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A Distributed Event-Triggered Approach for Decentralized Multi-Period Portfolio Optimization via the Alternating Direction Method of Multipliers

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Abstract

With the advent of the era of big data and the increasing demand for privacy protection, decentralized portfolio optimization has garnered significant attention in practical implementations. This paper addresses the problem of decentralized portfolio optimization within the mean-variance portfolio framework. A decentralized multi-period portfolio optimization model is established using the alternating direction method of multipliers (ADMM). The methodology incorporates a distributed event-triggered approach to imitate the professional investment manager for each sub-portfolio, where each investment manager independently triggers the rebalancing moment of the respective sub-portfolio. Empirical analysis is conducted on four well-known stock trading markets to demonstrate the performance of the decentralized multi-period portfolio optimization model.

Keywords

Decentralized Portfolio Optimization, Alternating Direction Method of Multipliers (ADMM), Multi-Period Portfolio, Event-Triggered Control

1. Introduction

In asset management, low risk and high returns are the most ideal investment types for every investor, but it is like a coin with two sides that cannot be achieved simultaneously. In investment activities, every investor's risk management efforts aim to seek a balance between risk and return. Allocating total wealth to various risk assets can effectively reduce unsystematic risk during the investment process. The key to portfolio optimization is to determine the optimal allocation proportions for each risky product. In the 1950s, Markowitz (1952) pioneered the famous portfolio theory, which introduced the mean and variance of risk product returns to measure return and risk, laying the foundation for the development of modern portfolio theory.

In financial institutions (e.g., banks, mutual funds, and pension fund investment management departments), there are two decision-making processes: centralized investment and decentralized investment (Sharpe, 1981). To put it simply, centralized investment is a portfolio made by one professional financial institution, while decentralized investment is a portfolio made by multiple professional financial institutions. Each institution rationally and independently makes decisions within its own area. In the past time, there has been a large body of work on centralized portfolio selection (Shi, Li, Leung, & So, 2022; Zhao & Liu, 2021; Dhaini & Mansour, 2021). However, centralized portfolio process requires a high degree of information sharing, which can easily lead to privacy leakage. In addition, centralized portfolio strategy ignores that certain investment products and wealth cannot be transferred (Lee, Kwon, & Lee, 2016), and centralized computing can easily lead to single points of failure with the growth of large global funds. Therefore, the application of centralized investment management is limited. There is an increasing focus on decentralized investment management (Cremers, Ferreira, Matos, & Starks, 2016), that is, the total assets are divided into multiple investment categories, and the portfolio of each category is constructed by a professional investment manager in the category. Decentralized investment management can not only diversify risks, but also provide more professional asset allocation strategies because each category has professional investment managers.

Decentralized optimization refers to decomposing the optimization problem into multiple subproblems and solving the overall optimization problem through collaboration and communication among multiple agents. In decentralized optimization, each agent can only access local data and does not exchange raw data with neighboring agents, thus providing privacy protection. The alternating direction method of multipliers (ADMM) (Lin, Li, & Fang, 2022) is a method for solving large-scale constrained optimization problems, which has become the preferred method for problems involving big data. And many variants of ADMM have appeared with the development (Zeng, Yao, & Xia, 2024). In ADMM framework, the optimization problem is decomposed into several subproblems, which are solved in a distributed manner. Its good scalability has been applied to many practical problems (Shi et al., 2022; Boyd et al., 2011). This paper will use the ℓ_1-norm with sparse properties as an auxiliary variable for implementing the ADMM algorithm to solve the decentralized portfolio optimization problem.

Portfolio models can be divided into single-period static models and multi-period dynamic models from another perspective. The single-period portfolio model solves the one-time asset allocation problem, using a fixed asset allocation weight throughout the entire investment period. For individual or institutional investors, they usually do not stick with a same portfolio strategy to the end of the investment period. Instead, they are more likely to dynamically adjust the portfolio based on changes in the financial market. Multi-period portfolio originates from these pioneering work (Mossin, 1968; Merton, 1969; Samuelson, 1975), and has received significant research attention in recent years (Gupta, Mehlawat, & Khan, 2021; Wei, Yang, Jiang, & Liu, 2021; Nesaz, Jasemi, & Monplaisir, 2020). For long-term stock investments, accurately establishing a portfolio model suitable for the entire investment period is highly challenging. The trading market is a complex system with market conditions constantly changing. Therefore, multi-period portfolio models that consider rebalancing asset allocation are a research topic with great practical value.

Event-triggered control is a way of control based on demand. Compared with the traditional communication strategy with a constant time period, event-triggered control has different triggering intervals. The event-triggered control uses a triggering function to replace the time constant in traditional periodic strategies. Event-triggered control has been studied in various fields such as communication, control, and optimization (Cao & Başar, 2020; Dai, Fang, & Chen, 2020; Ge, Han, & Wang, 2017). In multi-period portfolio selection, event-triggered strategies only rebalance the portfolio when certain events occur, and it is like a professional investment manager. Predefined event-triggered rules may consider any potentially valuable information from the market (Ding, Han, Ge, & Zhang, 2017). Centralized event-triggered control involves all agents jointly satisfying a global triggering function, while distributed event-triggered control is different. Each agent has its own local triggering function, and the agent can proceed to the next operation when the local information satisfies the function.

Based on the above discussion, the contribution of this work is to provide a new method for solving the decentralized portfolio optimization problem, that is, using alternating direction method of multipliers (ADMM) to solve the problem. In addition, in order to realize the dynamic adjustment of each sub-portfolio along with market information, we proposed two distributed event-triggered functions. Through distributed event-triggered functions, each sub-portfolio can be adjusted independently and dynamically. Distributed event-triggered functions are introduced into the decentralized portfolio optimization problem, so a new decentralized multi-period portfolio optimization model is established. This model not only fulfills the decentralized requirements of distributed investment management but also achieves dynamic adjustment of allocation weights based on market information.

2. Preliminaries

2.1. Portfolio Optimization

Assume that a portfolio model is constructed for n risky assets. Let $\boldsymbol{w} \in [0,1]^n$ be the proportions of a total wealth allocated to n risky assets, $\boldsymbol{\mu} \in \mathbb{R}^n$ be the expected rate of return, and $\boldsymbol{\Sigma}$ be the covariance matrix of n risky assets. The mean and covariance matrix of historical sample return rates are usually used as estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$. Markowitz's mean-variance portfolio model introduces estimates of the expected return vector and covariance matrix for given assets into the mean-variance model. The purpose is to find the minimum risk portfolio weight under the target return or the maximum return portfolio weight under the target risk. The formula is expressed as:

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \boldsymbol{w}^{\mathrm{T}} \boldsymbol{\Sigma} \boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{\mu}^{\mathrm{T}} \boldsymbol{w} \geq \mu_{\min}, \mathbf{1}^{\mathrm{T}} \boldsymbol{w} = 1, \boldsymbol{w} \geq 0 \end{aligned} \tag{1}$$

or

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \boldsymbol{\mu}^{\mathrm{T}} \boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{w}^{\mathrm{T}} \boldsymbol{\Sigma} \boldsymbol{w} \leq \sigma_{\max}, \mathbf{1}^{\mathrm{T}} \boldsymbol{w} = 1, \boldsymbol{w} \geq 0, \end{aligned} \tag{2}$$

where $\mu_{\min}$ is the minimum expected return of the portfolio, $\sigma_{\max}$ is is the maximum risk of the portfolio, and $\mathbf{1}$ is the column vector of all ones. $\mathbf{1}^{\mathrm{T}} \boldsymbol{w} = 1$ represents the budget constraint, and the nonnegativity constraint implies that no short selling is allowed.

In addition to the above basic model, the mean-variance portfolio model can also be expressed as a quadratic programming problem:

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \gamma \boldsymbol{w}^{\mathrm{T}} \boldsymbol{\Sigma} \boldsymbol{w} - \boldsymbol{\mu}^{\mathrm{T}} \boldsymbol{w} \\ \text{s.t.} \quad & \mathbf{1}^{\mathrm{T}} \boldsymbol{w} = 1, \boldsymbol{w} \geq 0, \end{aligned} \tag{3}$$

where $\gamma \in (0,1)$ is a predefined risk parameter. The smaller γ indicates that investors are more risk-taking. In Section 3.1, we will solve a decentralized portfolio optimization problem based on ADMM.

2.2. Alternating Direction Method of Multipliers (ADMM)

ADMM is an iterative algorithm for solving convex optimization problems. It combines the decomposability of the dual ascent method and the superior convergence property of the multiplier method. ADMM is usually used to solve optimization problems with constrained separable objective functions, as follows:

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) + g(\boldsymbol{y}) \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} = \boldsymbol{c}, \end{aligned} \tag{4}$$

where $\boldsymbol{x} \in \mathbb{R}^m$ and $\boldsymbol{y} \in \mathbb{R}^n$ are target variables. $\boldsymbol{c} \in \mathbb{R}^d$ is the constant vector, and $\boldsymbol{A} \in \mathbb{R}^{d \times m}$ and $\boldsymbol{B} \in \mathbb{R}^{d \times n}$. Use the augmented Lagrangian function to bring the constraints of the original optimization problem into the objective function, that is,

$$L_\rho(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + g(\boldsymbol{y}) + \boldsymbol{\lambda}^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} - \boldsymbol{c}) + \frac{\rho}{2}\left\|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} - \boldsymbol{c}\right\|^2, \tag{5}$$

where $\boldsymbol{\lambda}$ is the Lagrangian dual variable, and $\rho > 0$ is a penalty parameter. ADMM iteratively optimizes the problem through the following process:

$$x^{t+1} := \arg\min_{x} L_\rho\left(x, y^t, \lambda^t\right), \tag{6}$$

$$y^{t+1} := \arg\min_{y} L_\rho\left(x^{t+1}, y, \lambda^t\right), \tag{7}$$

$$\lambda^{t+1} := \lambda^t + \left(Ax^{t+1} + By^{t+1} - c\right). \tag{8}$$

In addition to the above form, ADMM can usually be written in a more convenient form, that is, using the scaled augmented Lagrangian function:

$$\mathcal{L}_\rho\left(x, y, \mu\right) = f\left(x\right) + g\left(y\right) + \frac{\rho}{2}\left\|Ax + By - c + \mu\right\|^2 - \frac{\rho}{2}\left\|\mu\right\|^2, \tag{9}$$

where $\mu = \frac{1}{\rho}\lambda$ is the scaled dual variable. After using the scaled form, the update process of ADMM is as follows:

$$x^{t+1} := \arg\min_{x} \mathcal{L}_\rho\left(x, y^t, \mu^t\right), \tag{10}$$

$$y^{t+1} := \arg\min_{y} \mathcal{L}_\rho\left(x^{t+1}, y, \mu^t\right), \tag{11}$$

$$\mu^{t+1} := \mu^t + \left(Ax^{t+1} + By^{t+1} - c\right). \tag{12}$$

3. Problem Formulation

3.1. Decentralized Portfolio Optimization

In decentralized management, a portfolio is divided among multiple professional investment institutions for joint construction. In other words, the portfolio is divided into K sub-portfolios. Let $\{\mu_1, \cdots, \mu_K\}$ represent the expected rates of return, $\{w_1, \cdots, w_K\}$ represent the weights of wealth invested to the risky assets, and $\{\Sigma_1, \cdots, \Sigma_K\}$ represent the covariance matrices under K sub-portfolios. Based on problem (3), a decentralized portfolio optimization problem is written as:

$$\begin{aligned} \min_{w_k} \quad & \sum_{k=1}^{K}\left(\gamma_k w_k^{\mathrm{T}}\Sigma_k w_k - \mu_k^{\mathrm{T}} w_k\right) \\ \text{s.t.} \quad & \sum_{k=1}^{K} \mathbf{1}_k^{\mathrm{T}} w_k = \sum_{k=1}^{K} c_k, \end{aligned} \tag{13}$$

where $\sum_{k=1}^{K} c_k = 1$, γ_k is the risk parameter for the kth sub-portfolio, which is constructed by each investment manager based on their own investment strategies and goals. $\mathbf{1}_k \in \mathbb{R}^{n_k}$ is the column vector where n_k is the number of risky assets in k sub-portfolio. $\gamma_k w_k^{\mathrm{T}}\Sigma_k w_k - \mu_k^{\mathrm{T}} w_k$ is called local objective function.

In order to solve the problem (13) in decentralized manner, we first introduce the ℓ_1-norm of the weight vector w_k into the problem, which can make the result sparse. Problem (13) is rewritten as:

$$\begin{aligned} \min_{w_k} \quad & \sum_{k=1}^{K}\left(\gamma_k w_k^{\mathrm{T}}\Sigma_k w_k - \mu_k^{\mathrm{T}} w_k\right) + \lambda\sum_{k=1}^{K}\left\|w_k\right\|_1 \\ \text{s.t.} \quad & \mathbf{1}_k^{\mathrm{T}} w_k = c_k, k = 1, \cdots, K. \end{aligned} \tag{14}$$

Problem (14) is reformulated into the following equivalent form through variable separation:

$$\begin{aligned}&\min_{\boldsymbol{w}_k} \quad \sum_{k=1}^{K}\left(\gamma_k \boldsymbol{w}_k^{\mathrm{T}}\boldsymbol{\Sigma}_k\boldsymbol{w}_k-\boldsymbol{\mu}_k^{\mathrm{T}}\boldsymbol{w}_k\right)+\lambda\sum_{k=1}^{K}\left\|\boldsymbol{z}_k\right\|_1 \\ &\text{s.t.} \quad \boldsymbol{1}_k^{\mathrm{T}}\boldsymbol{w}_k=c_k,\ \boldsymbol{w}_k=\boldsymbol{z}_k,\ k=1,\cdots,K,\end{aligned} \tag{15}$$

where $\lambda>0$ is a regularization parameter, and c_k is the weight of the total wealth given to sub-portfolio k.

The scaled augmented Lagrangian function L_ρ of problem (15) is as follows:

$$\begin{aligned}&L_\rho\left(\left\{\boldsymbol{w}_k\right\}_1^K,\left\{\boldsymbol{z}_k\right\}_1^K,\left\{u_k\right\}_1^K,\left\{\hat{\boldsymbol{u}}_k\right\}_1^K\right)\\ &=\sum_{k=1}^{K}\left(\gamma_k \boldsymbol{w}_k^{\mathrm{T}}\boldsymbol{\Sigma}_k\boldsymbol{w}_k-\boldsymbol{\mu}_k^{\mathrm{T}}\boldsymbol{w}_k\right)+\lambda\sum_{k=1}^{K}\left\|\boldsymbol{z}_k\right\|_1+\frac{\rho}{2}\sum_{k=1}^{K}\left\|\boldsymbol{1}_k^{\mathrm{T}}\boldsymbol{w}_k-c_k+u_k\right\|_2^2-\frac{\rho}{2}\sum_{k=1}^{K}\left\|u_k\right\|_2^2\\ &+\frac{\rho}{2}\sum_{k=1}^{K}\left\|\boldsymbol{w}_k-\boldsymbol{z}_k+\hat{\boldsymbol{u}}_k\right\|_2^2-\frac{\rho}{2}\sum_{k=1}^{K}\left\|\hat{\boldsymbol{u}}_k\right\|_2^2,\end{aligned} \tag{16}$$

where u and $\hat{\boldsymbol{u}}$ are dual variables of constraints in (15), and $\rho>0$ is the penalty parameter. Using the ADMM algorithm to solve this optimization problem, the alternating iterative optimization process is as follows:

$$\boldsymbol{w}_k^{t+1}=\left(\rho\boldsymbol{1}_k\boldsymbol{1}_k^{\mathrm{T}}+\rho\boldsymbol{I}_k+2\gamma_k\boldsymbol{\Sigma}_k\right)^{-1}\left(\rho\left(c_k\boldsymbol{1}_k-u_k^t\boldsymbol{1}_k+\boldsymbol{z}_k^t-\hat{\boldsymbol{u}}_k^t\right)+\boldsymbol{\mu}_k\right), \tag{17a}$$

$$\boldsymbol{z}_k^{t+1}=\mathcal{S}_{\frac{\lambda}{\rho}}\left(\boldsymbol{w}_k^{t+1}+\hat{\boldsymbol{u}}_k^t\right), \tag{17b}$$

$$u_k^{t+1}=u_k^t+\boldsymbol{1}_k^{\mathrm{T}}\boldsymbol{w}_k^{t+1}-c_k, \tag{17c}$$

$$\hat{\boldsymbol{u}}_k^{t+1}=\hat{\boldsymbol{u}}_k^t+\boldsymbol{w}_k^{t+1}-\boldsymbol{z}_k^{t+1}, \tag{17d}$$

where $\boldsymbol{I}_k\in\mathbb{R}^{n_k\times n_k}$ is the identity matrix of dimension n_k, and $\mathcal{S}$ is the soft threshold operator (Bredies & Lorenz, 2008):

$$\mathcal{S}_\kappa(a)=\begin{cases}a-\kappa, & a>\kappa\\ 0, & |a|\le\kappa\\ a+\kappa, & a<-\kappa\end{cases} \tag{18}$$

After achieving decentralized solving for the decentralized portfolio optimization problem, we will introduce appropriate distributed event-triggered functions to realize decentralized multi-period portfolio optimization in Section 3.2.

3.2. Distributed Event-Trigged Function

Stock prices are time-varying and markets are also dynamically changing. Obtaining valuable information from the market can enhance the performance of portfolio, and event-triggered functions can fuse these valuable information. In addition, a significant advantage of the event-triggered strategy is that its computational cost is very low. Inspired by (Skomorokhov, Wang, Ovchinnikov, Burnaev, & Oseledets, 2023), there are two types of distributed event-triggered functions available for portfolio problems. We propose two distributed event-triggered functions to implement decentralized multi-period portfolio optimization. The first is defined as:

$$\sum_{i=1}^{n_k} r_{k,i}(t) w_{k,i}(t) \le \lambda_1, \quad k = 1, \cdots, K, \tag{19}$$

where $r_{k,i}(t)$ is the sample return rate of stock *i* belonging to market *k* at moment *t*, which is defined as

$$r_{k,i}(t) = \frac{p_{k,i}(t) - p_{k,i}(t-1)}{p_{k,i}(t-1)}, \tag{20}$$

where $p_{k,i}(t)$ is the price of stock *i* belonging to market *k* at moment *t*. λ_1 is a predetermined threshold, usually μ or r_f, which means that the portfolio return at the moment is compared with the current average rate of return or the risk-free rate of return. Equation (19) is interpreted as if the current portfolio performance is below a certain threshold, then the portfolio should be restructured.

The second type of distributed event-triggered function is defined as:

$$\left| \sum_{i=1}^{n_k} r_{k,i}(t) w_{k,i}(t) - \sum_{i=1}^{n_k} r_{k,i}^{\text{prev}}(t) w_{k,i}(t) \right| \ge \lambda_2, \quad k = 1, \cdots, K, \tag{21}$$

where λ_2 is any positive number. $r_{k,i}^{\text{prev}}(t)$ is sample return rate of stock *i* belonging to market *k* from the initial investment moment to the current moment, and the formula can be written as:

$$r_{k,i}^{\text{prev}}(t) = \frac{p_{k,i}(t) - p_{k,i}(0)}{p_{k,i}(0)}. \tag{22}$$

Equation (21) takes into account the return at the triggering moment and the return of the entire portfolio. A major difference between the two distributed event-triggered functions is that the threshold λ_1 changes because the average return rate μ is different in each period. The risk-free return rate r_f, although usually assumed to be constant, can also be influenced by market factors, economic conditions, and other variables. The threshold λ_2 is fixed throughout the entire investment period.

A pseudocode for the decentralized event-triggered multi-period portfolio optimization is given in **Algorithm 1**.

4. Experiments

4.1. Experimental Setup

In our experiments, we choose datasets commonly used in decentralized portfolio optimization problems (Wang & Gan, 2023; Leung, Wang, & Che, 2022; Leung, Wang, & Li, 2021). At the same time, in order to reflect regional characteristics, we selected stock trading markets from different countries, and finally selected four stock trading markets: 1) HKSE; 2) NASDAQ; 3) LSE; 4) TSE. Selecting weekly adjusted price data of 100 stocks from each market, ranging from January 1, 2000, to January 1, 2018[1]. These data generate 4 datasets of different sizes, i.e. 10, 30, 60 and 100 stocks per market. Datasets are divided into in-sample

[1]https://finance.yahoo.com

Algorithm 1. Decentralized event-triggered multi-period portfolio optimization via ADMM.

Input: $r_k, \gamma_k, \lambda, \rho$
Output: $\mathbf{w}_k$
Initialize: $\mathbf{w}_k(0) = 0$, $\mathbf{z}_k(0) = 0$, $u_k(0) = 0$, $\hat{\mathbf{u}}_k(0) = 0$
for $j = 1, 2, \ldots, T$ **do**
  $\mu_{k,i}(j) = \frac{\sum_{t=1}^{j} r_{k,i}(t)}{j}$ for all assets $i \in 1, \ldots, n_k$ belonging to market k;
  $\mathbf{\Sigma}_{k,pq}(j) = \frac{\sum_{t=1}^{j}(r_{k,p}(t)-\mu_{k,p}(t))(r_{k,q}(t)-\mu_{k,q}(t))}{j-1}$ for all assets $p, q \in 1, \ldots, n_k$ belonging to market k;
  if $\sum_{i=1}^{n_k} r_{k,i}(j) w_{k,i}(j) \leq \lambda_1$ **then**
    for $t = 0, 1, \ldots, T_{\max}$ **do**
      $\mathbf{w}_k^{t+1} = \left(\rho \mathbf{1}_k \mathbf{1}_k^\top + \rho \mathbf{I}_k + 2\gamma_k \mathbf{\Sigma}_k\right)^{-1} \times \left(\rho\left(c_k \mathbf{1}_k - u_k^t \mathbf{1}_k + \mathbf{z}_k^t - \hat{\mathbf{u}}_k^t\right) + \boldsymbol{\mu}_k\right)$;
      $\mathbf{z}_k^{t+1} = \mathcal{S}_{\frac{\lambda}{\rho}}\left(\mathbf{w}_k^{t+1} + \hat{\mathbf{u}}_k^t\right)$;
      $u_k^{t+1} = u_k^t + \mathbf{1}_k^\top \mathbf{w}_k^{t+1} - c_k$;
      $\hat{\mathbf{u}}_k^{t+1} = \hat{\mathbf{u}}_k^t + \mathbf{w}_k^{t+1} - \mathbf{z}_k^{t+1}$.
    $\mathbf{w}_k(j) = \mathbf{w}_k^{T_{\max}}$
  else
    $\mathbf{w}_k(j) = \mathbf{w}_k(j-1)$
Return $\mathbf{w}_k$.

data and out-of-sample data. The in-sample data consists of 521 observations from 2000 to 2010, and the out-of-sample data consists of 417 observations from 2011 to 2018. In-sample data is used to select hyper-parameters and fit the model, while out-of-sample data are used to test model performance.

In our experiment, it is assumed that the risk preferences of professional investment managers in each market are the same, that is, $\gamma_k = 0.5$, $k = 1, \cdots, 4$. Referring to the experimental setting of (Leung et al., 2021), we assume that the proportion of total assets allocated to each market are the same, that is, $c_k = 0.25$, $k = 1, \cdots, 4$. We will determine the hyper-parameters through grid search during in-sample experiments.

This work will choose the well-known Sharpe ratio (SR) to measure the performance of a portfolio model. The Sharpe ratio comprehensively considers the risk and return of a portfolio, and it is simple to compute and widely used. The Sharpe ratio is defined as (Sharpe, 1998):

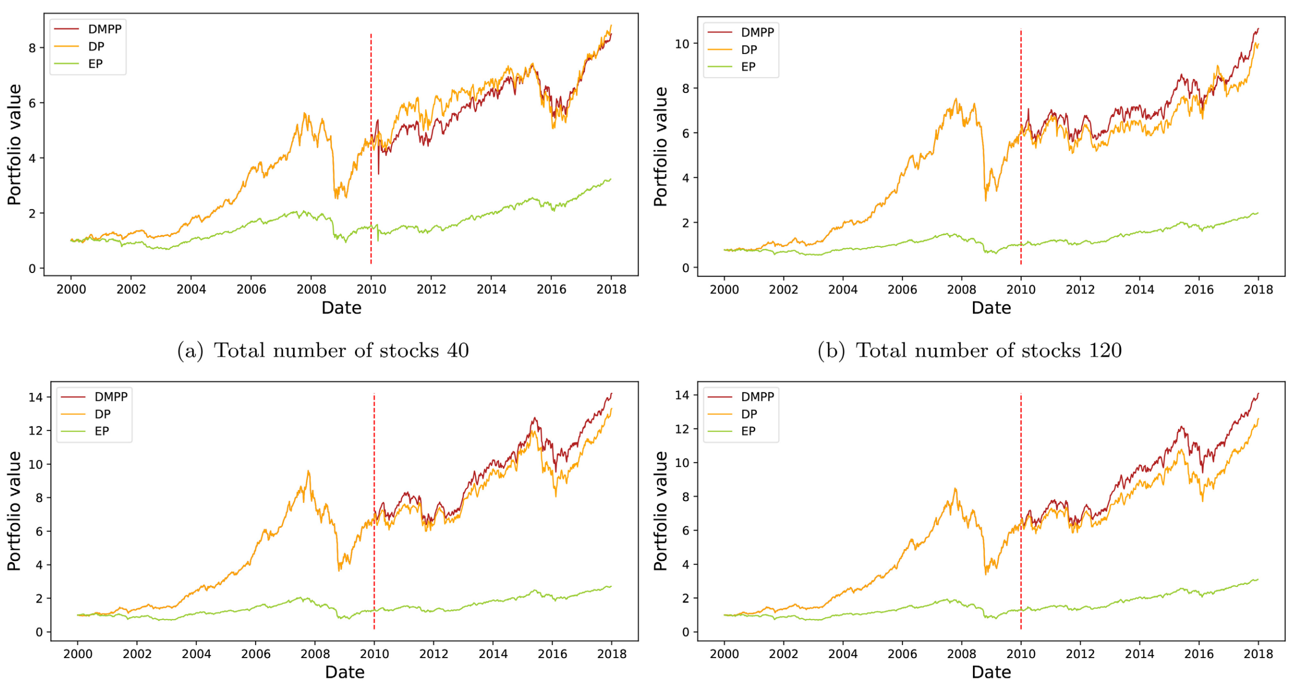
$$\mathrm{SR} = \frac{\boldsymbol{\mu}^{\mathrm{T}} \boldsymbol{w} - r_f}{\sqrt{\boldsymbol{w}^{\mathrm{T}} \boldsymbol{\Sigma} \boldsymbol{w}}}, \tag{23}$$

where r_f is the risk-free return rate, which is commonly chosen as 0.03. If the Sharpe ratio is positive, it means that the average return rate of the portfolio exceeds the risk-free rate. If it is a negative value, it means that the average return rate of the portfolio is lower than the risk-free interest rate, and there is a risk of loss. The higher the SR value, the better the portfolio.

4.2. Experimental Results

In this work, we compare three portfolio models: a distributed event-triggered decentralized multi-period portfolio optimization based on ADMM (DMPP), a decentralized single-period portfolio optimization based on ADMM (DP), and an equal-weighted portfolio model (EP). The equal-weight portfolio model is a highly competitive benchmark model and is widely used (Hsu, Han, Wu, & Cao, 2018). **Figure 1** shows the cumulative log returns for the three portfolio models. A vertical red dashed line divides the dataset into two parts. Data from 2000 to 2010 is used to train the model, and the training results are utilized as the initial asset allocation weights for a decentralized multi-period portfolio model. Data from 2011 to 2018 serves as the test set, and the decentralized multi-period portfolio model will dynamically allocate assets during this period. The results in the figure show that the cumulative log returns of two decentralized portfolio models, DMPP and DP, are significantly higher than the equally weighted portfolio model EP. In addition, the cumulative log returns of the dynamic decentralized multi-period portfolio model DMPP are higher than the static decentralized portfolio model DP, and the advantage becomes more pronounced as the data scale increases.

Table 1 provides the annualized returns and annualized risks of these three models. The results in the table show that in each experiment, DMPP has the highest annualized returns, and the lowest annualized risk is the equally weighted portfolio model EP. In the experiment with 30 stocks in each market, DMPP achieved the highest returns and lowest risk. Further comparing between DMPP and DP, in the experiment with 10 stocks in each market, DMPP achieved the highest annualized return at 0.144603, and DP exhibited the lowest annualized risk at 0.208614. However, as the data scale increases, DMPP has higher annualized returns and lower annualized risk than DP.



(a) Total number of stocks 40

(b) Total number of stocks 120

(c) Total number of stocks 240

(d) Total number of stocks 400

Figure 1. The cumulative log returns for the three portfolio models in different dataset sizes.

Table 1. Annualized risks and returns of three portfolio models in different dataset sizes.

Datasize	Model	Annualized return	Annualized risk
10	DMPP	**0.144603**	0.214312
	DP	0.141074	0.208614
	EP	0.086857	**0.200912**
30	DMPP	**0.165106**	**0.198289**
	DP	0.155512	0.231436
	EP	0.077837	0.209714
60	DMPP	**0.163731**	0.186914
	DP	0.160646	0.193069
	EP	0.069626	**0.176770**
100	DMPP	**0.162221**	0.178968
	DP	0.156120	0.184550
	EP	0.076429	**0.172620**

Table 2. SR value of three portfolio models in four datasets with 10, 30, 60 and 100 stocks per market.

	Datasize	Model	Overall	HKEX	JYP	LSE	Nasdaq
In-sample	10	DMPP	**0.941645**	**0.405476**	**0.424834**	**0.685359**	0.500077
		DP	0.920825	0.384902	0.312197	0.620250	**0.608229**
		EP	0.203181				
	30	DMPP	**1.111540**	**0.684962**	**0.545692**	**0.611723**	0.591732
		DP	1.006025	0.608240	0.437749	0.607806	**0.617202**
		EP	0.285610				
	60	DMPP	**1.318233**	**0.826391**	**0.447431**	**0.804308**	**0.830503**
		DP	1.234574	0.796821	0.415003	0.732514	0.827139
		EP	0.363972				
	100	DMPP	**1.465699**	**0.838616**	**0.529777**	0.908153	**0.919805**
		DP	1.419070	0.749867	0.494402	**0.987722**	0.858681
		EP	0.403217				
Out-of-sample	10	DMPP	0.415725	**0.136842**	**0.130012**	**0.020139**	**0.445197**
		DP	0.355467	0.102437	0.112504	0.015487	0.392548
		EP	**0.493651**				
	30	DMPP	0.604115	**0.201213**	**0.250172**	0.302175	**0.361124**
		DP	0.578124	0.189924	0.211842	**0.315487**	0.341102
		EP	**0.612245**				
	60	DMPP	0.641124	**0.005724**	**0.504975**	**0.466482**	0.501123
		DP	0.620003	0.004906	0.478852	0.431198	**0.512914**
		EP	**0.695142**				
	100	DMPP	0.532201	**0.037598**	**0.906291**	**0.038451**	**0.621031**
		DP	0.503251	0.020156	0.889245	0.032015	0.601937
		EP	**0.575139**				

Table 2 provides the Sharpe Ratio (SR) values of the three portfolio models, including the SR values of each sub-portfolio model in the decentralized portfolio model. In both in-sample and out-of-sample experiments, the SR values of DMPP are higher than DP, and the SR value of the sub-portfolios of the dynamic DMPP model are almost all greater than those of the static DP model. By introducing the distributed event-triggered strategies to decentralized portfolio model, each specialized investment institution has greater flexibility to independently adjust its sub-portfolio model based on the current market environment. In the in-sample experiments, DMPP has the highest SR value, and in the out-of-sample experiments, EP has the highest SR value. However, the SR value of DMPP can also be very close to EP.

5. Conclusion

This paper implements the solution of a decentralized portfolio optimization problem based on the alternating direction method of multipliers, and obtains four alternating optimization processes. The algorithm has no central coordinator, and each sub-portfolio model is constructed by local nodes. This approach protects data privacy and prevents single-point failures caused by large-scale data. Furthermore, considering that the market is a dynamic process, we introduce distributed event-triggered strategies into decentralized portfolio optimization problems, proposing a distributed event-triggered decentralized multi-period investment portfolio optimization problem. Experimental results show that the decentralized multi-period portfolio model based on ADMM and distributed event-triggered strategies performs better. Decentralized multi-period portfolio optimization can provide more professional investment decisions, leading to a wide range of practical applications.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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