

Computational Resolution of a Boolean Equation of 21 Variables

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Abstract

The Alienor method has been elaborated at the beginning of the 1980s by Yves Cherruault and Arthur Guillez (1983). The following people have also greatly contributed to the improvement of this new optimization method: Blaise Somé, Gaspar Mora, Balira Konfé, Jean Claude Mazza and Esther Claudine Bityé Mvondo. The basic idea consists in using a reducing transformation allowing us to simplify a multivariable optimization problem to a new optimization problem according to a single variable. The rational gestion of enterprises leads generally to the use of Operational Research, often called management science. The term Operational Research means a scientific approach to decision making, that seeks optimization in a system. Consequently, it is better to take the right decisions. Otherwise, fatal consequences can occur instantaneously [1]. Generally, we have to maximize the global profit margin, taking into account some constraints. For instance, in an integer programming problem, some or all the variables are required to be nonnegative integers. In this paper, we present new reducing transformations for global optimization in integer, binary and mixed variables as well as the applications in Boolean algebra by solving a Boolean Equation of 21 variables. The applications in Operational Research are presented on various examples, resolved by using the tabulator Excel of Microsoft.

Keywords

Alpha-Dense Curves, Boolean Equations, Diophantine Equations, Global Optimization, and Operational Research

1. Introduction: Reducing Transformations Applied to Global Optimization

This paper is concerned with computational resolution of a Boolean Equation of 21 variables. In this paper, it proposes new reducing transformations allowing us

to simplify a multivariable optimization problem to a new optimization problem according to a single variable. The Alienor method has been elaborated at the beginning of the 1980s by Yves Cherruault and Arthur Guillez (1983). The following people have also greatly contributed to the improvement of this new optimization method: Blaise Somé, Gaspar Mora, Balira Konfé, Jean Claude Mazza and Esther Claudine Bityé Mvondo. Let us recall the definition of an alpha-dense curve [2] [3].

1.1. Definition

A subspace K' of a compact $K \subset \mathbb{R}^n$ is said α -dense, if for any point $x \in K$, there exists $x' \in K'$ such that the Euclidian distance d satisfies $d(x, x') \leq \alpha$. In other words, we can approach any $x \in K$ by a point of K' to within α .

Our new method is described as follows. Let

$$x_i = h_i(\theta), \quad i = 1, \dots, n, \quad \theta \geq 0, \quad n \in \mathbb{N}^* \tag{1}$$

be a reducing transformation, α -dense curve in a compact set of \mathbb{R}^n and f a continuous function satisfying the condition

$$\lim_{x_1^2 + \dots + x_n^2 \rightarrow +\infty} f(x_1, \dots, x_n) \rightarrow +\infty \tag{2}$$

with this method the global minimization problem

$$\text{Glob min}_{x_1, \dots, x_n} f(x_1, \dots, x_n) \tag{3}$$

becomes

$$\min_{\theta \geq 0} f^*(\theta) \tag{4}$$

by using the α -dense curve

$$x_i = h_i(\theta), \quad i = 1, \dots, n, \quad \theta \geq 0 \tag{5}$$

where

$$f^*(\theta) = f(h_1(\theta), \dots, h_n(\theta)) \tag{6}$$

The global minimum of $f^*(\theta)$ has to be found in an interval $[0, \theta_{\max}]$ where θ_{\max} is bounded and independent of n . An approximation of this global minimum is obtained by choosing a step $\Delta\theta$ and by discretizing the bounded interval $[0, \theta_{\max}]$ by means of the points $i\Delta\theta$ where i is an integer, $i = 0, \dots, N$ with $N\Delta\theta = \theta_{\max}$. The discretized problem associated to our problem is therefore

$$\min_i \{f^*(i\Delta\theta), i = 0, \dots, N\} \tag{7}$$

when $\Delta\theta \rightarrow 0$, equation

$$f^*(\theta) = f(h_1(\theta), \dots, h_n(\theta)) \tag{8}$$

leads to approximations of

$$\min_{\theta \geq 0} f^*(\theta) \tag{9}$$

1.2. Theorem

Every (local or global) minimum of $f(x_1, \dots, x_n)$ can be approximated by a

minimum of $f^*(\theta)$.

Proof

Let (x'_1, \dots, x'_n) be a point realizing a minimum of f . Let θ^* be a point on the α -dense curve minimizing the distance between

$$(x'_1, \dots, x'_n) \text{ and } (x_1^* = h_1(\theta^*), \dots, x_n^* = h_n(\theta^*))$$

Furthermore, let θ' be the point ensuring the minimum of $f^*(\theta)$.

Recall that f^* is the restriction of f to the α -dense curve defined by

$$x_i = h_i(\theta), \quad i = 1, \dots, n, \quad \theta \geq 0 \tag{5}$$

We have the following inequality

$$f(x'_1, \dots, x'_n) \leq f^*(\theta')$$

Because f^* is a restriction of f to a subset of \mathbb{R}^n .

But f is continuous and (x_1^*, \dots, x_n^*) tends to (x'_1, \dots, x'_n) when the density parameter α tends to 0. Then we have

$$|f^*(\theta^*) - f(x'_1, \dots, x'_n)| \leq \varepsilon$$

involving:

$$\varepsilon \geq f^*(\theta^*) - f(x'_1, \dots, x'_n) \geq 0$$

with $\varepsilon > 0$, arbitrary small.

Moreover, we have

$$f^*(\theta') \leq f^*(\theta^*)$$

involving

$$\varepsilon \geq f^*(\theta^*) - f(x'_1, \dots, x'_n) \geq f^*(\theta') - f(x'_1, \dots, x'_n) \geq 0$$

We deduce

$$f(x'_1, \dots, x'_n) \leq f^*(\theta') \leq \varepsilon + f(x'_1, \dots, x'_n)$$

Consequently, $f^*(\theta')$ tends to $f(x'_1, \dots, x'_n)$ when α tends to 0.

Suppose that θ^* does not tend to θ' . The continuity of f implies that there exists $\varepsilon_0 > 0$ (ε_0 does not tend to 0) such that

$$|f^*(\theta^*) - f^*(\theta')| > \varepsilon_0$$

But we have

$$\begin{aligned} |f^*(\theta^*) - f^*(\theta')| &= |f^*(\theta^*) - f(x'_1, \dots, x'_n) + f(x'_1, \dots, x'_n) - f^*(\theta')| \\ &\leq |f^*(\theta^*) - f(x'_1, \dots, x'_n)| + |f(x'_1, \dots, x'_n) - f^*(\theta')| \\ &\leq \varepsilon + \varepsilon \leq \varepsilon_0 \end{aligned}$$

This contradiction proves the result.

1.3. Remark

If

$$x_i = h_i(\theta), \quad i = 1, \dots, n, \quad \theta \geq 0 \tag{5}$$

is a reducing transformation for real numbers then,

$$x_i = ABS(h_i(\theta)), \quad i = 1, \dots, n, \quad \theta \geq 0 \tag{10}$$

is a reducing transformation for positive real numbers,

$$x_i = ABS(INT(h_i(\theta))), \quad i = 1, \dots, n, \quad \theta \geq 0 \tag{11}$$

or

$$x_i = MOD(ABS(INT(h_i(\theta))), M), \quad i = 1, \dots, n, \quad \theta \geq 0 \tag{12}$$

are reducing transformations for positive integer numbers, and

$$x_i = MOD(ABS(INT(h_i(\theta))), 2), \quad i = 1, \dots, n, \quad \theta \geq 0 \tag{13}$$

is a reducing transformation for binary numbers.

Where *ABS* is the absolute value; *INT* is the integer part; *MOD* is the modulo function; θ varies from 0 to θ_{max} ; $M = 3, \dots$ is a positive integer number. We can use the function *INT* or *round*.

We will carry out the necessary demonstrations in the next papers. The main idea of our method consists in expressing n variables by means of a single variable. The Alpha-dense Curves generalize the Space-filling Curves. We have used the properties of the Archimedean Spiral to build reducing transformations.

1.4. The Archimedean Spiral

1.4.1. Dimension 2

$$X^2 + Y^2 = T^2$$

$$X = T \cos(T)$$

$$Y = T \sin(T)$$

$$T \geq 0$$

Let us draw this example of the Archimedean Spiral (**Figure 1**).

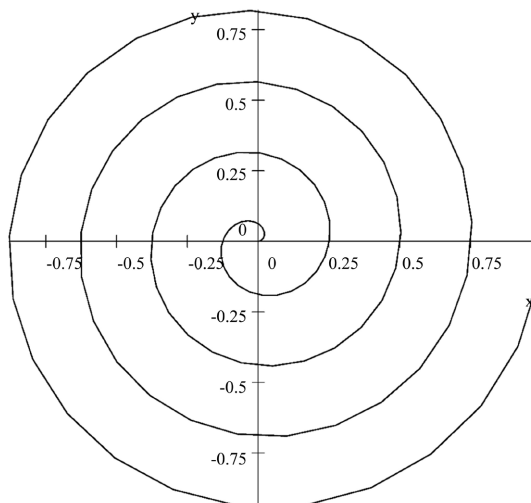


Figure 1. Archimedean spiral.

1.4.2. Dimension 3

$$X^2 + Y^2 + Z^2 = T^2$$

$$X^2 + Y^2 = K^2$$

$$K^2 + Z^2 = T^2$$

$$K = T \cos(T)$$

$$Z = T \sin(T)$$

So we have

$$X = K \cos(K) = T \cos(T) \cos(T \cos(T))$$

$$Y = K \sin(K) = T \cos(T) \sin(T \cos(T))$$

$$Z = T \sin(T) = T \sin(T)$$

It results

$$X = T \cos(T) \cos(T \cos(T))$$

$$Y = T \cos(T) \sin(T \cos(T))$$

$$Z = T \sin(T)$$

1.4.3. Dimension 4

$$X^2 + Y^2 + Z^2 + K^2 = T^2$$

$$X = T_1 \cos(T_1)$$

$$Y = T_1 \sin(T_1)$$

$$Z = T_2 \cos(T_2)$$

$$K = T_2 \sin(T_2)$$

$$T_1 = T \cos(T)$$

$$T_2 = T \sin(T)$$

Then

$$X = T \cos(T) \cos(T \cos(T))$$

$$Y = T \cos(T) \sin(T \cos(T))$$

$$Z = T \sin(T) \cos(T \sin(T))$$

$$K = T \sin(T) \sin(T \sin(T))$$

We use the following reducing transformations.

1.5. Reducing Transformations

1.5.1. For Real Numbers

$$x_i = A_i \cos(B_i T) \tag{14}$$

$$x_i = T \cos(T) \tag{15}$$

$$x_i = T \sin(T) \tag{16}$$

$$x_i = T \cos(T) \cos(T \cos(T)) \tag{17}$$

$$x_i = T \cos(T) \sin(T \cos(T)) \tag{18}$$

$$x_i = A_i \cos(B_i T) \cos(C_i T) \tag{19}$$

$$x_i = T \sin(T) \cos(T \sin(T)) \tag{20}$$

$$x_i = T \sin(T) \sin(T \sin(T)) \tag{21}$$

$$x_i = A_i (1 - \cos(B_i T)) \tag{22}$$

$$x_i = A_i \cos(B_i T + K) \tag{23}$$

Some curves

Let us consider this reducing transformation

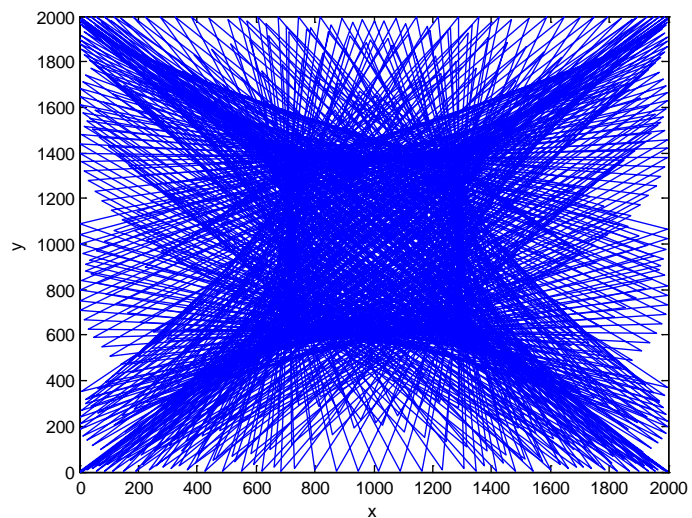
$$x_i = A_i (1 - \cos(B_i t))$$

For instance, let us draw the curve

$$\begin{cases} x = 1000(1 - \cos 5001(t)) \\ y = 1000(1 - \cos 5003(t)) \end{cases}$$

We obtain the curve below by setting with Matlab

```
close all;
t = 0 : .1 : 65
x = 1000(1 - cos 5001(t))
y = 1000(1 - cos 5003(t))
p1 = plot(x, y);
```



And for

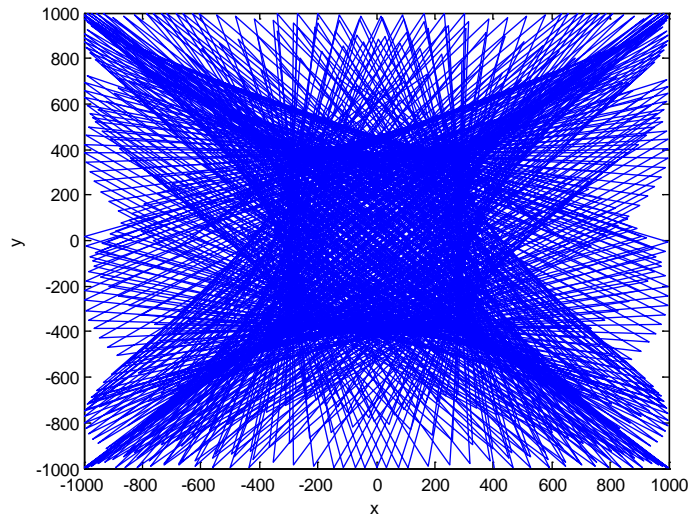
$$x_i = A_i \cos(B_i t + K)$$

For instance, let us draw the curve

$$\begin{cases} x = 1000(\cos 5001(t) + 7) \\ y = 1000(\cos 5003(t) + 7) \end{cases}$$

We obtain the curve below by setting with Matlab

```
close all;
t = 0 : .1 : 65
x = 1000(cos 5001(t) + 7)
y = 1000(cos 5003(t) + 7)
p2 = plot(x, y);
```



1.5.2. For Positive Real Numbers

$$x_i = ABS(A_i \cos(B_i T)) \tag{24}$$

$$x_i = ABS(T \cos(T)) \tag{25}$$

$$x_i = ABS(T \sin(T)) \tag{26}$$

$$x_i = ABS(T \cos(T) \cos(T \cos(T))) \tag{27}$$

$$x_i = ABS(T \cos(T) \sin(T \cos(T))) \tag{28}$$

$$x_i = ABS(A_i \cos(B_i T) \cos(C_i T)) \tag{29}$$

$$x_i = ABS(T \sin(T) \cos(T \sin(T))) \tag{30}$$

$$x_i = ABS(T \sin(T) \sin(T \sin(T))) \tag{31}$$

$$x_i = ABS(A_i (1 - \cos(B_i T))) \tag{32}$$

$$x_i = ABS(A_i \cos(B_i T + K)) \tag{33}$$

1.5.3. For Positive Integer Numbers

$$x_i = ABS(INT(A_i \cos(B_i T))) \tag{34}$$

$$x_i = ABS(INT(T \cos(T))) \tag{35}$$

$$x_i = ABS(INT(T \sin(T))) \tag{36}$$

$$x_i = ABS(INT(T \cos(T) \cos(T \cos(T)))) \tag{37}$$

$$x_i = ABS(INT(T \cos(T) \sin(T \cos(T)))) \tag{38}$$

$$x_i = ABS(INT(A_i \cos(B_i T) \cos(C_i T))) \tag{39}$$

$$x_i = ABS(INT(T \sin(T) \cos(T \sin(T)))) \tag{40}$$

$$x_i = ABS(INT(T \sin(T) \sin(T \sin(T)))) \tag{41}$$

$$x_i = ABS(INT(A_i (1 - \cos(B_i T)))) \tag{42}$$

$$x_i = ABS(INT(A_i \cos(B_i T + K))) \tag{43}$$

Let us consider this reducing transformation

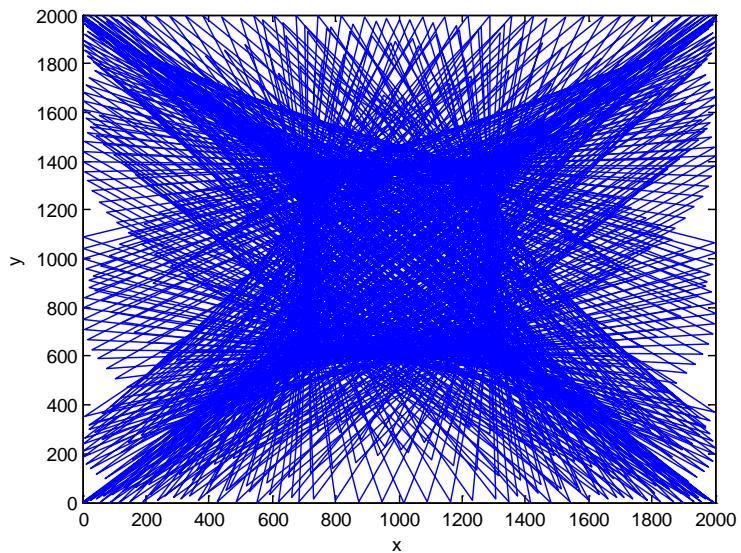
$$x_i = abs(round(A_i (1 - \cos(B_i t))))$$

For instance, let us draw the curve

$$\begin{cases} x = abs(round(1000(1 - \cos 5001(t)))) \\ y = abs(round(1000(1 - \cos 5003(t)))) \end{cases}$$

We obtain the curve below by setting with Matlab

```
close all;
t = 0 : .1 : 65
x = abs(round(1000(1 - cos 5001(t))))
y = abs(round(1000(1 - cos 5003(t))))
p3 = plot(x, y);
```



And for

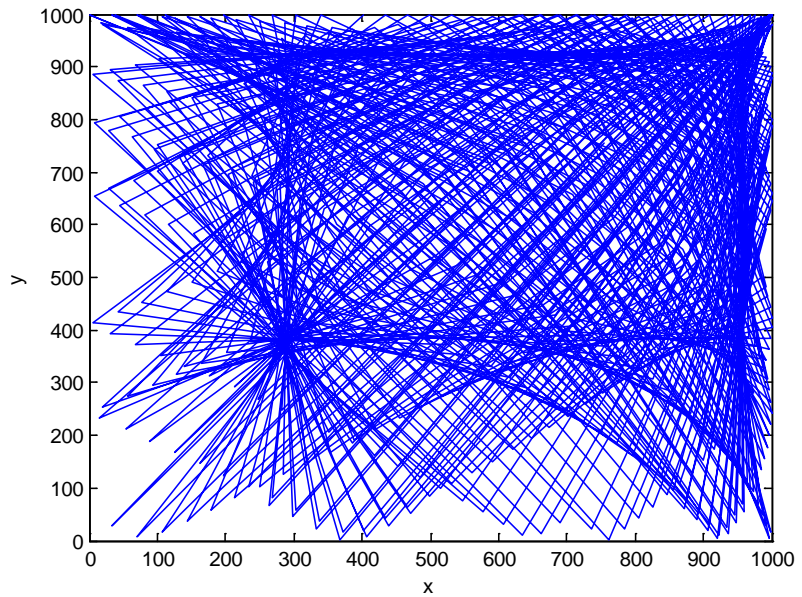
$$x_i = \text{abs}\left(\text{round}\left(A_i \cos(B_i t + K)\right)\right)$$

For instance, let us draw the curve

$$\begin{cases} x = \text{abs}\left(\text{round}\left(1000(\cos 5001(t) + 7)\right)\right) \\ y = \text{abs}\left(\text{round}\left(1000(\cos 5003(t) + 7)\right)\right) \end{cases}$$

We obtain the curve below by setting with Matlab

```
close all;
t = 0 : .1 : 65
x = abs(int(1000(cos 5001(t)+7)))
y = abs(int(1000(cos 5003(t)+7)))
p4 = plot(x, y);
```



We also have these reducing transformations:

$$x_i = \text{MOD}\left(\text{ABS}\left(\text{INT}\left(A_i \cos(B_i T)\right)\right); M\right) \tag{44}$$

$$x_i = \text{MOD}\left(\text{ABS}\left(\text{INT}\left(T \cos(T)\right)\right); M\right) \tag{45}$$

$$x_i = \text{MOD}\left(\text{ABS}\left(\text{INT}\left(T \sin(T)\right)\right); M\right) \tag{46}$$

$$x_i = \text{MOD}\left(\text{ABS}\left(\text{INT}\left(T \cos(T) \cos(T \cos(T))\right)\right); M\right) \tag{47}$$

$$x_i = \text{MOD}\left(\text{ABS}\left(\text{INT}\left(T \cos(T) \sin(T \cos(T))\right)\right); M\right) \tag{48}$$

$$x_i = \text{MOD}\left(\text{ABS}\left(\text{INT}\left(A_i \cos(B_i T) \cos(C_i T)\right)\right); M\right) \tag{49}$$

$$x_i = MOD\left(ABS\left(INT\left(T \sin(T) \cos(T \sin(T))\right)\right); M\right) \quad (50)$$

$$x_i = MOD\left(ABS\left(INT\left(T \sin(T) \sin(T \sin(T))\right)\right); M\right) \quad (51)$$

$$x_i = MOD\left(ABS\left(INT\left(A_i (1 - \cos(B_i T))\right)\right); M\right) \quad (52)$$

$$x_i = MOD\left(ABS\left(INT\left(A_i \cos(B_i T + K)\right)\right); M\right) \quad (53)$$

For binary numbers

$$x_i = MOD\left(ABS\left(INT\left(A_i \cos(B_i T)\right)\right); 2\right) \quad (54)$$

$$x_i = MOD\left(ABS\left(INT\left(T \cos(T)\right)\right); 2\right) \quad (55)$$

$$x_i = MOD\left(ABS\left(INT\left(T \sin(T)\right)\right); 2\right) \quad (56)$$

$$x_i = MOD\left(ABS\left(INT\left(T \cos(T) \cos(T \cos(T))\right)\right); 2\right) \quad (57)$$

$$x_i = MOD\left(ABS\left(INT\left(T \cos(T) \sin(T \cos(T))\right)\right); 2\right) \quad (58)$$

$$x_i = MOD\left(ABS\left(INT\left(A_i \cos(B_i T) \cos(C_i T)\right)\right); 2\right) \quad (59)$$

$$x_i = MOD\left(ABS\left(INT\left(T \sin(T) \cos(T \sin(T))\right)\right); 2\right) \quad (60)$$

$$x_i = MOD\left(ABS\left(INT\left(T \sin(T) \sin(T \sin(T))\right)\right); 2\right) \quad (61)$$

$$x_i = MOD\left(ABS\left(INT\left(A_i (1 - \cos(B_i T))\right)\right); 2\right) \quad (62)$$

$$x_i = MOD\left(ABS\left(INT\left(A_i \cos(B_i T + K)\right)\right); 2\right) \quad (63)$$

Let us consider this reducing transformation

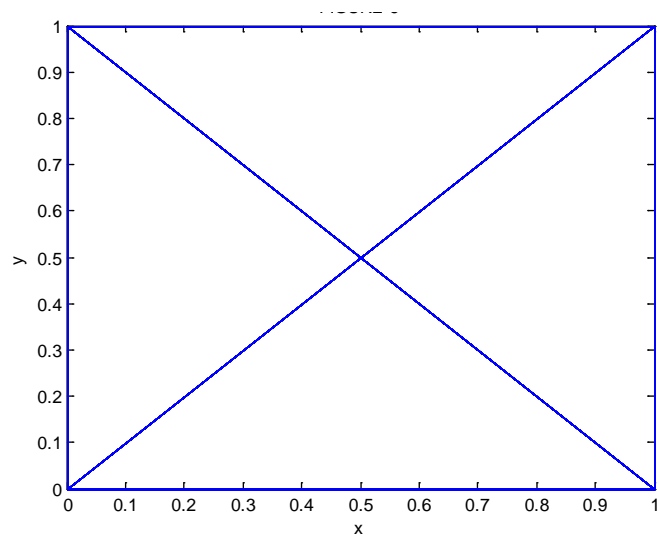
$$x_i = mod\left(abs\left(round\left(A_i (1 - \cos(B_i t))\right)\right); 2\right)$$

For instance, let us draw the curve

$$\begin{cases} x = mod\left(abs\left(round\left(1000(1 - \cos 5001(t))\right)\right); 2\right) \\ y = mod\left(abs\left(round\left(1000(1 - \cos 5003(t))\right)\right); 2\right) \end{cases}$$

We obtain the curve below by setting with Matlab

```
close all;
t = 0 : .1 : 65
x = mod(abs(round(1000(1 - cos 5001(t))), 2)
y = mod(abs(round(1000(1 - cos 5003(t))), 2)
p5 = plot(x, y);
```



And for

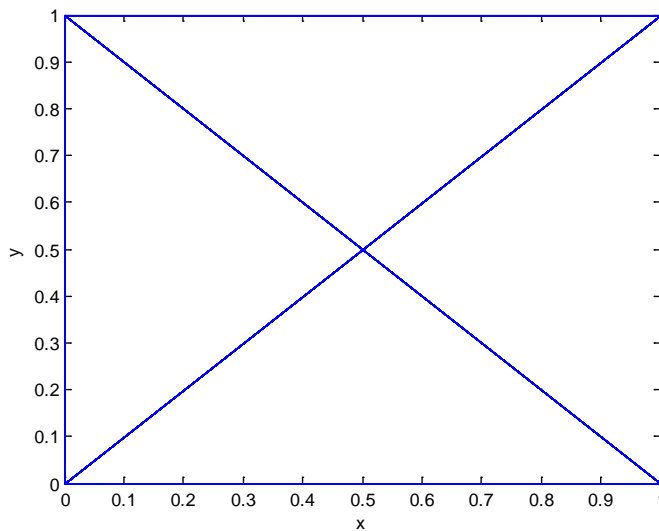
$$x_i = \text{mod}(\text{abs}(\text{round}(A_i \cos(B_i t + K))), 2)$$

For instance, let us draw the curve

$$\begin{cases} x = \text{mod}(\text{abs}(\text{round}(1000(\cos 5001(t) + 7))), 2) \\ y = \text{mod}(\text{abs}(\text{round}(1000(\cos 5003(t) + 7))), 2) \end{cases}$$

We obtain the curve below by setting with Matlab

```
close all;
t = 0 : .1 : 65
x = mod(abs(round(1000(cos 5001(t) + 7))), 2)
y = mod(abs(round(1000(cos 5003(t) + 7))), 2)
p6 = plot(x, y);
```



where ABS is the absolute value; INT is the integer part (we can also use the function round); MOD is the modulo function; A_i is a real number, B_i , C_i and K are prime numbers; T varies from 0 to T_{max} ; $M = 3, \dots$ is a positive integer number.

2. Applications

We can use different softwares like Microsoft Excel, Maple or others, to program this method.

2.1. With Excel

Excel is a software developed at the beginning of 1980s by Microsoft. The Excel tabulator helps to create spreadsheets, to view, to edit and share them with others quickly and easily.

2.1.1. Reducing Transformations

We use the following reducing transformations.

1) For real numbers

$$x_i = A_i * \cos(B_i * T)$$

$$x_i = T * \cos(T)$$

$$x_i = T * \sin(T)$$

$$x_i = T * \cos(T) * \cos(T * \cos(T))$$

$$x_i = T * \cos(T) * \sin(T * \cos(T))$$

$$x_i = A_i * \cos(B_i * T) * \cos(C_i * T)$$

$$x_i = T * \sin(T) * \cos(T * \sin(T))$$

$$x_i = T * \sin(T) * \sin(T * \sin(T))$$

$$x_i = A_i * (1 - \cos(B_i * T))$$

$$x_i = A_i * \cos(B_i * T + K)$$

2) For positive real numbers

$$x_i = ABS(A_i * \cos(B_i * T))$$

$$x_i = ABS(T * \cos(T))$$

$$x_i = ABS(T * \sin(T))$$

$$x_i = ABS(T * \cos(T) * \cos(T * \cos(T)))$$

$$x_i = ABS(T * \cos(T) * \sin(T * \cos(T)))$$

$$x_i = ABS(A_i * \cos(B_i * T) * \cos(C_i * T))$$

$$x_i = ABS(T * \sin(T) * \cos(T * \sin(T)))$$

$$x_i = ABS(T * \sin(T) * \sin(T * \sin(T)))$$

$$x_i = ABS(A_i * (1 - \cos(B_i * T)))$$

$$x_i = ABS(A_i * \cos(B_i * T + K))$$

3) For positive integer numbers

$$x_i = ABS(INT(A_i * \cos(B_i * T)))$$

$$x_i = ABS(INT(T * \cos(T)))$$

$$x_i = ABS(INT(T * \sin(T)))$$

$$x_i = ABS(INT(T * \cos(T) * \cos(T * \cos(T))))$$

$$x_i = ABS(INT(T * \cos(T) * \sin(T * \cos(T))))$$

$$x_i = ABS(INT(A_i * \cos(B_i * T) * \cos(C_i * T)))$$

$$x_i = ABS(INT(T * \sin(T) * \cos(T * \sin(T))))$$

$$x_i = ABS(INT(T * \sin(T) * \sin(T * \sin(T))))$$

$$x_i = ABS(INT(A_i * (1 - \cos(B_i * T))))$$

$$x_i = ABS(INT(A_i * \cos(B_i * T + K)))$$

$$x_i = MOD(ABS(INT(A_i * \cos(B_i * T))); M)$$

$$x_i = MOD(ABS(INT(T * \cos(T))); M)$$

$$x_i = MOD(ABS(INT(T * \sin(T))); M)$$

$$x_i = MOD(ABS(INT(T * \cos(T) * \cos(T * \cos(T)))); M)$$

$$x_i = MOD(ABS(INT(T * \cos(T) * \sin(T * \cos(T)))); M)$$

$$x_i = MOD(ABS(INT(A_i * \cos(B_i * T) * \cos(C_i * T)))); M)$$

$$x_i = MOD(ABS(INT(T * \sin(T) * \cos(T * \sin(T)))); M)$$

$$x_i = MOD(ABS(INT(T * \sin(T) * \sin(T * \sin(T)))); M)$$

$$x_i = MOD(ABS(INT(A_i * (1 - \cos(B_i * T)))); M)$$

$$x_i = MOD(ABS(INT(A_i * \cos(B_i * T + K)))); M)$$

4) For binary numbers

$$x_i = MOD(ABS(INT(A_i * \cos(B_i * T))); 2)$$

$$x_i = MOD(ABS(INT(T * \cos(T))); 2)$$

$$\begin{aligned}
 x_i &= MOD\left(ABS\left(INT\left(T * \sin(T)\right)\right);2\right) \\
 x_i &= MOD\left(ABS\left(INT\left(T * \cos(T) * \cos\left(T * \cos(T)\right)\right)\right);2\right) \\
 x_i &= MOD\left(ABS\left(INT\left(T * \cos(T) * \sin\left(T * \cos(T)\right)\right)\right);2\right) \\
 x_i &= MOD\left(ABS\left(INT\left(A_i * \cos(B_i * T) * \cos(C_i * T)\right)\right);2\right) \\
 x_i &= MOD\left(ABS\left(INT\left(T * \sin(T) * \cos\left(T * \sin(T)\right)\right)\right);2\right) \\
 x_i &= MOD\left(ABS\left(INT\left(T * \sin(T) * \sin\left(T * \sin(T)\right)\right)\right);2\right) \\
 x_i &= MOD\left(ABS\left(INT\left(A_i * (1 - \cos(B_i * T))\right)\right);2\right) \\
 x_i &= MOD\left(ABS\left(INT\left(A_i * \cos(B_i * T + K)\right)\right);2\right)
 \end{aligned}$$

where *ABS* is the absolute value; *INT* is the integer part (we can also use the function round); *MOD* is the modulo function; A_i is a real number, B_i , C_i and K are prime numbers; T varies from 0 to T_{max} ; $M = 3, \dots$ is a positive integer number and $*$ is the sign “multiply by”.

2.1.2. Application

With Excel, we have the following table

T	Variables	Constraints	Test	Values
0 to T_{max}	X_i	C_i	0 or 1 if Constraints verified	Max. Min or value

with this powerful technique, we can solve:

- Boolean Equations.
- Diophantine Equations.
- 0 - 1 integer programming problems.
- Mixed integer programming problems.
- Integer programming problems.
- Partial Differential Equations.
- Etc.

2.1.3. Examples

Let us look at some examples.

1) Resolution of a Boolean Equation of 3 variables and 21 variables

Boolean Algebra is a deductive mathematical system closed over the values 0 and 1 (false and true). Boolean Logic forms the basis for computation in modern binary computer systems. We can represent any electronic computer circuit by using a system of Boolean Equations. We usually represent Boolean Functions by means of truth tables. A statement with n logical variables requires a table with 2^n rows.

a) Resolution of a Boolean Equation of 3 variables

Let us solve [1]

$$X_1 * (X_2 + X_3) + X_2 * X_3 = 1$$

We set

$$X_1 = MOD\left(ABS\left(INT\left(T * \cos(T) * \cos(T * \cos(T))\right)\right); 2\right)$$

$$X_2 = MOD\left(ABS\left(INT\left(T * \cos(T) * \sin(T * \cos(T))\right)\right); 2\right)$$

$$X_3 = MOD\left(ABS\left(INT\left(T * \sin(T)\right)\right); 2\right)$$

We calculate

$$F = X_1 * (X_2 + X_3) + X_2 * X_3$$

T from 0 to 6.5514 step 0.0001

$$IF (F = 1; 1; 0)$$

And we have four solutions, the first solution

$$(X_1, X_2, X_3) = (1, 0, 1)$$

is first obtained when $T = 1.5708$;

The second solution

$$(X_1, X_2, X_3) = (1, 1, 1)$$

is first obtained when $T = 2.5312$;

The third solution

$$(X_1, X_2, X_3) = (0, 1, 1)$$

is first obtained when $T = 2.7284$;

And the fourth solution

$$(X_1, X_2, X_3) = (1, 1, 0)$$

is first obtained when $T = 3.4369$.

We obtain the following table

T	X_1	X_2	X_3	F
0	0	0	0	0
0.0001	0	0	0	0
...
1.5708	1	0	1	1
...
2.5312	1	1	1	1
...
2.7284	0	1	1	1
...
3.4369	1	1	0	1
...

It is easy to represent the truth table, because we have 2^3 (8) rows. The truth table is given below.

X_3	X_2	X_1	$X_1(X_2 + X_3) + X_2X_3$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

b) Resolution of a Boolean Equation of 21 variables

For twenty one variables, it is difficult to represent the truth table ($2^{21} = 2,097,152$ rows).

Let us solve

$$(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16}) * X_1 + X_{17} * X_{18} + X_{19} * (X_{20} + X_{21}) = 1$$

We set

$$X_1 = MOD\left(ABS\left(INT\left(1000 * \cos(5003 * T)\right)\right), 2\right)$$

$$X_2 = MOD\left(ABS\left(INT\left(1000 * \cos(6007 * T)\right)\right), 2\right)$$

$$X_3 = MOD\left(ABS\left(INT\left(1000 * \cos(7001 * T)\right)\right), 2\right)$$

$$X_4 = MOD\left(ABS\left(INT\left(1000 * \cos(8009 * T)\right)\right), 2\right)$$

$$X_5 = MOD\left(ABS\left(INT\left(1000 * \cos(8233 * T)\right)\right), 2\right)$$

$$X_6 = MOD\left(ABS\left(INT\left(1000 * \cos(9283 * T)\right)\right), 2\right)$$

$$X_7 = MOD\left(ABS\left(INT\left(1000 * \cos(9461 * T)\right)\right), 2\right)$$

$$X_8 = MOD\left(ABS\left(INT\left(1000 * \cos(9721 * T)\right)\right), 2\right)$$

$$X_9 = MOD\left(ABS\left(INT\left(1000 * \cos(9739 * T)\right)\right), 2\right)$$

$$X_{10} = MOD\left(ABS\left(INT\left(1000 * \cos(9743 * T)\right)\right), 2\right)$$

$$X_{11} = MOD\left(ABS\left(INT\left(1000 * \cos(9749 * T)\right)\right), 2\right)$$

$$X_{12} = MOD\left(ABS\left(INT\left(1000 * \cos(9767 * T)\right)\right), 2\right)$$

$$X_{13} = MOD\left(ABS\left(INT\left(1000 * \cos(9769 * T)\right)\right), 2\right)$$

$$X_{14} = MOD\left(ABS\left(INT\left(1000 * \cos(9781 * T)\right)\right), 2\right)$$

$$X_{15} = MOD\left(ABS\left(INT\left(1000 * \cos(9787 * T)\right)\right), 2\right)$$

$$X_{16} = MOD\left(ABS\left(INT\left(1000 * \cos(9791 * T)\right)\right), 2\right)$$

$$X_{17} = MOD\left(ABS\left(INT\left(1000 * \cos(9803 * T)\right)\right), 2\right)$$

$$X_{18} = MOD\left(ABS\left(INT\left(1000 * \cos(9811 * T)\right)\right), 2\right)$$

$$X_{19} = MOD\left(ABS\left(INT\left(1000 * \cos(9817 * T)\right)\right), 2\right)$$

$$X_{20} = MOD\left(ABS\left(INT\left(1000 * \cos(9829 * T)\right)\right), 2\right)$$

$$X_{21} = MOD\left(ABS\left(INT\left(1000 * \cos(9833 * T)\right)\right), 2\right)$$

We calculate

$$F = (X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16}) * X_1 + X_{17} * X_{18} + X_{19} * (X_{20} + X_{21})$$

T from 0 to 6.5514 step 0.0001

$$IF(F = 1; 1; 0)$$

And we have 11,192 solutions, the first solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1)$$

is obtained when $T = 0.0008$;

The second solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1)$$

is obtained when $T = 0.0011$;

The third solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1)$$

is obtained when $T = 0.0015$;

The fourth solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0)$$

is obtained when $T = 0.0018$;

The fifth solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0)$$

is obtained when $T = 0.0029$;

The sixth solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1)$$

is obtained when $T = 0.0034$;

...

The before last solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1)$$

is obtained when $T = 6.551$;

The last solution

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}) = (0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0)$$

is obtained when $T = 6.5514$.

2) Resolution of a Diophantine Equation

Let us solve the system

$$\begin{cases} 2 * X_1^3 - X_2^2 - 1 = 0 \\ X_1 * X_2^3 - X_2 - 4 = 0 \end{cases}$$

We set

$$F = (2 * X_1^3 - X_2^2 - 1)^2 + (X_1 * X_2^3 - X_2 - 4)^2$$

$$X_1 = 1.3 * \cos(18753 * T)$$

$$X_2 = 1.7 * \cos(35221 * T)$$

T from 0 to 6.5514 step 0.0001

We calculate

$$F = (2 * X_1^3 - X_2^2 - 1)^2 + (X_1 * X_2^3 - X_2 - 4)^2$$

We obtain the following table

T	X_1	X_2	F
0	1.3	1.7	0.72584761
0.0001	-0.38976563	-1.57840976	13.8207581
0.0002	-1.06628115	1.23103221	76.5361338
0.0003	1.02914987	-0.70755878	13.8352353
0.0004	0.44916231	0.08287094	17.3494185
0.0005	-1.2984853	0.55367136	55.11329
...
2.079	1.23311618	1.65997358	0.00041371
...

And we find

$$F = 0.00041371$$

$$X_1 = 1.23311618$$

$$X_2 = 1.65997358$$

when $T = 2.079$.

3) Resolution of a 0 - 1 integer programming problem

Let us maximize

$$Z = 3 * X_1 - 2 * X_2 + 5 * X_3$$

Subject to

$$X_1 + 2 * X_2 - X_3 \leq 2$$

$$X_1 + 4 * X_2 + X_3 \leq 4$$

$$X_1 + X_3 \leq 3$$

$$4 * X_2 + X_3 \leq 6$$

$$X_i = 0 \text{ or } 1, i = 1, 2, 3$$

We set

$$Z = 3 * X_1 - 2 * X_2 + 5 * X_3$$

$$X_1 = MOD\left(ABS\left(INT\left(T * \cos(T) * \cos(T * \cos(T))\right)\right); 2\right)$$

$$X_2 = MOD\left(ABS\left(INT\left(T * \cos(T) * \sin(T * \cos(T))\right)\right); 2\right)$$

$$X_3 = MOD\left(ABS\left(INT\left(T * \sin(T)\right)\right); 2\right)$$

T from 0 to 6.5514 step 0.0001

$$C_1 = X_1 + 2 * X_2 - X_3 - 2$$

$$C_2 = X_1 + 4 * X_2 + X_3 - 4$$

$$C_3 = X_1 + X_3 - 3$$

$$C_4 = 4 * X_2 + X_3 - 6$$

$$TEST = IF\left(AND\left(C_1 \leq 0; C_2 \leq 0; C_3 \leq 0; C_4 \leq 0\right); 1; 0\right)$$

$$F = IF\left(TEST = 1; 3 * X_1 - 2 * X_2 + 5 * X_3; 0\right)$$

We obtain the following table

T	X_1	X_2	X_3	C_1	C_2	C_3	C_4	$TEST$	F
0	0	0	0	-2	-4	-3	-6	1	0
0.0001	0	0	0	-2	-4	-3	-6	1	0
0.0002	0	0	0	-2	-4	-3	-6	1	0
0.0003	0	0	0	-2	-4	-3	-6	1	0
0.0004	0	0	0	-2	-4	-3	-6	1	0
0.0005	0	0	0	-2	-4	-3	-6	1	0
...
1.5708	1	0	1	-2	-2	-1	-5	1	8
...

And we find

$$\begin{aligned} \max Z &= 8 \\ X_1 &= 1 \\ X_2 &= 0 \\ X_3 &= 1 \end{aligned}$$

when $T = 1.5708$.

4) Resolution of a mixed integer programming problem

Let us minimize

$$Z = X_1 + X_2 + X_3$$

Subject to

$$6.4 * X_1 + 3.2 * X_2 \leq 6$$

$$2 * X_1 + 3 * X_2 + 3 * X_3 \geq 4$$

$$X_2 \leq 3$$

X_1 a real; X_2 and X_3 are integers; $X_1, X_2, X_3 \geq 0$

We set

$$X_1 = ABS(T * \cos(T) * \cos(T * \cos(T)))$$

$$X_2 = ABS(INT(T * \cos(T) * \sin(T * \cos(T))))$$

$$X_3 = ABS(INT(T * \sin(T)))$$

T from 0 to 6.5514 step 0.0001

$$Z = X_1 + X_2 + X_3$$

$$C_1 = 6.4 * X_1 + 3.2 * X_2 - 6$$

$$C_2 = 2 * X_1 + 3 * X_2 + 3 * X_3 - 4$$

$$C_3 = X_2 - 3$$

$$TEST = IF(AND(C_1 \leq 0; C_2 \geq 0; C_3 \leq 0); 1; 0)$$

$$F = IF(TEST = 1; X_1 + X_2 + X_3; 10000)$$

We obtain the following table

T	X_1	X_2	X_3	C_1	C_2	C_3	$TEST$	F
0	0	0	0	-6	-4	-3	0	10000
0.0001	0.0001	0	0	-5.99936	-3.9998	-3	0	10000
0.0002	0.0002	0	0	-5.99872	-3.9996	-3	0	10000
0.0003	0.0003	0	0	-5.99808	-3.9994	-3	0	10000
0.0004	0.0004	0	0	-5.99744	-3.9992	-3	0	10000
0.0005	0.0005	0	0	-5.9968	-3.999	-3	0	10000
...
1.8981	0.50008968	0	1	-2.79942606	0.00017936	-3	1	1.50008968
...

And we find

$$\begin{aligned} \min Z &= 1.50008968 \\ X_1 &= 0.50008968 \\ X_2 &= 0 \\ X_3 &= 1 \end{aligned}$$

when $T = 1.8981$.

5) Resolution of an integer programming problem

Let us maximize

$$Z = 7 * X_1 + 5 * X_2$$

Subject to

$$6 * X_1 + 9 * X_2 \leq 54$$

$$7 * X_1 + 6 * X_2 \leq 42$$

$$X_1 \leq 4$$

X_1 and X_2 are integers; $X_1, X_2 \geq 0$

We set

$$Z = 7 * X_1 + 5 * X_2$$

$$X_1 = ABS(INT(T * cos(T)))$$

$$X_2 = ABS(INT(T * sin(T)))$$

T from 0 to 6.5514 step 0.0001

$$C_1 = 6 * X_1 + 9 * X_2 - 54$$

$$C_2 = 7 * X_1 + 6 * X_2 - 42$$

$$C_3 = X_1 - 4$$

$$TEST = IF(AND(C_1 \leq 0; C_2 \leq 0; C_3 \leq 0); 1; 0)$$

$$F = IF(TEST = 1; 7 * X_1 + 5 * X_2; 0)$$

We obtain the following table

T	X_1	X_2	C_1	C_2	C_3	$TEST$	F
0	0	0	-54	-42	-4	1	0
0.0001	0	0	-54	-42	-4	1	0
0.0002	0	0	-54	-42	-4	1	0
0.0003	0	0	-54	-42	-4	1	0
0.0004	0	0	-54	-42	-4	1	0
0.0005	0	0	-54	-42	-4	1	0
...
5.5225	4	3	-3	-12	0	1	43
...

And we find

$$\max Z = 43$$

$$X_1 = 4$$

$$X_2 = 3$$

when $T = 5.5225$.

3. Conclusion

This article is the continuation of two previous papers: Computational resolution of Diophantine Equations by means of Alpha-dense Curves (2012) and Global Optimization with Alpha-dense Curves: resolution of Boolean Equations (2012) from Esther Claudine Bityé Mvondo, Yves Cherruault, and Jean Claude Mazza. We have proposed new reducing transformations allowing us to simplify a multivariable optimization problem to a new optimization problem according to a single variable. We have shown, in different examples, how to use Alpha-dense Curves to solve different Mathematical Programming problems by means of the tabulator Microsoft Excel such as a Boolean Equation of three variables, and a Boolean Equation of twenty-one variables. We will carry out the necessary demonstrations in the next papers. Differential Equations and Partial Differential Equations can also be solved with this powerful technique. We can also easily solve problems involving a large number of variables with the tabulator Excel of Microsoft. We can use different softwares like Microsoft Excel, Maple or others, to program our method.

Dedicated

This paper is dedicated to my ALMIGHTY FATHER.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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