American Journal of Operations Research, 2020, 10, 138-145
https://www.scirp.org/journal/ajor
ISSN Online: 2160-8849
ISSN Print: 2160-8830

Role of Examples and Interpretation of Results in Developing Multi-Objective Optimization Techniques

Chandra Sen

Department of Agricultural Economics, Institute of Agricultural Sciences, Banaras Hindu University, Varanasi, India
Email: chandra_sen@rediffmail.com

How to cite this paper: Sen, C. (2020) Role of Examples and Interpretation of Results in Developing Multi-Objective Optimization Techniques. *American Journal of Operations Research*, **10**, 138-145.
https://doi.org/10.4236/ajor.2020.104010

Received: June 9, 2020
Accepted: July 21, 2020
Published: July 24, 2020

Abstract

The paper evaluates the suitability of examples used in developing averaging techniques of multi-objective optimization (MOO). Most of the examples used for proposing these techniques were not suitable. The results of these examples have also not been interpreted correctly. An appropriate example has also been solved with existing and improved averaging techniques of multi-objective optimization.

Keywords

Multi-Objective Optimization, Averaging Multi-Objective Optimization Techniques, Improved Averaging Multi-Objective Optimization Techniques

1. Introduction

The importance of examples in understanding the mathematical theories or mathematical interpretations is very well recognised. Examples are the principle devices used to illustrate and communicate concepts to the learner. Examples are quite relevant for making any mathematical theory or concept more realistic and acceptable. The present study evaluates the suitability of the examples used in the development of new averaging MOO techniques. After Sen's MOO technique [1], several averaging MOO techniques [2]-[11] have been proposed during last three decades. Many examples have been used for testing the applicability of these techniques. Seven examples used in these MOO techniques have been selected for the present analysis. The presence of conflicts amongst objectives is the main characteristic of an appropriate example. The results of these examples and their interpretations have also been reviewed. The achievement of the objectives using MOO techniques has been compared with the results of individual optimization. The results of the existing averaging MOO techniques using these examples have not been interpreted correctly. An appropriate example has also been solved using existing and improved averaging MOO techniques [12] for comparison.

2. Multi-Objective Optimization Techniques

The mathematical forms of Sen's MOO technique, existing and improved averaging MOO techniques are described as:

Optimize $Z = [\text{Max. } Z_1, \text{Max. } Z_2, \cdots, \text{Max. } Z_r, \text{Min. } Z_{r+1}, \cdots, \text{Min. } Z_s]$

Subject to:

$$AX = b \quad \text{and} \quad X \geq 0$$

The individual optima are obtained by optimizing each objective separately as:

$$Z_{\text{optima}} = [\theta_1, \theta_2, \cdots, \theta_s]$$

The Sen's Multi-Objective Function [1] is formulated as:

Maximize $Z = \dfrac{\sum_{j=1}^{r} Z_j}{|\theta_j|} - \dfrac{\sum_{j=r+1}^{s} Z_j}{|\theta_{r+1}|}$

Subject to:

$$AX = b \quad \text{and} \quad X \geq 0$$

$$\theta_j \neq 0 \quad \text{for} \quad j = 1, 2, \cdots, s.$$

where,

θ_j is the optimal value of j^{th} objective function.

The Multi-Objective Function for the existing averaging MOO technique [2]-[11] is formulated as under:

Maximize $Z = \dfrac{\sum_{j=1}^{r} Z_j}{|\theta_{1av}|} - \dfrac{\sum_{j=r+1}^{s} Z_j}{|\theta_{2av}|}$

Subject to:

$$AX = b \quad \text{and} \quad X \geq 0$$

$$\theta_j \neq 0 \quad \text{for} \quad j = 1, 2, \cdots, s.$$

where,

θ_{1av} is the average of optimal values of the maximization objective functions and

θ_{2av} is the average of optimal values of the minimization objective functions.

An improved averaging MOO technique is also proposed for comparative analysis.

The Multi-Objective function for improved averaging MOO technique [12] is formulated as:

Maximize $Z = \dfrac{\sum_{j=1}^{r} Z_j}{|\theta_{1av}|} - \dfrac{\sum_{j=r+1}^{s} Z_j}{|\theta_{2av}|}$

Subject to:

$$AX = b \text{ and } X \ge 0$$

$$\theta_j \ne 0 \text{ for } j = 1,2,\cdots,s.$$

Where,

θ_{1av} is the average value of optimal and sub optimal values of the maximization objective functions and θ_{2av} is the average value of optimal and sub optimal values of the minimization objective functions.

3. Examples

The following seven examples used in existing averaging MOO technique are given below:

Example 1: [2] [6]

$$\text{Max. } Z_1 = X_1 + 2X_2$$

$$\text{Max. } Z_2 = X_1$$

$$\text{Min. } Z_3 = -2X_1 - 3X_2$$

$$\text{Min. } Z_4 = -X_2$$

Subject to:

$$6X_1 + 8X_2 \le 48$$

$$X_1 + X_2 \ge 3$$

$$X_1 \le 4$$

$$X_2 \le 3$$

Example 2: [7]

$$\text{Max. } Z_1 = (3X_1 + 2X_2 + 4)(2X_1 + 5X_2 + 3)/(2X_1 + 2X_2 + 2)(X_1 + X_2 + 1)$$

$$\text{Max. } Z_2 = (6X_1 + 4X_2 + 8)(6X_1 + 15X_2 + 9)/(3X_1 + 3X_2 + 3)(5X_1 + 5X_2 + 5)$$

$$\text{Max. } Z_3 = (12X_1 + 8X_2 + 16)(4X_1 + 10X_2 + 6)/(X_1 + X_2 + 1)(4X_1 + 4X_2 + 4)$$

$$\text{Min. } Z_4 = (9X_1 + 6X_2 + 12)(-8X_1 - 20X_2 - 12)/(4X_1 + 4X_2 + 4)(2X_1 + 2X_2 + 2)$$

$$\text{Min. } Z_5 = (-15X_1 + 10X_2 - 20)(12X_1 + 30X_2 + 18)/(3X_1 + 3X_2 + 3)(6X_1 + 6X_2 + 6)$$

Subject to:

$$6X_1 + 5X_2 \le 30$$

$$2X_1 + X_2 \le 8$$

$$5X_1 + 9X_2 \le 45$$

Example 3: [3] [4] [5] [11]

$$\text{Max. } Z_1 = (3X_1 - 2X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_2 = (9X_1 + 3X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_3 = (3X_1 - 5X_2)/(2X_1 + 2X_2 + 2)$$

$$\text{Min. } Z_4 = (-6X_1 + 2X_2)/(2X_1 + 2X_2 + 2)$$

$$\text{Min. } Z_5 = (-3X_1 - X_2)/(X_1 + X_2 + 1)$$

Subject to:

$$X_1 + X_2 \le 2$$

$$9X_1 + X_2 \le 9$$

Example 4: [7]

$$\text{Max. } Z_1 = (X_1 + 3X_2 + 1)(4X_1 + 2X_2 + 2)/(3X_1 + 3X_2 + 3)(2X_1 + 2X_2 + 2)$$

$$\text{Max. } Z_2 = (2X_1 + 6X_2 + 2)(12X_1 + 6X_2 + 6)/(5X_1 + 5X_2 + 5)(X_1 + X_2 + 1)$$

$$\text{Max. } Z_3 = (4X_1 + 12X_2 + 4)(20X_1 + 10X_2 + 10)/(4X_1 + 4X_2 + 4)(2X_1 + 2X_2 + 2)$$

$$\text{Min. } Z_4 = (-6X_1 - 8X_2 - 6)(16X_1 + 8X_2 + 8)/(3X_1 + 3X_2 + 3)(4X_1 + 4X_2 + 4)$$

$$\text{Min. } Z_5 = (5X_1 + 15X_2 + 5)(-8X_1 - 4X_2 - 4)/(6X_1 + 6X_2 + 6)(X_1 + X_2 + 1)$$

$$\text{Min. } Z_6 = (-3X_1 - 9X_2 - 3)(24X_1 + 12X_2 + 12)/(2X_1 + 2X_2 + 2)(5X_1 + 5X_2 + 5)$$

Subject to:

$$2X_1 + X_2 \le 4$$

$$5X_1 + 2X_2 \le 25$$

Example 5: [3] [11]

$$\text{Max. } Z_1 = (5X_1 + 3X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_2 = (9X_1 + 5X_2)/(3X_1 + 3X_2 + 3)$$

$$\text{Max. } Z_3 = (3X_1 - 4X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_4 = (3X_1 + 2X_2)/(2X_1 + 2X_1 + 2)$$

Subject to:

$$2X_1 + 4X_2 \ge 8$$

$$X_1 + X_2 \le 3$$

$$X_1 + 2X_2 \le 10$$

$$2X_1 + X_2 \le 5$$

$$X_1 \le 2$$

Example 6: [8] [10]

$$\text{Max. } Z_1 = (2X_1 + X_2 + 1)(2X_1 + X_2 + 2)/(2X_1 + 2X_2 + 2)$$

$$\text{Max. } Z_2 = (4X_1 + 2X_2 + 2)(6X_1 + 3X_2 + 6)/(3X_1 + 3X_2 + 3)$$

$$\text{Max. } Z_3 = (4X_1 + 2X_2 + 2)(6X_1 + 3X_2 + 6)/(6X_1 + 6X_2 + 6)$$

$$\text{Min. } Z_4 = (-8X_1 - 4X_2 - 4)(6X_1 + 3X_2 + 6)/(5X_1 + 5X_2 + 5)$$

$$\text{Min. } Z_5 = (-4X_1 - 2X_2 - 2)(10X_1 + 5X_2 + 10)/(2X_1 + 2X_2 + 2)$$

Subject to:

$$X_1 + 2X_2 \le 4$$

$$3X_1 + X_2 \le 6$$

Example 7: [6]

$$\text{Max. } Z_1 = X_1$$

$$\text{Max. } Z_2 = 2 + X_1 + 2X_2$$

$$\text{Max. } Z_3 = 3 + X_2$$

$$\text{Min. } Z_4 = -3X_2$$

$$\text{Min. } Z_5 = -X_1 - 3X_2$$

Subject to:

$$2X_1 + 3X_2 \le 6$$

$$X_1 \le 4$$

$$X_1 + 2X_2 \le 2$$

4. Interpretation of the Results

The solutions of all the above mentioned examples are presented in **Table 1**.

The solutions of the individual optimizations of all the objectives were unique as given in the X_i row. Hence, none of the examples requires the application of any MOO technique. However, all these examples have been solved using Sen's MOO technique and averaging technique. The solutions of MOO techniques were all the same as individual optimization and given in 2nd and 3rd row of X_i. The values of multi-objective function Z_{av} and Z_{sen} were not exactly the same for most of the examples due to difference in the formulation of multi-objective functions. The achievements of the real objectives of the examples have been evaluated for the efficiency of the MOO techniques The values of decision variable X_i were exactly same in all the solutions of individual as well as MOO techniques. The MOO techniques with higher values of multi-objective function

Table 1. Individual and multi-objective optimization.

Example	1	2	3	4	5	6	7
Individual Opt. X_i	4, 3	0, 5	1, 0	0, 4	2, 1	2, 0	0, 1
Multi-obj. opt.$_{av}$. X_i	4, 3	0, 5	1, 0	0, 4	2, 1	2, 0	0, 1
Multi-obj. opt.$_{sen}$ X_i	4, 3	0, 5	1, 0	0, 4	2, 1	2, 0	0, 1
Z_1	10	5.44	1.5	0.8	3.25	5.0	0
Z_2	4	4.35	4.5	6.2	1.92	20.0	4
Z_3	−17	21.7	0.75	13	0.5	10	4
Z_4	−3	−16.3	−1.5	−10.4	1	−24	−3
Z_5		−18.1	−1.5	−8.6		−50	−3
Z_6				−9.3			
$Z_{av.}$	3.39	4.35	5.2	6.25*	8	5	4.4
Z_{sen}	3.39	1.0	5	0.98	4	5	4

were declared superior over the MOO technique with lower values of multi-objective functions in the most of the studies, which is not appropriate. An appropriate example has been solved using the existing and improved MOO techniques.

5. Appropriate Example

Further a new example for testing existing and improved averaging MOO techniques is mentioned below:

Example 8:

$$\text{Max. } Z_1 = 12500X_1 + 25100X_2 + 16700X_3 + 23300X_4 + 20200X_5$$

$$\text{Max. } Z_2 = 21X_1 + 15X_2 + 13X_3 + 17X_4 + 11X_5$$

$$\text{Min. } Z_3 = 370X_1 + 280X_2 + 350X_3 + 270X_4 + 240X_5$$

$$\text{Min. } Z_4 = 1930X_1 + 1790X_2 + 1520X_3 + 1690X_4 + 1720X_5$$

Subject to:

$$X_1 + X_2 + X_3 + X_4 + X_5 = 4.5$$

$$2X_1 \geq 1.0$$

$$3X_4 \geq 1.5$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

The example was solved for achieving each objective and the results are presented in Table 2 of individual optimization matrix. Each column gives the optimal value of the mentioned objective function and suboptimal values of the remaining objective functions. The values of decision variables are given in X_i row. All the solutions of individual optimizations are different. None of the solution optimizes all the objective functions simultaneously. The conflicts amongst objective functions are very clear and necessitate the need of multi-objective optimization.

The existing averaging MOP techniques using mean, geometric mean and harmonic mean have been applied for solving the above example. The example was also solved using improved techniques of mean, geometric mean and harmonic mean and the results are presented in Table 3.

The results of multi-objective optimization with existing averaging techniques

Table 2. Individual optimization matrix.

Item	Individual Optimization			
	Max. Z_1	**Max. Z_2**	**Min. Z_3**	**Min. Z_4**
X_i	0.5, 3.5, 0, 0.5, 0	4, 0, 0, 0.5, 0	0.5, 0, 0, 0.5, 3.5	0.5, 0, 3.5, 0.5, 0
Z_1	105,750	61,650	88,600	76,350
Z_2	71.5	92.5	57.5	64.5
Z_3	1300	1615	1160	1545
Z_4	8075	8565	7830	7130

Table 3. Multi-objective optimization.

Item	Existing Average Techniques			Improved Average Techniques		
	Mean	Harmonic Mean	Geometric Mean	Mean	Harmonic Mean	Geometric Mean
X_i	0.5, 3.5, 0, 0.5, 0	0.5, 3.5, 0, 0.5, 0	0.5, 3.5, 0, 0.5, 0	0.5, 0, 0, 4, 0	0.5, 0, 0, 4, 0	0.5, 0, 0, 4, 0
Z_1	105,750	105,750	105,750	99,450	99,450	99,450
Z_2	71.5	71.5	71.5	78.5	78.5	78.5
Z_3	1300	1300	1300	1265	1265	1265
Z_4	8075	8075	8075	7725	7725	7725

using mean, geometric and harmonic mean are all the same and achieving first objective only. The remaining three objectives are ignored. The value Z_1 is 105,750 which is the optimal value of first objective. Values of remaining objectives Z_2, Z_3 and Z_4 are 71.4, 1300, and 8075 respectively which are sub optimal. However all the improved averaging techniques have also generated the unique solution but achieved all the objectives simultaneously. The improved averaging techniques have generated the compromised and more acceptable solutions than the existing averaging techniques.

6. Conclusion

The present analysis reveals that the examples used for testing existing averaging MOO techniques in many studies were not suitable for the purpose. The individual optimization revealed that all the examples were with non conflicting objectives and thus unsuitable in the application of MOO technique. The results have also not been interpreted appropriately. The values of multi-objective functions have been considered as achievements of all the objectives which are not correct. The values of basic objectives should have been considered for any conclusion. The study has been extended by adding an appropriate example and improved MOO techniques. The eighth example was found suitable for the validation of existing and improved averaging MOO techniques. The existing MOO techniques have been found inefficient in solving MOO problems.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Sen, C. (1983) A New Approach for Multi-Objective Rural Development Planning. *The Indian Economic Journal*, **30**, 91-96.

[2] Sulaiman, N.A. and Hamadameen, A.-Q.O. (2008) Optimal Transformation Technique to Solve Multi-Objective Linear Programming Problem (MOLPP). *Journal of Kirkuk University—Scientific Studies*, **3**, 158-168.

[3] Sulaiman, N.A. and Abdulrahim, B.K. (2013) Using Transformation Technique to

Solve Multi-Objective Linear Fractional Programming Problem. *International Journal of Research and Reviews in Applied Sciences*, **14**, 559-567.

[4] Sulaiman, N.A., Sadiq, G.W. and Abdulrahim, B.K. (2014) New Arithmetic Average Technique to Solve Multi-Objective Linear Fractional Programming: Problem and Its Comparison with Other Techniques *International Journal of Research and Reviews in Applied Sciences*, **18**, 122-131.

[5] Sulaiman, N.A. and Nawkhass, M.A. (2015) Using Short-Hierarchical Method to Solve Multi-Objective Linear Fractional Programming Problems. *Journal of Garmian University*, 1-15.

[6] Sulaiman, N.A. and Mustafa, R.B. (2016) Using Harmonic Mean to Solve Multi-Objective Linear Programming Problems. *American Journal of Operations Research*, **6**, 25-30. https://doi.org/10.4236/ajor.2016.61004

[7] Sulaiman, N.A. and Nawkhass, M.A. (2016) Using Standard Division to Solve Multi-Objective Quadratic Fractional Programming. *Journal of Zankoy Sulaimani*, **18**, 157-163. https://doi.org/10.17656/jzs.10544

[8] Akhtar, H., Modi, G. and Duraphe, S. (2017) An Appropriate Approach for Transforming and Optimizing Multi-Objective Quadratic Fractional Programming Problem. *International Journal of Mathematics Trends and Technology*, **50**, 80-83. https://doi.org/10.14445/22315373/IJMTT-V50P511

[9] Nahar, S. and Alim, Md.A. (2017) A New Statistical Averaging Method to Solve Multi-Objective Linear Programming Problem. *International Journal of Science and Research*, **6**, 623-629.

[10] Huma, A., Geeta, M. and Sushma, D. (2017) Transforming and Optimizing Multi-Objective Quadratic Fractional Programming Problem. *International Journal of Statistics and Applied Mathematics*, **2**, 1-5.

[11] Abdulrahim, B.K. and Abdulla, S.O. (2019) Using Interactive Techniques and New Geometric Average Techniques to Solve MOLFPP. *Journal of the University of Garmian*, **6**. https://doi.org/10.24271/garmian.196363

[12] Sen, C. (2019) Improved Scalarizing Techniques for Solving Multi-Objective Optimization Problems. *American Journal of Operational Research*, **9**, 8-11.