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Using Return and Risk Model for Choosing Perfect Portfolio Applied Study in Cairo Stock Exchange

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Abstract

Modern financial theory, commonly known as portfolio theory, provides an analytical framework for the investment decision to be made under uncertainty. It is a well-established proposition in portfolio theory that whenever there is an imperfect correlation between returns risk is reduced by maintaining only a portion of wealth in any asset, or by selecting a portfolio according to expected returns and correlations between returns. The major improvement of the portfolio approaches over prior received theory is the incorporation of 1) the riskiness of an asset and 2) the addition from investing in any asset. The theme of this paper is to discuss how to propose a new mathematical model like that provided by Markowitz, which helps in choosing a nearly perfect portfolio and an efficient input/output. Besides applying this model to reality, the researcher uses game theory, stochastic and linear programming to provide the model proposed and then uses this model to select a perfect portfolio in the Cairo Stock Exchange. The results are fruitful and the researcher considers this model a new contribution to previous models.

Keywords

Game Theory, Stochastic and Linear Programming, Perfect Portfolio, Portfolio Theory, Returns and Risks

1. The Theoretical Introduction

Recent history has shown us that many problems of our technically oriented society yield mathematical descriptions and solutions [1].

This research is concerned with three specific fields of mathematics, stochastic programming, linear programming and game theory that offer insights into certain problems of the real world and techniques for solving some of these problems.

Game theory is a mathematical framework that is used to study decision-making in situations of strategic interaction. It is used to model and analyze situations where the outcome depends not only on the actions of an individual but also on the actions of other individuals. Game theory is widely used in many fields of science and mathematics [2].

The methodology of this research is based on a game theory and stochastic programming model that select portfolio positions which perform well on a variety of scenarios generated through statistical modelling and optimization, necessary to reach the proposed model in this research.

The interest of accountants in quantity methods, like those of economists, goes back to the inefficiency in the concept of competitive equilibrium, which states that every decision-maker ignores the behaviour of others when making decisions. Therefore, game theory is used to analyze the actions of players in a strategic manner, and to further analyze how these players make decisions after creating a pre-image of strategies and interests of others.

Although game theory differs from the concept of competitive equilibrium, they both share the following:

1) Each player in the game should be considered rational in making decisions, should have limited preferences, and choose his perfect strategy according to these preferences.

2) The player predicts in game theory and the concept of competitive equilibrium the situation he faces and realizes to what extent he can depend on the results of his decision.

3) The player must know the suitable environment variables in order to interpret some concepts of theoretical solutions in games.

Regarding how to choose a perfect portfolio, operational research is required to a perfect solution for the problem of choosing perfect investments.

We can use an accounting information system that provides the following:

1) Information about stocks regarding prices, analysis, previous direction of prices, and predicting the returns and risks in the future.

2) Predicting and estimating returns and risks in the future by using previous data as input in the proposed model for choosing the perfect portfolio.

2. A Proposed Model

The portfolio model introduced by Markowitz [3] assumes an investor has two considerations when constructing an investment portfolio: expected return and variance in return (*i.e.*, risk). Variance measures the variability in realized return around the expected return, giving equal weight to realizations below the expected and above the expected return.

The Markowitz model might be, mildly, criticized in this regard because the typical investor is probably concerned only with variability below the expected return, so-called downside risk. The Markowitz model requires two major kinds of information: 1) the estimated expected return for each candidate investment and 2) the covariance matrix of returns. The covariance matrix characterizes not only the individual variability of the return on each investment, but also how each investment's return tends to move with other investments.

The proposed model is based on using game theory to formulate a linear programming model that deals with the problem of uncertain variables that do not probably exist.

It is created through the relationship between returns and risks in the portfolio. The goal of this model is to create a portfolio with different returns and risks by determining the perfect percentage of any stock in the portfolio.

We use quantity measurements to calculate the expected benefits of investments as a return or an expected average return. The measurement of risks in any investment is represented through variant (V), or standard deviation. These variables are considered to be quantity bases for the utility function of the portfolio.

2.1. The Basic Problem

The development of the mathematical model consists of translating the problem into mathematical terms, that is, into the language and concepts of mathematics.

Stochastic programming deals with a class of optimization models and algorithms in which some of the data may be subject to significant uncertainty. Such models are appropriate when data evolve over time, and decisions need to be made prior to observing the entire data stream.

This paper is dedicated to the problem of portfolio optimization through the following question:

How to distribute a limited amount of money as capital in a perfect manner between many available investments or stocks? This problem is considered the basic question in the portfolio model since the contribution of Markowitz.

2.2. The Target of the Model

Game Theory is important to enhance one's reasoning and decision-making skills in a complex world. It is a framework for understanding choice in situations among competing players.

It can help players reach optimal decision-making when confronted by independent and competing actors in a strategic setting [2].

The goal of a proposed model agrees with other models which are used for making decisions.

The target is:

1) Maximizing the returns of the portfolio.

2) Minimizing the risks of the portfolio.

3) Choosing stocks that provide high returns and low risks

3. Hypothesis of the Model

If the known theory does provide a complete theoretical solution to the problem, the specific answer to the problem at hand must still be calculated. It could very well be that further analysis does not provide any simplification of the problem, and only through involved computations can an estimate of the solution be made. Thus, finding a solution to a problem could mean determining a technique to approximate a solution that is financially feasible to implement within a given computer's capabilities and provides error estimates within given tolerance limits [1].

The model proposed is based on the following realistic hypotheses:

1) There is an uncertainty in the expected results of alternatives concerning the decision maker.

2) The alternative is embodied in choosing bonds, shares, derivatives and shares of mutual funds.

3) The previous returns are used as expected returns for the future without any possible occurrence.

4. The Proposed Model

The literature on financial optimization models dates back to the ground breaking application of Markowitz on optimizing a portfolio of financial products by concentrating on the mean return and taking the variance of the return as a measure of the risk.

In the proposed model the game consists of two players. The first player is called Returns (R), and the other is called Variance (V). Accordingly, the goal of portfolio is:

1) Maximizing the present value of expected returns.

2) Minimizing the present value of expected risks.

By matching the variables of preferences for expected risks, with the variables of preferences for expected returns, we get the perfect return value, as we do in the situation of the game theory (the value of the game).

Variances (risks) (V)

			S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_n
			Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_n
Returns (R)	P_1	X_1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{1n}
	P_2	X_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{2n}
	P_3	X_3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{3n}
	P_4	X_4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{4n}
	P_5	X_5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{5n}
	P_6	X_6	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{6n}
	P_7	X_7	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{7n}
	P_8	X_8	a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{8n}
	P_n	X_n	an_1	a_{n2}	a_{n3}	a_{n4}	a_{n5}	a_{n6}	A_{n7}	a_{n8}	a_{nn}

a_{ij}: Means the annual Dividends of stock (share—bond…).

X_i: Means the possibilities to choose alternatives (strategies x_i). In other words, it means the possibilities as percentages and substitutes (shares—bonds), regarding expected returns.

Y_i: Means the possibilities to choose alternatives (strategies e_i), as percentages regarding expected risks.

P_i: Means time periods to achieve return.

S_i: Means sort of stocks (shares—bonds…).

F: Final target (game value): means (return to portfolio).

As the target of the first player (expected return) is maximizing present value, we can rewrite the target of the first player (expected returns) as maximizing present value in the light of condition (≤) because we will maximize the return value as the following:

$$\left(a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{14}X_4 + a_{15}X_5 + a_{1n}X_n\right)(1+i)^{-1} \le F \quad (1)$$

$$\left(a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + a_{24}X_4 + a_{25}X_5 + a_{2n}X_n\right)(1+i)^{-2} \le F$$

$$\left(a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + a_{34}X_4 + a_{35}X_5 + a_{3n}X_n\right)(1+i)^{-3} \le F$$

$$\left(a_{41}X_1 + a_{42}X_2 + a_{43}X_3 + a_{44}X_4 + a_{45}X_5 + a_{4n}X_n\right)(1+i)^{-4} \le F$$

$$\left(a_{51}X_1 + a_{52}X_2 + a_{53}X_3 + a_{54}X_4 + a_{55}X_5 + a_{5n}X_n\right)(1+i)^{-5} \le F$$

$$\left(a_{61}X_1 + a_{62}X_2 + a_{63}X_3 + a_{64}X_4 + a_{65}X_5 + a_{6n}X_n\right)(1+i)^{-6} \le F$$

$$\left(a_{n1}X_1 + a_{n2}X_2 + a_{n3}X_3 + a_{n4}X_4 + a_{n5}X_5 + a_{nn}X_n\right)(1+i)^{-n} \le F$$

When we use alternatives (strategies r_i), and transfer values of returns from stochastic (possibilities) values to true values we get:

$$X_1 + X_2 + X_3 + X_4 + \cdots + X_n = 1$$

$$X_1, X_2, X_3, X_4, \cdots, X_n \ge 0$$

Then we can write the target of minimizing present value of risks in the light of (≥). As a result, we determine the expected value of the second player (expected risks) as the following:

$$\left(a_{11}Y_1 + a_{21}Y_2 + a_{31}Y_3 + a_{41}Y_4 + a_{51}Y_5 + a_{n1}Y_n\right)(1+i)^{-1} \ge F \quad (2)$$

$$\left(a_{12}Y_1 + a_{22}Y_2 + a_{32}Y_3 + a_{42}Y_4 + a_{52}Y_5 + a_{n2}Y_n\right)(1+i)^{-2} \ge F$$

$$\left(a_{13}Y_1 + a_{23}Y_2 + a_{33}Y_3 + a_{43}Y_4 + a_{53}Y_5 + a_{n3}Y_n\right)(1+i)^{-3} \ge F$$

$$\left(a_{14}Y_1 + a_{24}Y_2 + a_{34}Y_3 + a_{44}Y_4 + a_{54}Y_5 + a_{n4}Y_n\right)(1+i)^{-4} \ge F$$

$$\left(a_{15}Y_1 + a_{25}Y_2 + a_{35}Y_3 + a_{45}Y_4 + a_{55}Y_5 + a_{n5}Y_n\right)(1+i)^{-5} \ge F$$

$$\left(a_{16}Y_1 + a_{26}Y_2 + a_{36}Y_3 + a_{46}Y_4 + a_{56}Y_5 + a_{n6}Y_n\right)(1+i)^{-6} \ge F$$

$$\left(a_{1n}Y_1 + a_{2n}Y_2 + a_{3n}Y_3 + a_{4n}Y_4 + a_{5n}Y_5 + a_{nn}Y_n\right)(1+i)^{-n} \ge F$$

When we use alternatives (strategies e_i) according to the position in the first situation we get:

$$Y_1 + Y_2 + Y_3 + Y_4 + \cdots + Y_n = 1$$
$$Y_1, Y_2, Y_3, Y_4, \cdots, Y_n \geq 0$$

By dividing (1) and (2) by (F), we find that the possibility to invest in each stock equals the percentage of the expected return of this stock (x_i), which is then compared to the return on the portfolio as a whole (F).

To simplify the model, the right side will be (1) as the following:

$$\left(a_{11}X_1/F + a_{12}X_2/F + a_{13}X_3/F + a_{14}X_4/F + a_{1n}X_n/F\right)(1+i)^{-1} \leq F/F$$
$$\left(a_{21}X_1/F + a_{22}X_2/F + a_{23}X_3/F + a_{24}X_4/F + a_{2n}X_n/F\right)(1+i)^{-2} \leq F/F$$
$$\left(a_{31}X_1/F + a_{32}X_2/F + a_{33}X_3/F + a_{34}X_4/F + a_{3n}X_n/F\right)(1+i)^{-3} \leq F/F$$
$$\left(a_{41}X_1/F + a_{42}X_2/F + a_{43}X_3/F + a_{44}X_4/F + a_{4n}X_n/F\right)(1+i)^{-4} \leq F/F$$
$$\left(a_{51}X_1/F + a_{52}X_2/F + a_{53}X_3/F + a_{54}X_4/F + a_{5n}X_n/F\right)(1+i)^{-5} \leq F/F$$
$$\left(a_{61}X_1/F + a_{62}X_2/F + a_{63}X_3/F + a_{64}X_4/F + a_{6n}X_n/F\right)(1+i)^{-n} \leq F/F$$
$$\left(a_{1n}X_1/F + a_{2n}X_2/F + a_{3n}X_3/F + a_{4n}X_4/F + a_{nn}X_n/F\right)(1+i)^{-n} \leq F/F$$

For the second player (risks):

$$\left(a_{11}Y_1/F + a_{21}Y_2/F + a_{31}Y_3/F + a_{41}Y_4/F + a_{n1}Y_n/F\right)(1+i)^{-1} \geq F/F \qquad (3)$$
$$\left(a_{12}Y_1/F + a_{22}Y_2/F + a_{32}Y_3/F + a_{42}Y_4/F + a_{n2}Y_n/F\right)(1+i)^{-2} \geq F/F$$
$$\left(a_{13}Y_1/F + a_{23}Y_2/F + a_{33}Y_3/F + a_{43}Y_4/F + a_{n3}Y_n/F\right)(1+i)^{-3} \geq F/F$$
$$\left(a_{14}Y_1/F + a_{24}Y_2/F + a_{34}Y_3/F + a_{44}Y_4/F + a_{n4}Y_n/F\right)(1+i)^{-4} \geq F/F$$
$$\left(a_{15}Y_1/F + a_{25}Y_2/F + a_{35}Y_3/F + a_{45}Y_4/F + a_{n5}Y_n/F\right)(1+i)^{-5} \geq F/F$$
$$\left(a_{16}Y_1/F + a_{26}Y_2/F + a_{36}Y_3/F + a_{46}Y_4/F + a_{n6}Y_n/F\right)(1+i)^{-6} \geq F/F$$
$$\left(a_{1n}Y_1/F + a_{2n}Y_2/F + a_{3n}Y_3/F + a_{4n}Y_4/F + a_{nn}Y_n/F\right)(1+i)^{-6} \geq F/F$$
$$Y_1/F + Y_2/F + Y_3/F + Y_4/F + Y_5/F + Y_6/F + Y_n/F = 1/F$$

Then we define new variables as the following:

$$W_1 = X_1/F, \quad W_2 = X_2/F, \quad W_3 = X_3/F, \quad W_4/F = X_4, \quad W_n = X_n/F$$
$$Z_1 = Y_1/F, \quad Z_2 = Y_2/F, \quad Z_3 = Y_3/F, \quad Z_4 = Y_4/F, \quad Z_n = Y_n/F$$

By compensating for the first player (expected return):

$$\left(a_{11}W_1 + a_{12}W_2 + a_{13}W_3 + a_{14}W_4 + a_{15}W_5 + a_{1n}W_n\right)(1+i)^{-1} \leq 1 \qquad (4)$$
$$\left(a_{21}W_1 + a_{22}W_2 + a_{23}W_3 + a_{24}W_4 + a_{25}W_5 + a_{2n}W_n\right)(1+i)^{-2} \leq 1$$
$$\left(a_{31}W_1 + a_{32}W_2 + a_{33}W_3 + a_{34}W_4 + a_{35}W_5 + a_{3n}W_n\right)(1+i)^{-3} \leq 1$$
$$\left(a_{41}W_1 + a_{42}W_2 + a_{43}W_3 + a_{44}W_4 + a_{45}W_5 + a_{4n}W_n\right)(1+i)^{-4} \leq 1$$
$$\left(a_{51}W_1 + a_{52}W_2 + a_{53}W_3 + a_{54}W_4 + a_{55}W_5 + a_{5n}W_n\right)(1+i)^{-5} \leq 1$$

$$\left(a_{61}W_1 + a_{62}W_2 + a_{63}W_3 + a_{64}W_4 + a_{65}W_5 + a_{6n}W_n\right)\left(1+i\right)^{-6} \le 1$$

$$\left(a_{n1}W_1 + a_{n2}W_2 + a_{n3}W_3 + a_{n4}W_4 + a_{n5}W_5 + a_{nn}W_n\right)\left(1+i\right)^{-n} \le 1$$

And by compensating for the second player (expected risks):

$$\left(a_{11}Z_1 + a_{21}Z_2 + a_{31}Z_3 + a_{41}Z_4 + a_{51}Z_5 + a_{n1}Z_n\right)\left(1+i\right)^{-1} \ge 1 \quad (5)$$

$$\left(a_{12}Z_1 + a_{22}Z_2 + a_{32}Z_3 + a_{42}Z_4 + a_{52}Z_5 + a_{n2}Z_n\right)\left(1+i\right)^{-2} \ge 1$$

$$\left(a_{13}Z_1 + a_{23}Z_2 + a_{33}Z_3 + a_{43}Z_4 + a_{53}Z_5 + a_{n3}Z_n\right)\left(1+i\right)^{-3} \ge 1$$

$$\left(a_{14}Z_1 + a_{24}Z_2 + a_{34}Z_3 + a_{44}Z_4 + a_{54}Z_5 + a_{n4}Z_n\right)\left(1+i\right)^{-4} \ge 1$$

$$\left(a_{15}Z_1 + a_{25}Z_2 + a_{35}Z_3 + a_{45}Z_4 + a_{55}Z_5 + a_{n5}Z_n\right)\left(1+i\right)^{-5} \ge 1$$

$$\left(a_{16}Z_1 + a_{26}Z_2 + a_{36}Z_3 + a_{46}Z_4 + a_{56}Z_5 + a_{n6}Z_n\right)\left(1+i\right)^{-6} \ge 1$$

$$\left(a_{1n}Z_1 + a_{2n}Z_2 + a_{3n}Z_3 + a_{4n}Z_4 + a_{5n}Z_5 + a_{nn}Z_n\right)\left(1+i\right)^{-n} \ge 1$$

$$\text{Maximization of } \; Z_1 + Z_2 + Z_3 + Z_4 + \cdots + Z_n = 1/F$$

According to the following constraints:

$$\left(a_{11}Z_1 + a_{21}Z_2 + a_{31}Z_3 + a_{41}Z_4 + a_{51}Z_5 + a_{n1}Z_n\right)\left(1+i\right)^{-1} \le 1$$

$$\left(a_{12}Z_1 + a_{22}Z_2 + a_{32}Z_3 + a_{42}Z_4 + a_{52}Z_5 + a_{n2}Z_n\right)\left(1+i\right)^{-2} \le 1$$

$$\left(a_{13}Z_1 + a_{23}Z_2 + a_{33}Z_3 + a_{43}Z_4 + a_{53}Z_5 + a_{n3}Z_n\right)\left(1+i\right)^{-3} \le 1$$

$$\left(a_{14}Z_1 + a_{24}Z_2 + a_{34}Z_3 + a_{44}Z_4 + a_{54}Z_5 + a_{n4}Z_n\right)\left(1+i\right)^{-4} \le 1$$

$$\left(a_{15}Z_1 + a_{25}Z_2 + a_{35}Z_3 + a_{45}Z_4 + a_{55}Z_5 + a_{n5}Z_n\right)\left(1+i\right)^{-5} \le 1$$

$$\left(a_{16}Z_1 + a_{26}Z_2 + a_{36}Z_3 + a_{46}Z_4 + a_{56}Z_5 + a_{n6}Z_n\right)\left(1+i\right)^{-6} \le 1$$

$$\left(a_{1n}Z_1 + a_{2n}Z_2 + a_{3n}Z_3 + a_{4n}Z_4 + a_{5n}Z_5 + a_{nn}Z_n\right)\left(1+i\right)^{-n} \le 1$$

We have also:

$$X_1 + X_2 + X_3 + X_4 + X_5 + \cdots + X_n = 1/F$$

$$Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + \cdots + Y_n = 1/F$$

We go back to the first player whose target is to maximize the present value of expected return which is equal to minimizing ($1/F$).

Now, we can write the problem of linear programming for the first player (minimizing present value of expected returns) as the following:

$$\text{Minimizing: } \; W_1 + W_2 + W_3 + W_4 + W_5 + \cdots + W_n = 1/F$$

According to the following constraints:

$$\left(a_{11}W_1 + a_{12}W_2 + a_{13}W_3 + a_{14}W_4 + a_{15}W_5 + a_{1n}W_n\right)\left(1+i\right)^{-1} \ge 1$$

$$\left(a_{21}W_1 + a_{22}W_2 + a_{23}W_3 + a_{24}W_4 + a_{25}W_5 + a_{2n}W_n\right)\left(1+i\right)^{-2} \ge 1$$

$$\left(a_{31}W_1 + a_{32}W_2 + a_{33}W_3 + a_{34}W_4 + a_{35}W_5 + a_{3n}W_n\right)\left(1+i\right)^{-3} \ge 1$$

$$\left(a_{41}W_1 + a_{42}W_2 + a_{43}W_3 + a_{44}W_4 + a_{45}W_5 + a_{4n}W_n\right)\left(1+i\right)^{-4} \ge 1$$

$$\left(a_{51}W_1 + a_{52}W_2 + a_{53}W_3 + a_{54}W_4 + a_{55}W_5 + a_{5n}W_n\right)\left(1+i\right)^{-5} \geq 1$$

$$\left(a_{61}W_1 + a_{62}W_2 + a_{63}W_3 + a_{64}W_4 + a_{65}W_5 + a_{6n}W_n\right)\left(1+i\right)^{-6} \geq 1$$

$$\left(a_{n1}W_1 + a_{n2}W_2 + a_{n3}W_3 + a_{n4}W_4 + a_{n5}W_5 + a_{nn}W_n\right)\left(1+i\right)^{-n} \geq 1$$

At the same time the target of the second player is to minimize present value of risks, which equals maximizing value of ($1/F$).

Then we can write the problem of linear programming as the following:

$$\text{Maximization of } Z_1 + Z_2 + Z_3 + Z_4 + \cdots + Z_n = 1/F$$

According to the following constraints:

$$\left(a_{11}Z_1 + a_{21}Z_2 + a_{31}Z_3 + a_{41}Z_4 + a_{51}Z_5 + a_{n1}Z_n\right)\left(1+i\right)^{-1} \leq 1$$

$$\left(a_{12}Z_1 + a_{22}Z_2 + a_{32}Z_3 + a_{42}Z_4 + a_{52}Z_5 + a_{n2}Z_n\right)\left(1+i\right)^{-2} \leq 1$$

$$\left(a_{13}Z_1 + a_{23}Z_2 + a_{33}Z_3 + a_{43}Z_4 + a_{53}Z_5 + a_{n3}Z_n\right)\left(1+i\right)^{-3} \leq 1$$

$$\left(a_{14}Z_1 + a_{24}Z_2 + a_{34}Z_3 + a_{44}Z_4 + a_{54}Z_5 + a_{n4}Z_n\right)\left(1+i\right)^{-4} \leq 1$$

$$\left(a_{15}Z_1 + a_{25}Z_2 + a_{35}Z_3 + a_{45}Z_4 + a_{55}Z_5 + a_{n5}Z_n\right)\left(1+i\right)^{-5} \leq 1$$

$$\left(a_{16}Z_1 + a_{26}Z_2 + a_{36}Z_3 + a_{46}Z_4 + a_{56}Z_5 + a_{n6}Z_n\right)\left(1+i\right)^{-6} \leq 1$$

$$\left(a_{1n}Z_1 + a_{2n}Z_2 + a_{3n}Z_3 + a_{4n}Z_4 + a_{5n}Z_5 + a_{nn}Z_n\right)\left(1+i\right)^{-n} \leq 1$$

We have to note that (4) is binary to (5) when we solve one problem of linear programming, that is the basic linear programming problem, and we get the value of $Z_1, Z_2, Z_3, Z_4, \cdots, Z_n$.

By using the computer, we calculate the percentages of the perfect investments in the portfolio, and we get the value of the target (F) through:

$$W_1 + W_2 + W_3 + W_4 + W_5 + \cdots + W_n = 1/F$$

By going back to (3) we find that:

$$X_1 = F \cdot W_1, X_2 = F \cdot W_2, X_3 = F \cdot W_3, \cdots, X_n = F \cdot W_n$$

This means that the alternatives (percentages of stocks we choose) are ranging between 1% till 100%.

At the same time, we calculate the following:

$$Y_1 = F \cdot Z_1, Y_2 = F \cdot Z_2, Y_3 = F \cdot Z_3, \cdots, Y_n = F \cdot Z_n$$

This means that the alternatives of the second player (Risks) range between 1% and 100%.

Accordingly, we choose the percentages of high returns (Rs) of stocks, and low risks of stocks (Vs), compared with the returns and risk of portfolio (F) as a whole.

As a result, the high return on stocks represents the amount of money we have to invest in every kind of stock because it provides both a high return and a low risk simultaneously.

The researcher tested the proposed model to choose the perfect portfolio in Cairo Stock Exchange.

5. The Samples Chosen Were Active Shares of Companies Who Traded Their Shares at Cairo

Stock Exchange (Egypt) (Table 1):

Table 1. Active companies who traded their shares at Cairo Stock Exchange (Egypt) [4] [5] [6].

VAR (1)	Suez Canal Bank
VAR (2)	Commercial International Bank (Egypt)
VAR(3)	Oriental Weavers
VAR (4)	Egypt Telecommunications
VAR (5)	Edita Food Industry
VAR (6)	Cement Egypt
VAR (7)	Orascom Investment Holding
VAR (8)	Ezz Steel
VAR (9)	Abou Kir Fertilizers & Chemical
VAR (10)	B Investment Holding
VAR (11)	Elsewedy Electric
VAR (12)	Egyptian Transport & Commercial Services CO.
VAR (13)	Sidi Kerir Petrochemicals CO.
VAR (14)	Arabian Food Industries company
VAR (15)	Minapharm Pharmaceuticals
VAR(16)	Glaxo Smithkline
VAR (17)	Delta for Sugar
VAR (18)	Juhayna Food Industries
VAR(19)	Arab Aluminum CO.
VAR (20)	Madinat Naser for Housing and Construction
VAR (21)	Misr Fertilizers production company
VAR (22)	Pyramisa Hotels & Resorts CO.
VAR (23)	Saudi Egyptian Investments & Finance Co.
VAR(24)	Egyptian International Pharmaceutical Industries CO.
VAR (25)	Delta for Printing & Packaging CO.
VAR (26)	KZ for Pesticides & Chemicals Kafr Az Zayat
VAR (27)	Housing & Development Bank
VAR (28)	Al Ezz Ceramics and Porcelain CO.
VAR(29)	Egyptian Arabian Themar Securities Brokerage
VAR (30)	Al Tawfeek for Financial Lease
VAR (31)	Egyptian Financial & Industries CO.
VAR (32)	Egypt Gas CO.
VAR (33)	Faisal Islamic Bank
VAR (34)	Ferchem Masr Fertilizers and Chemicals
VAR (35)	Real Estate Egyptian Consortium

1) Time period Pi from year (1) to year (10) (2014-2023).

2) The developments of the discount rate issued by Egyptian Central Bank (2014-2023) are shown in **Table 2** [4] [5] [6].

3) Discount rates and present value of one Egyptian pound after (n) year are shown in **Table 3** [4] [5] [6].

4) Data of monetary distributions of Dividends are shown in **Tables 4-38** [4] [5] [6].

5) Processing data through Linear and Integer Goal Programming and the results of the program are shown in **Tables 39-41**.

6. Interpreting the Output of the Program (Linear Programming Program)

1) Shares of companies represent the perfect portfolio according to the proposed model.

Table 2. Discount rate 2014-2023.

Years	Discount Rate
2014	2.75
2015	9.75
2016	11.25
2017	16.75
2018	17.75
2019	12.75
2020	9.25
2021	8.75
2022	11.75
2023	18.75

Table 3. Present value of one Egyptian pound.

Years	Discount Rate	Present Value of One Egyptian Pound
2014	2.75	0.906329119
2015	9.75	0.2563691716
2016	11.25	0.726273072
2017	16.75	0.5382356292
2018	17.75	045126925
2019	12.75	0486744100
2020	9.25	0.010209039
2021	8.75	0.0065462083
2022	11.75	0.3679359417
2023	18.75	0.1793344879

Table 4. Distributions of dividends of Suez Canal Bank VAR (1).

Suez Canal Bank	Years	Dividends Per Share
	2014	0.53
	2015	0.53
	2016	0.63
	2017	0.64
	2018	0.65
	2019	0.66
	2020	0.60
	2021	0.62
	2022	1.61
	2023	2.79

Table 5. Distributions of dividends of Commercial International Bank (Egypt) VAR (2).

Commercial International Bank (Egypt)	Years	Dividends Per Share
	2014	1.20
	2015	0.75
	2016	0.75
	2017	0.750
	2018	0.500
	2019	1
	2020	1
	2021	1.250
	2022	1.3538758
	2023	0.5378478

Table 6. Distributions of dividends of Oriental Weavers VAR (3).

Oriental Weavers	Years	Dividends Per Share
	2014	2
	2015	0.4
	2016	0.5
	2017	1.4
	2018	1.5
	2019	1.5
	2020	0.65
	2021	1
	2022	1
	2023	0.6

Table 7. Distributions of dividends of Egypt Telecommunications VAR (4).

Egypt Telecommunications	Years	Dividends Per Share
	2014	1
	2015	0.20
	2016	0.75
	2017	1
	2018	2.5
	2019	0.25
	2020	0.25
	2021	0.75
	2022	1
	2023	1

Table 8. Distributions of dividends of Edita Food Industry VAR (5).

Edita Food Industry	Years	Dividends Per Share
	2014	0.20
	2015	0.20
	2016	0.223
	2017	0.149851
	2018	0.1171
	2019	0.208
	2020	0.2
	2021	0.207452
	2022	0.275
	2023	0.277

Table 9. Distributions of dividends of Cement Egypt VAR (6).

Cement Egypt	Years	Dividends Per Share
	2014	0.990
	2015	0730
	2016	0.650
	2017	0.53
	2018	0.53
	2019	0.47
	2020	0.66
	2021	0.950
	2022	0.66
	2023	0.950

Table 10. Distributions of dividends of Orascom Investment Holding VAR (7).

Orascom Investment Holding	Years	Dividends Per Share
	2014	0.257
	2015	0.010
	2016	0.074
	2017	0.10
	2018	0.04
	2019	0.01
	2020	0.01
	2021	0.01
	2022	0.01
	2023	0.08

Table 11. Distributions of dividends of Ezz Steel VAR (8).

Ezz Steel	Years	Dividends Per Share
	2014	0.27
	2015	0.25
	2016	0.21
	2017	0.98
	2018	0.36
	2019	1.99
	2020	1.9
	2021	1.44
	2022	1.95
	2023	1.07

Table 12. Distributions of dividends of Abou Kir Fertilizers & Chemical VAR (9).

Abou Kir Fertilizers & Chemical	Years	Dividends Per Share
	2014	13
	2015	3
	2016	3
	2017	3
	2018	0.8
	2019	3
	2020	0.30
	2021	1
	2022	1
	2023	3

Table 13. Distributions of dividends of B Investment Holding VAR (10).

B Investment Holding	Years	Dividends Per Share
	2014	0.500
	2015	0.500
	2016	0.400
	2017	0.500
	2018	0.400
	2019	0.500
	2020	0.400
	2021	1.250
	2022	0.500
	2023	0.1297249

Table 14. Distributions of dividends of Elsewedy Electric VAR (11).

Elsewedy Electric	Years	Dividends Per Share
	2014	1
	2015	1
	2016	1
	2017	1
	2018	4
	2019	1.60
	2020	1.60
	2021	0.80
	2022	0.400
	2023	0.400

Table 15. Distributions of dividends of Egyptian Transport & Commercial Services Co. VAR (12).

Egyptian Transport & Commercial Services Co.	Years	Dividends Per Share
	2014	0.30
	2015	0.40
	2016	0.50
	2017	0.70
	2018	1
	2019	1
	2020	1
	2021	1
	2022	0.750
	2023	0.130

Table 16. Distributions of dividends of Sidi Kerir Petrochemicals Co. VAR (13).

Sidi Kerir Petrochemicals Co.	Years	Dividends Per Share
	2014	1
	2015	1
	2016	1.15
	2017	1.20
	2018	1.10
	2019	1.60
	2020	1.40
	2021	0.75
	2022	0.30
	2023	0.90

Table 17. Distributions of dividends of Arabian Food Industries Company VAR (14).

Arabian Food Industries Company	Years	Dividends Per Share
	2014	0.05
	2015	0.100
	2016	0.200
	2017	0.200
	2018	0.100
	2019	0.200
	2020	0.200
	2021	0.500
	2022	0.500
	2023	0.500

Table 18. Distributions of dividends of Minapharm Pharmaceutical VAR (15).

Minapharm Pharmaceutical	Years	Dividends Per Share
	2014	0.45
	2015	2.25
	2016	4.500
	2017	4.500
	2018	4.500
	2019	1.25
	2020	6.750
	2021	6
	2022	2.700
	2023	2.921806918

Table 19. Distributions of dividends of Glaxo Smithklin VAR (16).

Glaxo Smithklin	Years	Dividends Per Share
	2014	0.75
	2015	0.400
	2016	0.45
	2017	0.500
	2018	0.500
	2019	0.400
	2020	0.500
	2021	0.500
	2022	0.75
	2023	0.750

Table 20. Distributions of dividends of delta for Sugar VAR (17).

Delta for Sugar	Years	Dividends Per Share
	2014	1
	2015	0.875
	2016	1.75
	2017	2
	2018	0.500
	2019	0.25
	2020	2
	2021	0.25
	2022	0.500
	20234	2.550

Table 21. Distributions of dividends of Juhayna Food Industries VAR (18).

Juhayna Food Industries	Years	Dividends Per Share
	2014	0.100
	2015	0.100
	2016	0.150
	2017	0.150
	2018	0.100
	2019	0.20
	2020	0.20
	2021	0.20
	2022	0.550
	2023	0.150

Table 22. Distributions of dividends of Arab Aluminum Co. VAR (19).

Arab Aluminum Co.	Years	Dividends Per Share
	2014	5
	2015	1.5
	2016	1.5
	2017	1
	2018	0.75
	2019	1.250
	2020	1
	2021	1.50
	2022	1.750
	2023	1.750

Table 23. Distributions of dividends of Madinat Naser for Housing and Construction VAR (20).

Madinat Naser for Housing and Construction	Years	Dividends Per Share
	2014	0.15
	2015	0.25
	2016	1.3
	2017	0.317499
	2018	1.11
	2019	1.24
	2020	0.45
	2021	0.45
	2022	0.45
	2023	2.075

Table 24. Distributions of dividends of Misr Fertilizers Production Company VAR (21).

Misr Fertilizers Production Company	Years	Dividends Per share
	2014	0.5
	2015	2
	2016	0.5
	2017	2
	2018	2
	2019	2.25
	2020	2.50
	2021	3.50
	2022	1
	2023	15

Table 25. Distributions of dividends of Pyramids Hotels & Resorts Co. VAR (22).

Pyramids Hotels & Resorts Co.	Years	Dividends Per Share
	2014	1
	2015	2
	2016	3
	2017	1
	2018	2
	2019	1
	2020	1
	2021	1
	2022	1
	2023	1

Table 26. Distributions of dividends of Saudi Egyptian Investments & Finance Co. VAR (23).

Saudi Egyptian Investments & Finance Co.	Years	Dividends Per Share
	2014	3
	2015	4
	2016	17
	2017	2
	2018	2
	2019	2
	2020	2.25
	2021	2
	2022	1.5
	2023	1.5

Table 27. Distributions of dividends of Egyptian International Pharmaceutical Industries Co. VAR (24).

Egyptian International Pharmaceutical Industries Co.	Years	Dividends per Share
	2014	3.3
	2015	3.5
	2016	3.5
	2017	3.5
	2018	4
	2019	4
	2020	4.0655
	2021	2.95155985
	2022	1.5
	2023	1

Table 28. Distributions of dividends of Delta for Printing & Packaging Co. VAR (25).

Delta for Printing & Packaging Co.	Years	Dividends Per Share
	2014	2
	2015	1
	2016	1.5
	2017	1
	2018	1
	2019	1.5
	2020	1.5
	2021	3
	2022	1
	2023	1

Table 29. Distributions of dividends of KZ for Pesticides & Chemicals Kafr Az Zayat VAR (26).

KZ for Pesticides & Chemicals Kafr Az Zayat	Years	Dividends Per Share
	2014	2.400
	2015	2.400
	2016	2.400
	2017	3
	2018	2
	2019	2.300
	2020	1.500
	2021	2.500
	2022	4
	2023	0.525

Table 30. Distributions of dividends of Housing & Development Bank VAR (27).

Housing & Development Bank	years	Dividends Per Share
	2014	1
	2015	1
	2016	1.50
	2017	5.01
	2018	2
	2019	1.500
	2020	5
	2021	2.5
	2022	2.5
	2023	1

Table 31. Distributions of dividends of Al Ezz Ceramics and Porcelain Co. VAR (28).

Al Ezz Ceramics and Porcelain Co.	Years	Dividends Per share
	2014	0.400
	2015	0.400
	2016	0.250
	2017	0.250
	2018	0.250
	2019	0.400
	2020	0.225
	2021	1
	2022	0.550
	2023	0.900

Table 32. Distributions of dividends of Egyptian Arabian Themar Securities Brokerage VAR (29).

Egyptian Arabian Themar Securities Brokerage	Years	Dividends Per share
	2014	0.050
	2015	0.040
	2016	0.3636
	2017	0.050
	2018	0.040
	2019	0.080
	2020	0.080
	2021	0.080
	2022	0.120
	2023	0.100

Table 33. Distributions of dividends of Al Tawfeek for Financial Leas VAR (30).

Al Tawfeek for Financial Leas	Years	Dividends Per share
	2014	0.100
	2015	0.112
	2016	0.3019
	2017	0.23382
	2018	0.3019
	2019	0.5708188
	2020	0.253818
	2021	0.1959420
	2022	0.9292724
	2023	0.424078

Table 34. Distributions of dividends of Egyptian Financial & Industries Co. VAR (31).

Egyptian Financial & Industries Co.	Years	Dividends Per Share
	2014	0.25
	2015	0.30
	2016	0.50
	2017	0.50
	2018	0.75
	2019	0.250
	2020	0.750
	2021	2
	2022	2
	2023	3

Table 35. Distributions of dividends of Egypt Gas CO. VAR (32).

Egypt Gas Co.	Years	Dividends Per Share
	2014	1
	2015	1.250
	2016	2
	2017	2
	2018	1
	2019	1.250
	2020	3
	2021	1
	2022	1.500
	2023	1.250

Table 36. Distributions of dividends of Faisal Islamic Bank VAR (33).

Faisal Islamic Bank	Years	Dividends
	2014	1.10
	2015	1
	2016	1.278
	2017	1.63
	2018	0.o8
	2019	1.433
	2020	0.08
	2021	1.28332
	2022	1.257
	2023	1.979

Table 37. Distributions of dividends of Ferchem Masr Fertilizers and Chemicals VAR (34).

Ferchem Masr Fertilizers and Chemicals	Years	Dividends
	2014	0.100
	2015	0.160
	2016	0.350
	2027	2
	2018	2
	2019	2.500
	2020	2.500
	2021	1.350
	2022	1.350
	2023	1.50

Table 38. Distributions of dividends of Real Estate Egyptian Consortium VAR (35).

Real Estate Egyptian Consortium	Years	Dividends
	2014	0.50
	2015	0.51
	2016	0.5
	2017	0.53
	2018	1.25
	2019	0.14
	2020	0.5
	2021	0.22
	2022	0.03
	2023	0.01

Table 39. Combined report for using computer for choosing perfect portfolio in Cairo Stock Exchange.

	Goal Level	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Allowable Min. c(j)	Allowable Max. c(j)
1	G1	X_1	0	1.00	0	0.91	0.09	M
2	G1	X_2	0	1.00	0	0.85	0.15	M
3	G1	X_3	0	1.00	0	0.90	0.10	M
4	G1	X_4	0	1.00	0	0.96	0.04	M
5	G1	X_5	0	1.00	0	0.97	0.03	M
6	G1	X_6	0	1.00	0	0.90	0.10	M
7	G1	X_7	0	1.00	0	1.00	0.00	M
8	G1	X_8	0	1.00	0	0.97	0.03	M
9	G1	X_9	0	1.00	0	0.95	0.05	M
10	G1	X_{10}	0	1.00	0	0.94	0.06	M

Continued

11	G1	X_{11}	0	1.00	0	0.77	0.23	M
12	G1	X_{12}	0	1.00	0	0.85	0.15	M
13	G1	X_{13}	0	1.00	0	0.80	0.20	M
14	G1	X_{14}	0	1.00	0	0.97	0.03	M
15	G1	X_{15}	14.43	1.00	14.43	0	0.04	M
16	G1	X_{16}	0	1.00	0	0.93	0.07	M
17	G1	X_{17}	0.44	1.00	0.44	0	0.19	M
18	G1	X_{18}	0	1.00	0	0.97	0.03	M
19	G1	X_{19}	0	1.00	0	0.85	0.15	M
20	G1	X_{20}	0	1.00	0	0.93	0.07	M
21	G1	X_{21}	0	1.00	0	0.62	0.38	M
22	G1	X_{22}	0	1.00	0	0.85	0.15	M
23	G1	X_{23}	0	1.00	0	0.67	0.33	M
24	G1	X_{24}	0	1.00	0	0.40	0.60	M
25	G1	X_{25}	0	1.00	0	0.77	0.23	M
26	G1	X_{26}	0	1.00	0	0.77	0.23	M
27	G1	X_{27}	0	1.00	0	0.27	0.73	M
28	G1	X_{28}	0	1.00	0	0.96	0.04	M
29	G1	X_{29}	0	1.00	0	0.99	0.01	M
30	G1	X_{30}	0	1.00	0	0.96	0.04	M
31	G1	X_{31}	0	1.00	0	0.88	0.12	M
32	G1	X_{32}	0	1.00	0	0.56	0.44	M
33	G1	X_{33}	0	1.00	0	0.98	0.02	M
34	G1	X_{34}	0	1.00	0	0.63	0.37	M
35	G1	X_{35}	0	1.00	0	0.93	0.07	M

Objective function value = 14.78.

Table 40. Optimal solution—detailed report.

Shadow Price	Allowable Max. RHS	Allowable Min. RHS	Right Hand Side Direction	Left Hand Side Direction	Goal 1	Slack or Surplus	Constraint
0	6.28	-M	1.00	5.28	=> 6.28	C1	1
0	8.42	-M	1.00	7.42	=> 8.42	C2	2
0	47.71	-M	1.00	46.71	=> 47.71	C3	3
0	35.42	-M	1.00	34.42	=> 35.42	C4	4
0	29.40	-M	1.00	28.40	=> 29.40	C5	5
0	8.83	-M	1.00	7.83	=> 8.83	C6	6
14.04	14.04	0.11	1.00	0.11	=> 1.00	C7	7
0.82	0.82	0.57	1.00	0.57	=> 1.00	C8	8
0	14.41	-M	1.00	13.41	=> 14.41	C9	9
0	7.76	-M	1.00	6.76	=> 7.7	C10	10

Table 41. Solution summary for using computer for choosing perfect portfolio in Cairo Stock Exchange.

Goal 1	Decision Variable	Solution Value	Status	Reduced Cost
1	X_1	0	At bound	0.91
2	X_2	0	At bound	0.85
3	X_3	0	At bound	0.90
4	X_4	0	At bound	0.96
5	X_5	0	At bound	0.97
6	X_6	0	At bound	0.95
7	X_7	0	At bound	1.00
8	X_8	0	At bound	0.97
9	X_9	0	At bound	0.95
10	X_{10}	0	At bound	0.94
11	X_{11}	0	At bound	0.77
12	X_{12}	0	At bound	0.85
13	X_{13}	0	At bound	0.80
14	X_{14}	0	At bound	0.97
15	X_{15}	14.43	Basic	0
16	X_{16}	0	At bound	0.93
17	X_{17}	0.44	Basic	0
18	X_{18}	0	At bound	0.97
19	X_{19}	0	At bound	0.85
20	X_{20}	0	At bound	0.93
21	X_{21}	0	At bound	0.62
22	X_{22}	0	At bound	0.85
23	X_{23}	0	At bound	0.67
24	X_{24}	0	At bound	0.40
25	X_{25}	0	At bound	0.77
26	X_{26}	0	At bound	0.77
27	X_{27}	0	At bound	0.27
28	X_{28}	0	At bound	0.96
29	X_{29}	0	At bound	0.99
30	X_{30}	0	At bound	0.96
31	X_{31}	0	At bound	0.88
32	X_{32}	0	At bound	0.56
33	X_{33}	0	At bound	0.98
34	X_{34}	0	At bound	0.63
35	X_{35}	0	At bound	0.93

$C_{15} = X_{15} = \text{VAR}_{15}$ = Minapharm Pharmaceuticals (Table 18).

Minapharm Pharmaceuticals: A leading pharmaceutical company in Egypt, Africa and the Middle East and the premier biopharmaceutical company in the region, with multiple subsidiaries in Berlin and Cairo. Minapharm commercializes over 100 life-saving and life-enhancing products ranging from small molecules with advanced galenic formulations to complex genetically engineered proteins and other advanced therapies.

$C_{17} = X_{17} = \mathrm{VAR}_{17}$ = Delta for Sugar (**Table 20**).

Delta Sugar: The company engages in the manufacture and sale of beet sugar in Egypt. It offers sugar, dry beet pulp, and molasses for use in the animal feed industry. The company was founded in 1978 and is headquartered in the 6th of October City, Egypt. Delta Sugar Company is a subsidiary of Egyptian Sugar and Integrated Industries Company SAE.

2) The whole percentages of stock return represent the return of the portfolio. At the same time, they represent the perfect percentages of investments according to the model proposed as the following:

$C_{15} = X_{15} = \mathrm{VAR}_{15}$ = the percentage of investing in a share of Minapharm Pharmaceuticals.

$C_{17} = X_{17} = \mathrm{VAR}_{17}$ = the percentage of investing in a share of Delta for Sugar.

We calculate the percentages as follows:

$$1/F = 14.87 \quad (\textbf{Table 40})$$

$$F = 1/14.87 = 0.0672494956$$

$$X_{15} = W_{15} * F = 14.43 * 0.0672494956 = 0.9704102219 \quad (\textbf{Table 39})$$

$$X_{17} = W_{17} * F = 0.44 * 0.0672494956 = 0.0295897781$$

With regard to risks (according to the binary method in linear programming), the value of basic variables, in the perfect solution of the binary method, represents the shadow prices of slack variables in the basic method as follows:

$$\mathrm{VAR}_7 = Y_7 = Z_7 * F = 14.04 * 0.0672494956 = 0.9441829182 \quad (\textbf{Table 40})$$

$$\mathrm{VAR}_8 = Y_8 = Z_8 * F = 0.82 * 0.0672494956 = 0.0551445864$$

These percentages range between 1% and 100% which meet the conditions of the proposed model.

7. Conclusions

The target of this proposed model is achieved, as the risks increase by expanding the size of the portfolio because of the positive direct relationship between risks and returns.

Accordingly, we note that the increase of risks in variables (7) and (8) represents the high percentages of investments in the perfect portfolio containing (2) shares as follows:

$$(0.9704102219, 0.9441829182) = \mathrm{VAR}_{15}(X_{15}), \mathrm{VAR}_7(Y_7)$$

$$(0.0295897781, 0.0551445864) = \mathrm{VAR}_{17}(X_{17}), \mathrm{VAR}_8(Y_8)$$

Thus, the proposed hypotheses have been tested and proven. The first hypothesis states that it is difficult to estimate an expected return because of its changing features. Therefore, the researcher used the series of distributions of dividends from 2014 to 2023, and calculated the present value of these dividends. The problem was how to transfer values from stochastic values to real values. It is solved by using game theory and stochastic programming, then linear programming to calculate expected returns.

With regard to the second hypothesis, the model accepts any kind of stock, but because of the shortage of data about bonds and derivatives and mutual funds, the researcher used shares only.

In the third hypothesis, the data was tested and found to have met the conditions of the Model, where the summation of returns equals 100% and the summation of risks equals 100%. Finally, the researcher considers this model, a scientific contribution to previous models, which have dealt with choosing perfect portfolios.

Processing data through Linear and Integer Goal Programming and the results of the program are shown in interpreting the output of the program.

Acknowledgements

I am honored to Provide an original research article entitled "Using Return and Risk Model For Choosing Perfect Portfolio, Applied Study in Cairo Stock Exchange" for consideration by the Journal of Portfolio Management.

I confirm that this work is original and has not been published elsewhere, nor under any consideration for publication elsewhere nowadays. Besides the Model proposed is the contribution of a PH.D. degree in Finance and Accounting in Egypt.

The theme of this paper is how to propose a new mathematical model like that provided by Markowitz which helps in choosing a perfect portfolio and the input/output of this model, besides applying this model in reality, the researcher used the game theory with stochastic and linear programming to provide the model proposed and then used this model to select a perfect portfolio in Cairo Stock Exchange. The results were fruitful and the researcher considers this model a new scientific addition to previous models.

As your Journal is interested in Academic research, including new thoughts, I hope that my research will be up to your standards. I think it would be interesting by the readers of your journal.

Conflicts of Interest

I have no conflicts of interest to disclose.

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