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Computational Prooving of Riemann's Hypothesis

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Abstract

Formulated in 1859 by the mathematician Bernhard Riemann, the Riemann hypothesis is a conjecture. She says that the Riemann's Zeta function non-trivial zeros of all have real part $\frac{1}{2}$. This demonstration would improve the prime numbers distribution knowledge. This conjecture constitutes one of the most important mathematics unsolved problems of the 21st century: it is one of the famous Hilbert problems proposed in 1900. In this article, a method for solving this conjecture is given. This work has been started by finding an analytical function which gives a best accurate 10^{-8} of particular zeros sample that this number has increased gradually and finally prooving that this function is always irrational. This demonstration is important as allows Riemann's zeta function to be a model function in the Dirichlet series theory and be at the crossroads of many other theories. Also, it is going to serve as a motivation and guideline for new studies.

Keywords

Complex Function, Differential Equation, Riemann Zeta Function

1. Introduction

Published in 1859, Reimann Hypothesis attempts to predict the occurrence of prime numbers using a mathematical function. Prime numbers do not follow a pattern of occurrence. After you find one, it is impossible to predict the occurrence of the next prime number. Mathematical greats like Euclid, Euler, and Gauss are among many who attempted to address this problem. Bernhard Riemann, a student of Gauss, found a pattern in the frequency of prime numbers. He found them to follow a pattern that could be explained with a function, which he called Riemann zeta function. The Riemann's function formulation is defined as [1] [2]:

$$\zeta(z)=\sum_{1}^{\infty}\frac{1}{n^{z}} \tag{1}$$

- The zeta function plays an important role in mathematical research. It constitutes a first link between arithmetic and analysis. It was used by Euler, Dirichlet, Tchebychev and Riemann to study the distribution of prime numbers.
- The Riemann zeta function and the Dirichlet L-functions are powerful analytical tools for studying the distribution of prime numbers. It seems that these functions are also revealing of the most hidden properties of number theory. They are far from being well understood!
- In 1737, Leonhard Euler (1707-1783) studies the zeta function and discovers Euler's identity between prime numbers and integers.
- In 1900, David Hilbert (1862-1943) places the Riemann hypothesis among the great mathematical challenges of the 20th century
- Since 1920, Number theory and algebraic geometry tend to be unified. These functions are perhaps only the fragmentary elements of a more general theory to be discovered. Dedekind generalized these functions and relations to integer ideals and prime ideals.
- The German Edmund Landau assumed the Riemann conjecture to be true, and showed that a large number of conclusions would be drawn from it.
- The study of the complex zeta function of Riemann shows that it passes through the value zero. It exists:

- trivial (uninteresting) zeros like −2, −4, 6 … and
- Particular zeros which seem to line up on the line of the real $\frac{1}{2}$.

- The Riemann hypothesis or conjecture consists in asserting that all non-trivial zeros are on this line $\frac{1}{2}$ [3]. Novely, Mercedes Orús-Lacort and al has been performed a detailed analysis of Riemann's hypothesis, dealing with the zeros of the analytically-extended zeta function [4].
- The complex issue of the Riemann's Hypothesis and ultimately its elementary proof was explained by Jan Feliksiak [5], the numerically and computationally provable was been provided by Suhaas Pediredla [6]. We are going to proove that all zeros are on the line $\frac{1}{2}$ with the first is in $\frac{1}{2}+i.14.13$. And all imaginary values are irrational [7].

The gap in all these researches is found in their accurates, that we are going to perform in this work.

Firstly we are going to take a sample of particular zeros that the are going to increase gradually it number and establishing a function that accurate the whole and finally prooving that this function is always irrational.

2. Methods

Let variable $X\in\mathbb{Z}$ in which each element x_i the index of Riemann zeta function non trivial zero y_i element of variable $Y \in \mathbb{R} \setminus \{\mathbb{Q}\}$ the Riemann zeta function non trivial zero, as $(x_1, y_1), \cdots, (x_i, y_i)$ [8] [9] [10]. We wish to fit the model for $200 < data < 10^{22}$

$$Y = \xi_0 + \xi_1 X + \xi_2 X^2 + \cdots + \xi_n X^n + \varepsilon \tag{2}$$

where $\mathbb{E}[\varepsilon \mid X = x] = 0$, $Var[\varepsilon \mid X = x] = \sigma^2$, and ε is uncorrelated across measurements. The sum of squares for n known data points is given by:

$$RSS = \sum_{i=0}^{n} \left[y_i - \left(\xi_0 + \xi_1 x_i + \xi_2 x_i^2 + \cdots + \xi_n x_i^n \right) \right] \tag{3}$$

As you can see we have n + 1 coefficients $\xi_{i \in \mathbb{N}}$ in the equation. Partials derivate of a is given by:

$$\frac{\partial RSS}{\partial \xi_i} = \sum_{i=0}^{n} \left[2\left(\xi_0 + \xi_1 x_i + \xi_2 x_i^2 + \cdots + \xi_n x_i^n - y_i \right) \frac{\partial \left(\xi_0 + \xi_1 x_i + \xi_2 x_i^2 + \cdots + \xi_n x_i^n - y_i \right)}{\partial \xi_i} \right] \tag{4}$$

Partial derivate of ξ_i is given as:

$$\frac{\partial RSS}{\partial \xi_i} = 2\sum_{i=0}^{n} x_i^i \left(\xi_0 + \xi_1 x_i + \xi_2 x_i^2 + \cdots + \xi_n x_i^n - y_i \right) \tag{5}$$

Now to find the minima, we will set the partial derivatives to 0.

$$\sum_{i=0}^{n} x_i^i \left(\xi_0 + \xi_1 x_i + \xi_2 x_i^2 + \cdots + \xi_n x_i^n - y_i \right) = 0 \tag{6}$$

we get a generalized matrix [11] [12] [13]:

$$\begin{pmatrix} \sum_{i=0}^{n} x_i^{2n} & \cdots & \sum_{i=0}^{n} x_i^{n+2} & \sum_{i=0}^{n} x_i^{n+1} & \sum_{i=0}^{n} x_i^{n} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \sum_{i=0}^{n} x_i^{n+2} & \cdots & \sum_{i=0}^{n} x_i^{4} & \sum_{i=0}^{n} x_i^{3} & \sum_{i=0}^{n} x_i^{2} \\ \sum_{i=0}^{n} x_i^{n+1} & \cdots & \sum_{i=0}^{n} x_i^{3} & \sum_{i=0}^{n} x_i^{2} & \sum_{i=0}^{n} x_i \\ \sum_{i=0}^{n} x_i^{n} & \cdots & \sum_{i=0}^{n} x_i^{2} & \sum_{i=0}^{n} x_i & n \end{pmatrix} \times \begin{bmatrix} \xi_0 \\ \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{n} y_i x_i^n \\ \vdots \\ \sum_{i=0}^{n} y_i x_i^2 \\ \sum_{i=0}^{n} y_i x_i \\ \sum_{i=0}^{n} y_i \end{bmatrix}$$

An overdetermined system is solved by first creating a residual function, summing the square of the residual which forms a parabola/paraboloid, and then finding the coefficients by finding the minimum of the parabola/paraboloid using partial derivatives. It give that since the set $\mathbb{Z}$ is the infinity countable numbers, every n th non-trivial zero of the Riemann zeta function exists, which means that all non-trivial zeros lie on the line of real $\frac{1}{2}$ of the riemann zeta. We are going now to proove that for nth no-trivial zero existing, this zero is $\in \mathbb{R} \setminus \{\mathbb{Q}\}$.

For this we are going to calculate all polynomial coefficients in prooved that at least these one of all is real [14] [15].

3. Results and Discussion

Frisly we are going to compute a Riemann Zéta Non trivial zeros number data with $dim(data) = \tau < \infty$ by using this code:

```
import numpy as np
import matplotlib.pyplot as plt
#The next library contains the zeta(), zetazero(),and siegelz() functions from
mpmath import *
mp.dps = 25; mp.pretty = True
D=[]
def graph_zeta(real, image_name):
    A,B,C = [], [], []
    for i in np.arange(0.1, , 0.1):
        function = zeta(real + 1j*i)
        function1 = siegelz(i)
        A.append(abs(function))
        B.append(function1)
        C.append(i)
    return A,B,C
A,B,C=graph_zeta(0.5, "Z(t)_Plot.png")
fig = plt.figure()
ax = fig.add_subplot(111)
ax.grid(True)
ax.plot(C,A,label='modulus of Riemann zeta function along critical line, s =
1/2 + it', lw=0.8)
ax.plot(C,B, label='Riemann-Siegel Z-function, Z(t)', lw=0.8)
ax.set_title("Riemann Zeta function - re(s)=1/2")
ax.set_ylabel("Z(t)")
ax.set_xlabel("t")
D.append(zero.imag)
#Include legend
leg = ax.legend(shadow=True)
#Edit font size of legend to make it fit into chart
for t in leg.get_texts():
    t.set_fontsize('small')
#Edit the line width in the legend
for l in leg.get_lines():
    l.set_linewidth(2.0)
#Plot the zeroes of zeta
for i in range(1, \tau):
    zero = zetazero(i)
    ax.plot(zero.imag, [0.0], "ro")
#save plot and print that it was saved
ax.set_ylim(-7, 7)
plt.savefig("Z(t)_Plot.png")
print("Successfully plotted %s !" % "Z(t)_Plot.png")
show
```

Let's begining with $\tau = 2000$, Figure 1.

Here is a code to save data:

```
D=[] a=np.linspace(1, 100, 2000, endpoint=True)
for i in a:
    zero = zetazero(i)
    D.append(zero.imag)
plt.plot(a,D,'b*',label='Mark')
plt.xlabel('n')
plt.ylabel('zero image')
plt.legend(loc='upper left')
plt.show(
```

An exemple of 2000 data points are been plotted in Figure 2.

Evidently, our study is based on much data as $200 < \tau < 10^{22}$ with $\tau = dim(data)$. Accordingly, evaluating those data points with polynomial regression, we obtain a $\varepsilon = 0.0000000E^0$ residual for polynomial degree $n = 5$ for Riemann Zéta Non trivial zeros data $\tau < \infty$ (Figure 3).

However, we are going to generalize for all existing zeros. Firstly let's define mathematical expression of each coefficient:

$$\begin{pmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} \\ \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} \\ \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} \\ \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} \\ \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i & 5 \end{pmatrix} \times \begin{bmatrix} \xi_0 \\ \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{n} y_i x_i^5 \\ \sum_{i=0}^{n} y_i x_i^4 \\ \sum_{i=0}^{n} y_i x_i^3 \\ \sum_{i=0}^{n} y_i x_i^2 \\ \sum_{i=0}^{n} y_i x_i \\ \sum_{i=0}^{n} y_i \end{bmatrix}$$

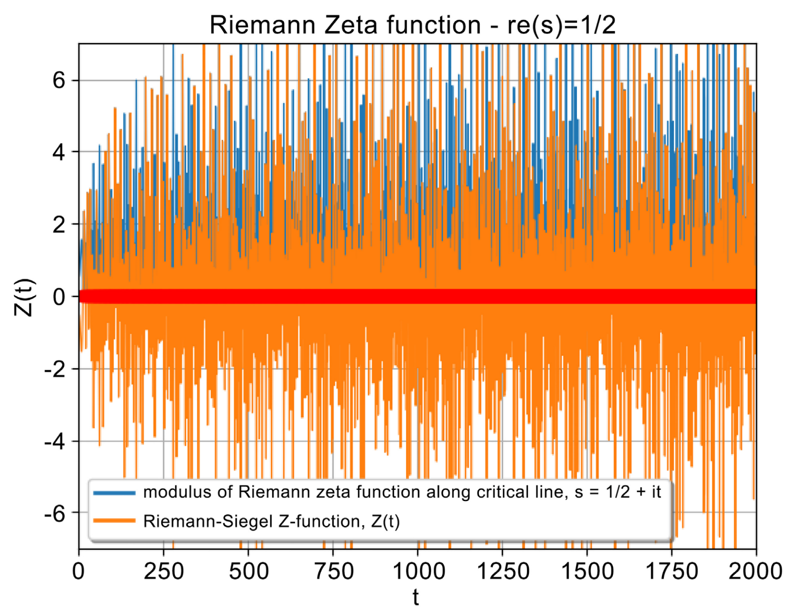

Figure 1. For tau = 2000 data points (non trivial zero in red).

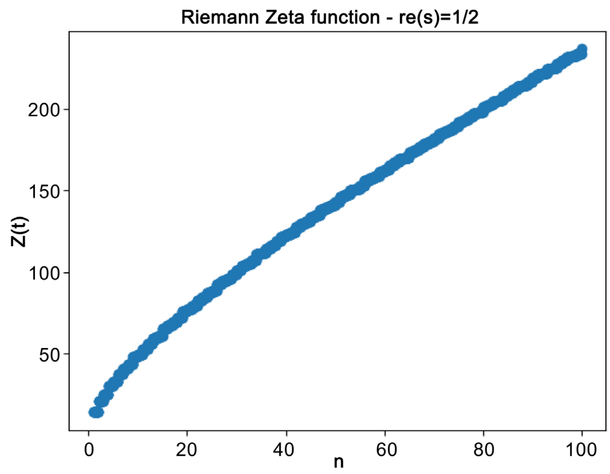

Figure 2. An exemple of 2000 data points plotted.

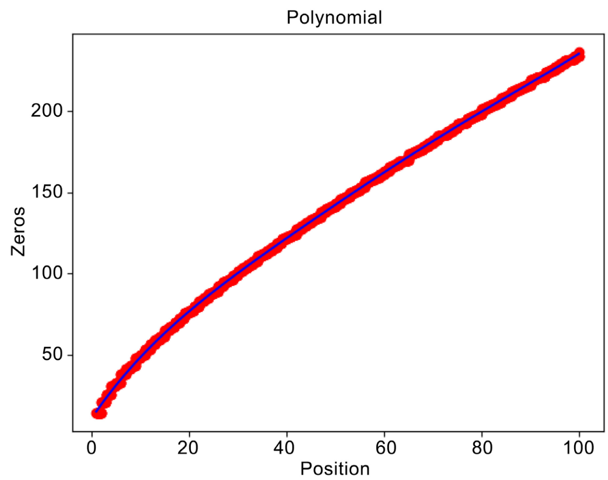

Figure 3. An exemple of one Simulation.

Using Cramer methods we get:

$$\xi_0 = \frac{\begin{vmatrix} \sum_{i=0}^{n} y_i x_i^5 & \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 \\ \sum_{i=0}^{n} y_i x_i^4 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 \\ \sum_{i=0}^{n} y_i x_i^3 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 \\ \sum_{i=0}^{n} y_i x_i^2 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 \\ \sum_{i=0}^{n} y_i x_i & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{n} y_i & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}}{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 \\ \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 \\ \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 \\ \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 \\ \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}} \quad (7)$$

Let's

$$\xi_0 = \frac{A}{W} \tag{8}$$

$$\xi_0 = \frac{\sum_{j=1}^{6}(-1)^{1+j} a_{1,j} det\left(A_{1,j}\right)}{\sum_{j=1}^{6}(-1)^{1+j} w_{1,j} det\left(W_{1,j}\right)} \tag{9}$$

$$\xi_1 = \frac{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{n} y_i x_i^5 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 \\ \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{n} y_i x_i^4 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 \\ \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{n} y_i x_i^3 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 \\ \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{n} y_i x_i^2 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 \\ \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{n} y_i x_i & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{n} y_i & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}}{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 \\ \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 \\ \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 \\ \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 \\ \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}} \tag{10}$$

Let's

$$\xi_1 = \frac{B}{W} \tag{11}$$

$$\xi_1 = \frac{\sum_{j=1}^{6}(-1)^{1+j} b_{1,j} det\left(B_{1,j}\right)}{\sum_{j=1}^{6}(-1)^{1+j} w_{1,j} det\left(W_{1,j}\right)} \tag{12}$$

$$\xi_2 = \frac{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{n} y_i x_i^5 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 \\ \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{n} y_i x_i^4 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 \\ \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{n} y_i x_i^3 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 \\ \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{n} y_i x_i^2 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 \\ \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{n} y_i x_i & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{n} y_i & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}}{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 \\ \sum_{i=0}^{5} x_i^9 & \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 \\ \sum_{i=0}^{5} x_i^8 & \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 \\ \sum_{i=0}^{5} x_i^7 & \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 \\ \sum_{i=0}^{5} x_i^6 & \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^5 & \sum_{i=0}^{5} x_i^4 & \sum_{i=0}^{5} x_i^3 & \sum_{i=0}^{5} x_i^2 & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}} \tag{13}$$

Let's

$$\xi_2 = \frac{C}{W} \tag{14}$$

$$\xi_2 = \frac{\sum_{j=1}^{6} (-1)^{1+j} c_{1,j} det(C_{1,j})}{\sum_{j=1}^{6} (-1)^{1+j} w_{1,j} det(W_{1,j})} \tag{15}$$

$$\xi_3 = \frac{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{n} y_i x_i^{5} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} \\ \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{n} y_i x_i^{4} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} \\ \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{n} y_i x_i^{3} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} \\ \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{n} y_i x_i^{2} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} \\ \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{n} y_i x_i & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}}{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} \\ \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} \\ \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} \\ \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} \\ \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}} \tag{16}$$

Let's

$$\xi_3 = \frac{D}{W} \tag{17}$$

$$\xi_3 = \frac{\sum_{j=1}^{6} (-1)^{1+j} d_{1,j} det(D_{1,j})}{\sum_{j=1}^{6} (-1)^{1+j} w_{1,j} det(W_{1,j})} \tag{18}$$

$$\xi_4 = \frac{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{n} y_i x_i^{5} & \sum_{i=0}^{5} x_i^{5} \\ \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{n} y_i x_i^{4} & \sum_{i=0}^{5} x_i^{4} \\ \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{n} y_i x_i^{3} & \sum_{i=0}^{5} x_i^{3} \\ \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{n} y_i x_i^{2} & \sum_{i=0}^{5} x_i^{2} \\ \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{n} y_i x_i & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i^{2} & 5 \end{vmatrix}}{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} \\ \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} \\ \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} \\ \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} \\ \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}} \tag{19}$$

Let's

$$\xi_4 = \frac{E}{W} \tag{20}$$

$$\xi_4 = \frac{\sum_{j=1}^{6}(-1)^{1+j} e_{1,j} det\left(E_{1,j}\right)}{\sum_{j=1}^{6}(-1)^{1+j} w_{1,j} det\left(W_{1,j}\right)} \tag{21}$$

$$\xi_5 = \frac{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{n} y_i x_i^{5} \\ \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{n} y_i x_i^{4} \\ \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{n} y_i x_i^{3} & \sum_{i=0}^{5} x_i^{3} \\ \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{n} y_i x_i^{2} \\ \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{n} y_i x_i \\ \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i & \sum_{i=0}^{5} x_i^{2} \end{vmatrix}}{\begin{vmatrix} \sum_{i=0}^{5} x_i^{10} & \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} \\ \sum_{i=0}^{5} x_i^{9} & \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} \\ \sum_{i=0}^{5} x_i^{8} & \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} \\ \sum_{i=0}^{5} x_i^{7} & \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} \\ \sum_{i=0}^{5} x_i^{6} & \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i \\ \sum_{i=0}^{5} x_i^{5} & \sum_{i=0}^{5} x_i^{4} & \sum_{i=0}^{5} x_i^{3} & \sum_{i=0}^{5} x_i^{2} & \sum_{i=0}^{5} x_i & 5 \end{vmatrix}} \tag{22}$$

Let's

$$\xi_5 = \frac{F}{W} \tag{23}$$

$$\xi_5 = \frac{\sum_{j=1}^{6}(-1)^{1+j} f_{1,j} det\left(F_{1,j}\right)}{\sum_{j=1}^{6}(-1)^{1+j} w_{1,j} det\left(W_{1,j}\right)} \tag{24}$$

Evidently each $\xi_{i\in[0,1,2,3,4,5]}$ varies everytime the τ increase. We must then establish for each $\xi_{i\in[0,1,2,3,4,5]}$ a function describing it evolution. By increasing τ value, consequently,

ξ_0 coefficient is described as for $\alpha \in \mathbb{R} \setminus \{\mathbb{Q}\}$:

$$\xi_0 = x\left[1-\exp\left(-\frac{x}{\alpha}\right)\right] \tag{25}$$

ξ_1 coefficient is described as:

$$\xi_1 = x\left[1+\exp\left(-\frac{x}{\alpha}\right)\right] \tag{26}$$

ξ_2 coefficient is described as for $\gamma \in \mathbb{R} \setminus \{\mathbb{Q}\}$:

$$\xi_2 = x\left[1-\exp\left(-\frac{x}{\gamma}\right)\right] \tag{27}$$

ξ_3 coefficient is described:

$$\xi_3 = x\left[1+\exp\left(-\frac{x}{\gamma}\right)\right] \tag{28}$$

ξ_4 coefficient is described as for $\theta \in \mathbb{R} \setminus \{\mathbb{Q}\}$:

$$\xi_4 = x\left[1-\exp\left(-\frac{x}{\theta}\right)\right] \tag{29}$$

For any coefficient, It is define as: $y = x\left[1 - \exp\left(-\frac{x}{\omega}\right)\right]$. Necessarily therefor the best accurate polynomial function for non trivial Riemann zeta function zeros is defined as:

$$P^{\tau}(x) = \sum_{n=0}^{5} x^{n+1}\left[1 + (-1)^{n+1}\exp\left(-\frac{x}{\omega(n) = \omega(n+1) \neq \omega(n+2)}\right)\right] \tag{30}$$

Direct computational analysis verifies that for $x\left[1 + (-1)^{n+1}\exp\left(\frac{x}{\omega(n) = \omega(n+1) \neq \omega(n+2)}\right)\right] > 0$, the residual value increase and the polynomial degree must been decreased for $x\left[1 + (-1)^{n+1}\exp\left(\frac{x}{\omega(n) = \omega(n+1) \neq \omega(n+2)}\right)\right] = 0$, with an existing $\omega(n) \in \mathbb{R} \setminus \{\mathbb{Q}\}$ maintaining residual value constant for $x \in \mathbb{Z} \to \infty$, x being Riemann zeta function non trivial zero index. Consequently $\exists \eta \in \mathbb{R}$ such as $\forall$ $\omega(\eta)$ the polynomial function is generalized as:

$$P(x) = \sum_{0}^{\eta} x^{\eta+1}\left[1 + (-1)^{\eta+1}\exp\left(-\frac{x}{\omega(\eta) = \omega(\eta+1) \neq \omega(n+2)}\right)\right] \tag{31}$$

This polynomial function is defined from $\mathbb{Z}$ to $\mathbb{R} \setminus \{\mathbb{Q}\}$ for any Rieman zeta non trivial zero. That proove that all non trivial zeros are in $Re(s) = \frac{1}{2}$ and the each zero is only in $\mathbb{R} \setminus \{\mathbb{Q}\}$.

4. Conclusion

In this work, we have presented a method for solving the Riemann hypothesis conjecture. We began our study by computing and saving Riemann's zeta function non trivial zeros, then we fit each data points and studied the polynomial coefficients variations maintaining the best accurate constant and finished by solving the Riemann hypothesis conjecture. This work is one best demonstration of the validity of Riemman's conjecture. As such, we have proven here that the conjecture is true, up to the best of our numerical analysis. The demonstration of the work is important as it allows Riemann's zeta function to be a model function in the Dirichlet series theory and be at the crossroads of many other theories.

Scope of Future Work

The present investigation will be very helpful to the researchers who are engaged for those area research works in earth and in Universe [16] [17] [18] [19].

1) Mathematics. This is so because any number, when broken down into its factors at the end, can be defined as the multiplication of prime numbers.

2) Music, all the structures of tonal music like chords, scales, harmony, modality, it is possible to represent them all mathematically as rational structures of

prime numbers.

3) Nature, many interesting examples of prime numbers exist because some insects like cicadas only emerge from the underground habitat after the prime number of years, such as 17 years. Often flowers have an odd number of petals, and most often these are prime numbers. For example, five is a commonly found number of petals in flowers.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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List of Symbols

$P(x)$: Polynomial function;

$\zeta(z)$: Riemann's Zéta function;

z: Complex number;

RSS: Squares sum of n known data points;

$\mathbb{Z}$: Integers;

$\mathbb{Q}$: Rational Numbers;

$\mathbb{R}$: Real Numbers;

ε: Uncorrelated across measurements;

τ: Riemann Zéta Non trivial zeros number;

X: The index of Riemann zeta function non trivial zero numbers;

Y: The Riemann zeta function non trivial zero numbers;

var: The Variance;

∞: Infinity;

E: Exponent;

ξ_i: Polynomial coefficients;

$\sum$: Sum;

α: Parameter;

γ: Parameter;

θ: Parameter;

ω: Parameter;

$Re(\ )$: Real part;

∂: Partials derivate;

$\mathbb{E}$: expectation probability.