}{6}\right)(T-\eta)^2-\frac{\eta\theta(T-\eta)^3}{4}+\frac{\theta(T-\eta)^4}{10}\right)\right]\cdot\left[h_cT+d_c\right]\\
&-\frac{2(T-\eta)}{T^2}\left[\frac{a\theta(T-\eta)}{3}+b\left(\frac{1}{2}-\frac{\eta\theta(T-\eta)}{3}+\frac{3\theta(T-\eta)^2}{2}\right)\right.\\
&\left.+c\left(-\eta+\left(\frac{2+\theta\eta^2}{6}\right)(T-\eta)-\frac{3\eta\theta(T-\eta)^2}{4}+\frac{2\theta(T-\eta)^3}{5}\right)\right]\cdot\left[h_c\left(\frac{T+\eta}{2}\right)+d_c\right]\\
&+\frac{2d_c}{T^2}\left[a+b(T-2\eta)+c(T-2\eta)^2\right]+\frac{2}{T^2}\left(TC(T)\right)
\end{aligned}
$$