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Audit Risk Evaluation Method Based on TOPSIS and Choquet Fuzzy Integral

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Abstract

In view of the fuzziness of the existing audit risk evaluation methods, a comprehensive evaluation method based on TOPSIS and Choquet fuzzy integral is proposed. Using fuzzy set description and transformation language to evaluate information, based on TOPSIS method to evaluate audit risk, using Choquet fuzzy integral as information integration operator to measure the correlation between attributes. The case study shows that this method can measure the correlation between attributes more accurately and evaluate audit risk effectively.

Keywords

Audit Risk Evaluation, TOPSIS, Choquet Fuzzy Integral

1. Introduction

Modern audit is an audit mode based on risk assessment. Risk oriented audit requires auditors to assess the potential risk of material misstatement and then implement audit procedures on this basis. Therefore, the accuracy of audit risk evaluation will directly affect the audit results. Some techniques, such as dynamic programming (Zhong, 2016) and fuzzy set (Chang et al., 2016), are applied to audit risk assessment.

The influencing factors of audit risk are fuzzy, and the intermediate transition of objective risk difference is not clear. Moreover, there are many factors that affect audit risk. Many factors, such as professional ethics, are difficult to measure and describe quantitatively and accurately, and the impact effect is difficult to be verified. Audit risk assessment needs to describe these fuzzy factors, which has been a hot issue in academic research. (Wang, 2011) introduces the fuzzy comprehensive evaluation method, and tries to establish the level system of audit risk comprehensive evaluation. In the process of audit risk evaluation, the fuzziness of evaluation information and the correlation between evaluation attributes are the main factors affecting the evaluation and ranking results, and it is a complex multi-attribute mixed decision-making problem. The fuzzy comprehensive evaluation method is described by using the fuzzy set theory. The intermediate transition state is described by the fuzzy concept with clear concept connotation and unclear extension boundary. In the process of risk assessment, for quantitative indicators, such as amount, time, etc., interval number can be introduced instead of real number to reduce the information loss of quantitative indicators and increase the controllable range of data measurement; for qualitative indicators, such as risk preference, some can be quantified with simple language evaluation "high", "medium" and "low". However, due to many factors such as users' cognition of risk preference and decision-makers' professional knowledge, it is often unable to provide accurate information on such indicators, that is, there is a certain degree of fuzziness, so it is necessary to introduce a relatively comparative TOPSIS method to calculate the relative preference information of membership. Therefore, it can effectively evaluate multi-factor and multi-level complex problems, and has a wide range of applications in many fields. In this paper, we try to use TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) to improve the method of audit risk assessment. (Chen, 2000) extend the TOPSIS to the fuzzy environment.

In the actual audit risk evaluation, there is a certain degree of correlation between the risk elements. This correlation will destroy the additivity of risk information and lead to the failure of weighted integration operator. The existing research mainly uses Choquet integral to deal with the correlation between attributes, which has been applied in the evaluation of audit risk. (Meng & Tang, 2013) defined a new operator called the arithmetic interval-valued intuitionistic fuzzy Choquet aggregation operator. (Joshi & Kumar, 2016) defined the Choquet integral operator for interval-valued intuitionistic hesitant fuzzy sets and to extend the TOPSIS method using Choquet integral operator in interval-valued intuitionistic hesitant fuzzy environment.

On the basis of the above research, this paper comprehensively considers the characteristics of uncertain influence factors of audit risk, fuzzy evaluation content and insufficient evaluation information of auditors. On the basis of comprehensive analysis of modern risk oriented audit connotation and audit risk characteristics, fuzzy comprehensive evaluation method is introduced to study the quantitative evaluation of audit risk. First of all, in the process of audit risk evaluation, there are mutual feedback and mutual influence among the comprehensive evaluation indexes using TOPSIS method. Then, using the membership degree theory of fuzzy mathematics, quantitative analysis of qualitative indexes with fuzziness can avoid people's subjective and arbitrary weight value; thirdly, through Choquet integral as an information integration operator, the correlation degree between attribute indexes is calculated. Finally, the effectiveness of the audit risk evaluation method proposed in this paper is proved by case study.

2. Preliminaries

Here we give a brief review of some preliminaries.

2.1. TOPSIS

The TOPSIS is a very effective method to solve the multi-objective decision-making problem. Its concept is simple, but when it is used, it needs to define a measure in the target space to measure the degree that a solution is close to and far away from the ideal solution. The central idea is to first select an ideal solution, and then find the scheme which is the closest to the ideal solution and the farthest from the negative ideal solution as the optimal scheme.

The principle and steps of TOPSIS are as follows:

Step 1 Normalize the decision matrix.

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \tag{1}$$

Step 2 Determining positive and negative ideal points.

$$\begin{cases} D_j^+ = \max\limits_{1\le i\le n}\left(a_{ij}\right) \\ D_j^- = \min\limits_{1\le i\le n}\left(a_{ij}\right) \end{cases} \tag{2}$$

Step 3 Calculate the distance between items and positive and negative ideal points.

2.2. Fuzzy Theory and Fuzzy Integral

In order to measure the fuzzy information accurately, the interval fuzzy number is defined as follows:

Defifinition 1 (Zadeh, 1965) $a=\left[a^L,a^R\right]=\left\{x\middle|a^L\le x\le a^R,a^L,a^R\in\mathbb{R}\right\}$.

For the convenience of discussion, the definition of fuzzy measure μ is given as follows:

Defifinition 2 (Ludmila et al., 2001) $\mathbb{F}$ is algebra on set, set function is a fuzzy measure on set *X*, if and only if it satisfies the following conditions:

1) If $\varnothing\in\mathbb{F}$, then $\mu(x)=0$;

2) for $E\in\mathbb{F}$, $F\in\mathbb{F}$, if $E\subset F$, then $\mu(\mathrm{E})\le\mu(\mathrm{F})$;

3) if $E_j\in\mathbb{F}$, $E_1\subset E_2\subset\cdots\subset E_n$, $\bigcup\limits_{j=1}^{\infty}E_j\in\mathbb{F}$, then $\lim\limits_{j\to\infty}\mu\left(E_j\right)=\mu\left(\bigcup\limits_{j=1}^{\infty}E_j\right)$;

4) if, $E_j\in\mathbb{F}$, $E_1\supset E_2\supset\cdots\supset E_n$, $\bigcap\limits_{j=1}^{\infty}E_j\in\mathbb{F}$, then $\lim\limits_{j\to\infty}\mu\left(E_j\right)=\mu\left(\bigcap\limits_{j=1}^{\infty}E_j\right)$.

The λ law proposed by Sugeno is used to measure the correlation degree between attributes in audit risk. The definition of λ law is as follows:

Definition 3 (Tan, 2011) It exists $\lambda\in\left(-\frac{1}{\sup\mu},\infty\right)$. Among,

$\sup\mu=\sup\limits_{E\in\mathbb{F}}\mu\left(E\right)$ for any: $E\bigcup F\in\mathbb{F}$, $E\bigcap F=\varnothing$, it has

$\mu(E \cup F) = \mu(E) + \mu(F) + \lambda\mu(E)\mu(F)$.

In particular, if the $\mathbb{F}$ disjoint sequences $\{E_1, \cdots, E_n\}$ are finite and exist λ so that their union is also in $\mathbb{F}$, then

$$\mu\left(\bigcup_{j=1}^{n} \mu(\mathrm{E}_j)\right) = \begin{cases} \frac{1}{\lambda}\left\{\prod_{j=1}^{n}\left[1+\lambda\mu(\mathrm{E}_j)\right]-1\right\}, \lambda \neq 0 \\ \sum_{j=1}^{n} \mu(\mathrm{E}_\mathrm{j}) \end{cases} \tag{3}$$

In order to calculate the relationship between the attributes of audit risk, the Choquet integral which can reflect the relationship between indicators is used, and its definition is given as follows:

Definition 4 (Toshiaki & Michio, 1989) If it is a non-empty finite set $X = \{x_1, x_2, \cdots, x_r\}$, μ it is a fuzzy measure defined on *X*, and $f : X \to [0,1]$, the discrete Choquet integral of the fuzzy measure *X* is:

$$\int f \mathrm{d}\mu = \sum_{j=1}^{r}\left[\mu\left(P_{(j)}\right) - \mu\left(P_{(j+1)}\right)\right] x_{(j)} \tag{4}$$

where, (*j*) is the subscript after sorting $x_{(1)}$, $x_{(2)}$ and $x_{(r)}$.

3. Audit Risk Evaluation Method

Suppose that the index value is a multi index decision-making problem with interval number. There are evaluation indexes *m* of audit event $A_1, A_2, \cdots, A_m$, and *n* of the misstatement risks $G_1, G_2, \cdots, G_m$, the weight of evaluation index G_j is w_j, When attributes are independent of each other, $\sum_{j=1}^{n} w_j = 1$.

The value of audit A_i under the evaluation index G_j is the interval fuzzy measure value $\left[a_{ij}^L, a_{ij}^R\right]$, So, the decision matrix *A* is:

$$A = \begin{pmatrix} \left[a_{11}^L, a_{11}^R\right] & \cdots & \left[a_{1n}^L, a_{1n}^R\right] \\ \vdots & \ddots & \vdots \\ \left[a_{m1}^L, a_{m1}^R\right] & \cdots & \left[a_{mn}^L, a_{mn}^R\right] \end{pmatrix} \tag{5}$$

Step 1 Normalize the decision matrix.

Take the average value for each interval fuzzy index:

$$\bar{a}_j = \frac{1}{2m}\sum_{i=1}^{m}\left(a_{ij}^L + a_{ij}^R\right), j = 1, 2, \cdots, n. \tag{6}$$

Without losing generality, the following only focuses on the value with higher risk of misstatement.

$$\left[b_{ij}^L, b_{ij}^R\right] = \left[\frac{a_{ij}^L - \bar{a}_j}{\left|\bar{a}_j\right|}, \frac{a_{ij}^R - \bar{a}_j}{\left|\bar{a}_j\right|}\right] \tag{7}$$

Among, $i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$. Matrix $B = \left[b_{ij}^L, b_{ij}^R\right]_{m \times n}$.

Normalized:

$$\left[c_{ij}^L, c_{ij}^R\right] = \left[\frac{b_{ij}^L}{\max_i\left(\left|b_{ij}^L\right|, \left|b_{ij}^R\right|\right)}, \frac{b_{ij}^R}{\max_i\left(\left|b_{ij}^L\right|, \left|b_{ij}^R\right|\right)}\right] \tag{8}$$

Obviously, $c_{ij}^L, c_{ij}^R \in [-1,1]$.

Step 2 Weighted decision moment for interval fuzzy number $R = \left(\left[r_{ij}^L, r_{ij}^R\right]\right)_{m\times n}$.

$$\left[r_{ij}^L, r_{ij}^R\right] = w_j \cdot \left[c_{ij}^L, c_{ij}^R\right] = \left[w_j c_{ij}^L, w_j c_{ij}^R\right] \tag{9}$$

Step 3 Determining positive and negative ideal points.

Positive ideal points is:

$$y = \left[y_j^L, y_j^R\right] = \left[\max_i r_{ij}^L, \max_i r_{ij}^R\right] \tag{10}$$

Negative ideal points is:

$$z = \left[z_j^L, z_j^R\right] = \left[\min_i r_{ij}^L, \min_i r_{ij}^R\right] \tag{11}$$

Step 4 Calculate the distance between audit items and positive and negative ideal points:

$$\begin{cases} D_i^+ = \sqrt{\sum_{j=1}^{n}\left(d_{ij}^+\right)^2} \\ D_i^- = \sqrt{\sum_{j=1}^{n}\left(d_{ij}^-\right)^2} \end{cases} \tag{12}$$

Among,

$$\begin{cases} d_{ij}^+ = \max\left(\left|y_j^L - r_{ij}^L\right|, \left|y_j^R - r_{ij}^R\right|\right) \\ d_{ij}^- = \max\left(\left|r_{ij}^L - z_j^L\right|, \left|y_j^R - r_{ij}^R\right|\right) \end{cases} \tag{13}$$

Step 5 Using Choquet fuzzy integral to integrate positive and negative ideal distance information, the final evaluation is obtained.

Use the following formula to determine the degree of correlation between attributes:

$$\lambda + 1 = \prod_{j=1}^{r}\left[1 + \lambda\mu\left(P_j\right)\right] \tag{14}$$

where, $-1 < \lambda < \infty, \lambda \neq 0$.

Using fuzzy integral to integrate all the interrelations:

$$\int f\mathrm{d}\mu = \sum_{j=1}^{r}\left[\mu\left(P_{(j)}\right) - \mu\left(P_{(j+1)}\right)\right]x_{(j)} \tag{15}$$

4. Illustrative Example

Taking the expected audit risk assessment as an example, the application of fuzzy comprehensive evaluation method is illustrated. According to the level of audit risk quantification, it is usually divided into five levels of evaluation index. The interval fuzzy number is shown in **Table 1**.

Table 1. Correspondence between language variables and fuzzy numbers.

Define	Scale
Extremely Unimportant	[0.0, 0.2]
Slightly Unimportant	[0.2, 0.4]
No Difference	[0.4, 0.6]
Slightly Important	[0.6, 0.8]
Extremely Important	[0.8, 1.0]

The audit risk is evaluated by four dimensions of operational risk, internal control, financial performance and external environment, and its weight is as follows:

$$w = (0.3, 0.2, 0.3, 0.4)^{\mathrm{T}} \tag{16}$$

The detailed audit risk assessment information of the case is shown in Table 2. The table lists four audit items and the risk indicators of each audit item in four dimensions. The fuzzy information forms of these risk indicators include accurate number, interval number and language evaluation information.

The numerical value in the case has been normalized, so the problem of numerical normalization will not be considered here. According to the case information in the table above, the information matrix is constructed as follows:

$$A = \begin{pmatrix} 0.28 & [0.2, 0.5] & [0.4, 0.6] & [0.6, 0.8] \\ 0.45 & [0.25, 0.45] & [0.6, 0.8] & [0.8, 1.0] \\ 0.75 & [0, 0.5] & [0.4, 0.6] & [0.6, 0.8] \\ 0.45 & [0.1, 0.4] & [0.8, 1.0] & [0.4, 0.6] \end{pmatrix} \tag{17}$$

Calculate the distance between positive and negative ideal points, and get the advantage distance matrix:

$$Y = \begin{pmatrix} 0 & 0.22 & 0 & 0.28 \\ 0.17 & 0.25 & 0.28 & 0.57 \\ 0.47 & 0.1 & 0 & 0.28 \\ 0.17 & 0.1 & 0.57 & 0 \end{pmatrix} \tag{18}$$

and disadvantage distance matrix:

$$Z = \begin{pmatrix} 0.47 & 0.05 & 0.57 & 0.28 \\ 0.3 & 0.05 & 0.28 & 0 \\ 0 & 0.25 & 0.57 & 0.28 \\ 0.3 & 0.18 & 0 & 0.57 \end{pmatrix} \tag{19}$$

According to the evaluation dimension, $\sum_{j=1}^{4} w_j \neq 1$, We can know that these four evaluation dimensions are related to each other, so we need to deal with the evaluation indicators.

$$\lambda + 1 = (1 + 0.3\lambda)(1 + 0.2\lambda)(1 + 0.3\lambda)(1 + 0.4\lambda) \tag{20}$$

So, $\lambda \approx -0.41$.

According to the fuzzy integral of Choquet (shown in Table 3), get the advantage value of each audit item:

$$D^{+} = \begin{pmatrix} 0.149 \\ 0.370 \\ 0.254 \\ 0.231 \end{pmatrix} \tag{21}$$

And the disadvantage value of each audit item:

$$D^{-} = \begin{pmatrix} 0.387 \\ 0.171 \\ 0.307 \\ 0.330 \end{pmatrix} \tag{22}$$

Integrate the advantages and disadvantages to get the final risk assessment score of each audit item:

$$D = D^{+} - D^{-} = \begin{pmatrix} -0.238 \\ 0.199 \\ -0.053 \\ -0.099 \end{pmatrix} \tag{23}$$

Therefore, we can find that the risk assessment of audit item 2 is higher, and we should pay more attention to the risk of material misstatement of audit item 2.

Table 2. Audit risk case information.

Item of audit	Audit risk evaluation index			
	Operating risk	Internal Controls	Financial Performance	External Environment
Item 1	0.28	[0.2, 0.5]	No difference	Slightly important
Item 2	0.45	[0.25, 0.45]	Slightly important	extremely important
Item 3	0.75	[0, 0.5]	No difference	Slightly important
Item 4	0.45	[0.1, 0.4]	extremely important	No difference

Table 3. Fuzzy measure of each attribute.

Attribute Subset	Fuzzy measure	Attribute Subset	Fuzzy measure	Attribute Subset	Fuzzy measure
Ø	0	$\{x_1, x_2\}$	0.48	$\{x_1, x_2, x_3\}$	0.72
$\{x_1\}$	0.3	$\{x_1, x_3\}$	0.56	$\{x_1, x_2, x_4\}$	0.80
$\{x_2\}$	0.2	$\{x_1, x_4\}$	0.65	$\{x_1, x_3, x_4\}$	0.87
$\{x_3\}$	0.3	$\{x_2, x_3\}$	0.48	$\{x_2, x_3, x_4\}$	0.80
$\{x_4\}$	0.4	$\{x_2, x_4\}$	0.57	$\{x_1, x_2, x_3, x_4\}$	1
		$\{x_3, x_4\}$	0.65		

5. Conclusion

In order to improve the flexibility and accuracy of audit risk assessment process, this paper proposes an audit risk assessment method based on TOPSIS and Choquet integral. Compared with the existing audit risk evaluation methods, the evaluation method proposed in this paper has the following advantages: 1) on the basis of TOPSIS method, the fuzzy set theory is integrated, which reduces the subjectivity and uncertainty of attribute weight in the decision-making process, and is compatible with more fuzzy information. 2) The relevance between audit risk evaluation attributes is considered more accurately. The relevance information of attributes is integrated into the evaluation decision-making process by Choquet integral, which makes the evaluation process better reflect the measurement of evaluation attributes and more in line with the reality. The significance of this study is mainly reflected in the following two aspects: first, as a typical multi-attribute decision-making problem, the evaluation of audit risk has always been a hot topic in academia. The multi-attribute decision-making method of audit risk proposed in this study enriches the research of multi-attribute decision-making; secondly, this study uses the fuzzy integral theory to study the evaluation of audit risk, which is helpful to the research of other issues related to audit risk to a certain extent.

The audit risk assessment method proposed in this paper is suitable for the situation that the evaluation information of the scheme is described by interval fuzzy number, but in the actual evaluation process, there are other forms of fuzzy information, such as hesitation information in fuzzy number. Because the pursuit of high membership and low on membership of benefit oriented objective function will conflict with the goal of reducing the degree of hesitation to some extent, this information is not integrated into the framework of this method. In the future research, other fuzzy information processing methods can be further introduced into the calculation framework of this paper to expand the universality of application. In addition, the limitation of this paper is that it uses case analysis to verify the feasibility of the method, but does not use a certain amount of empirical research evidence to verify the method proposed in this paper.

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Author Contributions

Zhong Fangyuan developed the idea, motivation, and research question of the paper and contributed to the discussion. Deng yuqian outlined and revised the manuscript.

Main Contribution of Research

The main contribution of this paper is to propose an audit risk evaluation method based on TOPSIS and Choquet fuzzy integral, and to verify the feasibility of this method through case analysis.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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