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Three Dimensional Electric Circuits with Multiple Capacitors and Resistors

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Abstract

Two cubical 3D electric circuits with single and double capacitors and twelve ohmic resistors are considered. The resistors are the sides of the cube. The circuit is fed with a single internal emf. The charge on the capacitor(s) and the current distributions of all twelve sides of the circuit(s) vs. time are evaluated. The analysis requires solving twelve differential-algebraic intertwined symbolic equations. This is accomplished by applying a Computer Algebra System (CAS), specifically *Mathematica*. The needed codes are included. For a set of values assigned to the elements, the numeric results are depicted.

Keywords

3D Electric Circuits, Capacitors, Multiple Resistors, Differential-Algebraic Equations, Computer Algebra System, *Mathematica*

1. Introduction

The motivation for considering this project stems from our previous work [1]. In short [1], the issues of interest for an electric circuit composed of twelve ohmic resistors forming the sides of a 3D cube are addressed. For instance, 1) all seven equivalent resistors of the circuit symbolically were identified and 2) for a chosen set of numeric values assigned to the resistors, the current distributions were graphed. A point of clarification: the considered circuit(s) is an actual three-dimensional structure and does not need to be confused vs. a 3D representation of a 2D circuit customarily constructed with simulation programs e.g. (Lab Report) [2]. The current project in addition to including the twelve resistors embodies one, two, and potentially more capacitors, and the driver instead of being an external is an internal DC emf. The analysis paves the research road generalizing the issues by including numerous emfs with desired polarities. The former scenario [1] ought to be considered the simplest 3D circuit with its challenges. Irrespective of the detailed specifics of the case on hand, the solutions hinge heavily on the employment of a Computer Algebra System (CAS), namely *Mathematica*. To focus on the issue at hand see **Figure 1**.

This report embodies four sections. In addition to Section 1, Introduction, in Section 2, the needed equations conducive to charge and current distributions in all twelve sides of the cube are identified. Section 3 deals with the second 3D circuit leading to the identification of the currents. Section 4 is the Conclusion and Discussions embodying suggestions for generalizations.

Noting again the circuit depicted in **Figure 1** is an actual 3D structure, *i.e.* the design of the shown circuit deviates from the norm; this is not a 3D profile of a 2D circuit. A literature search [2] reveals the shown design is a fresh creation and is not addressed yet. The mentioned reference has a module converting a 2D circuit into 3D, this is not the intent of the project in hand. Comparing to [1], one realizes there are three major differences: 1) there is no outside feeder current, so the issue of the equivalent resistor, R_{eq}, is irrelevant, and that 2) an internal emf drives the circuit and 3) the circuit embodies a capacitor. As one may imagine, even the shown design with the embodied elements may be restructured by rearranging the elements e.g. by separating the capacitor from the emf and then inserting them separately on the different sides. This would change the character of the circuit, namely would change the current distributions, etc. In other words, since we may not consider all these interesting scenarios in one report, we rather focus only on one. However, our analysis provides clues for generalizations.

Nonetheless, the analysis hinges on applications of limited principles. These are Kirchhoff's laws, specifically, the nodal and loop laws. Paraphrasing the laws, they state: the node law, the algebraic sum of the currents at any node is null. And the loop law indicates in a closed circuit, the algebraic sum of the emfs equals the algebraic ohmic voltage drops across the resistors [3].

Although on the face, the mentioned laws should solve the issues on hand, namely the identification of the currents and charge on the capacitor, because the charge on the capacitor and the capacitor's feeder current differential-wise related to the set of governing equations instead of being coupled algebraic equations are differential-algebraic equations. The issue becomes more complicated because as shown in **Figure 1**, the eight nodes of the cube and all twelve currents are intertwined. The symbolic solution to the problem poses a mathematical challenge. That is why the solution calls for the deployment of a CAS. This is addressed in the next section.

With this introduction we processed, first, by applying Kirchhoff's laws, we form the needed equations. For the mentioned reasons, since there are eight corners and twelve sides to the cube, we foresee twelve coupled equations. However, because the master capacitor's equation is $q = (\Delta V)\ C$ with q being the charge on the capacitor, ΔV, the voltage across the cap, and because the diver current is related to the charge via $i = \mathrm{d}q/\mathrm{d}t$, the nodal law leads to differential equations. In short, the twelve equations become differential-algebraic intertwined

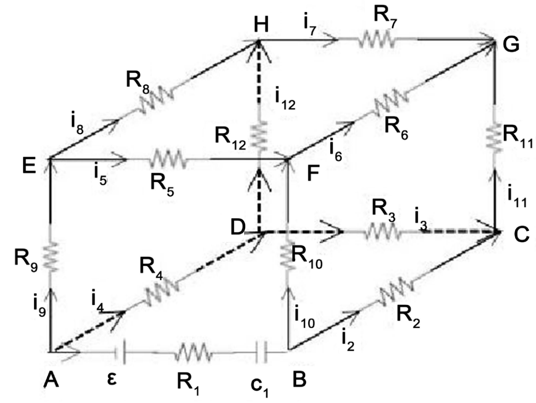

Figure 1. A 3D cubic electric circuit embodies one emf, one capacitor, and twelve ohmic resistors.

equations. We solve the equations by applying *Mathematica* [4]. The solutions are symbolic and for a set of assigned numeric values, the results are depicted graphically.

2. Applied Kirchhoff's Laws

All the resistors and associated currents are labeled as shown in **Figure 1**. The side $\overline{\mathrm{AB}}$ includes a single emf, ϵ, a capacitor, C and a resistor, R. Because of the character of the capacitor that deals with the charge rather than the current Kirchhoff's node law ought to be reformatted by underlying the conservation of charge rather than the currents at the nodes. Therefore, the nodal law for the circuit on hand applies not to the conservation of current but rather to the conservation of charge. These are embodied in (1).

$$\begin{cases} eq1 = q_1(t) + q_4(t) + q_9(t) = 0 \\ eq2 = q_1(t) - q_2(t) - q_{10}(t) = 0 \\ eq3 = q_2(t) + q_3(t) - q_{11}(t) = 0 \\ eq4 = q_4(t) - q_3(t) - q_{12}(t) = 0 \\ eq5 = q_9(t) - q_5(t) - q_8(t) = 0 \\ eq6 = q_5(t) + q_{10}(t) - q_6(t) = 0 \\ eq8 = q_8(t) + q_{12}(t) - q_7(t) = 0 \end{cases} \tag{1}$$

Applied Kirchhoff's Laws to Circuit Shown in Figure 3

By the same token, the applied loop law results in the set of Equation (2).

$$\begin{cases} eq9 = \epsilon - R_1 q_1'(t) - \dfrac{1}{c_1} q_1(t) - R_{10} q_{10}'(t) + R_5 q_5'(t) + R_9 q_9'(t) = 0 \\ eq10 = R_3 q_3'(t) + R_{11} q_{11}'(t) - R_7 q_7'(t) - R_{12} q_{12}'(t) = 0 \\ eq11 = R_2 q_2'(t) + R_{11} q_{11}'(t) - R_6 q_6'(t) - R_{10} q_{10}'(t) = 0 \\ eq12 = R_4 q_4'(t) + R_{12} q_{12}'(t) - R_8 q_8'(t) - R_9 q_9'(t) = 0 \\ eq13 = \epsilon - R_1 q_1'(t) - \dfrac{1}{C_1} q_1(t) - R_2 q_2'(t) + R_3 q_3'(t) + R_4 q_4'(t) = 0 \end{cases} \tag{2}$$

In (2), primes mean derivative w/time.

The set of Equations (1) and (2) contains twelve coupled differential-algebraic equations. Solving these equations for the unknowns $q_n[t]$ with the initial conditions $q_n[t = 0] = 0$ and the chosen values of R_n for $n = 1, 2, \cdots, 12$ yield $q_n[t]$ and the associated currents $i_n[t]$.

Applying *Mathematica* **DSolve**, we solve the equations symbolically. The output fills pages leading to no insight. On the other hand, we select a set of numeric values for the emf, capacitor, and resistors. For the case of twelve identical resistors, and $\{\epsilon, c, R\} = \{10.\ \text{V}, 1.0\ \text{F}, 1.0\ \Omega\}$, noting $c_1 = c$, and by applying **NDSolve** the deduced numeric solutions are displayed in twelve plates in **Figure 2**.

```
solqEqualR=NDSolve[{eq1,eq2,eq3,eq4,eq5,eq6,eq8,eq9,eq10,eq11,eq12,eq13,q1[0]==0,q2[0]==0,q3[0]==0,q4[0]==0,q5[0]==0,q6[0]==0,q7[0]==0,q8[0],q9[0]==0,q10[0]==0,q11[0]==0,q12[0]==0}/.{R1->R,R2->R,R3->R,R4->R,R5->R,R6->R,R7->R,R8->R,R9->R,R10->R,R11->R,R12->R}/.{ϵ->10.,c->1.,R->1.0},{q1,q2,q3,q4,q5,q6,q7,q8,q9,q10,q11,q12},{t,0,10}]        (3)
```

As shown the capacitor is being charged and the feeder current, the red curve exponentially fades away as expected. This is expected and is referred to as the "gold standard".

Each plate in **Figure 2** contains two curves, a blue and a red. The blue curves are the time-dependent charge, and the red curves are their slopes, *i.e.* the currents. For instance, the first plate confirms the accuracy of the solution, *i.e.* this represents the "gold standard" of a charging RC-series circuit. The charge accumulates gradually and the feeder current fades out. The feature is true for all the twelve sides, although the "sign" of the charge and the direction of the current in some cases, e.g. Plate 3, is opposite of Plate 1. Meaning in an actual circuit current runs in the opposite direction. The character of the shown curves in **Figure 2** does make physics sense although without providing our results they would not have been predictable. Note also each frame shows all the currents fade as expected.

There are general variations to the presented analysis. 1) With the above- mentioned code we may simulate: a) for any desired set of chosen non-identical resistors and b) for any chosen value of the capacitor. The former is completed but not included in this report. 2) The RC may be inserted in any one of the sides of the circuit. This will change the characters in all twelve plates in **Figure 2**, expecting no extraordinary surprises. 3) The three elements in the AB side, namely, ϵ, R, and C may be separated, and each element may be inserted on different sides of the cube! …there are many possibilities. As a potential research project, the author wishes to leave the investigation to the interest of the reader. To investigate the latter, one may tweak the given (1), (2) and code (3) expecting no major code alterations to produce results.

Another topic of interest: the individual who is reading this report and is familiar with the "gold standard" of the *RC* circuit maybe interested in asking

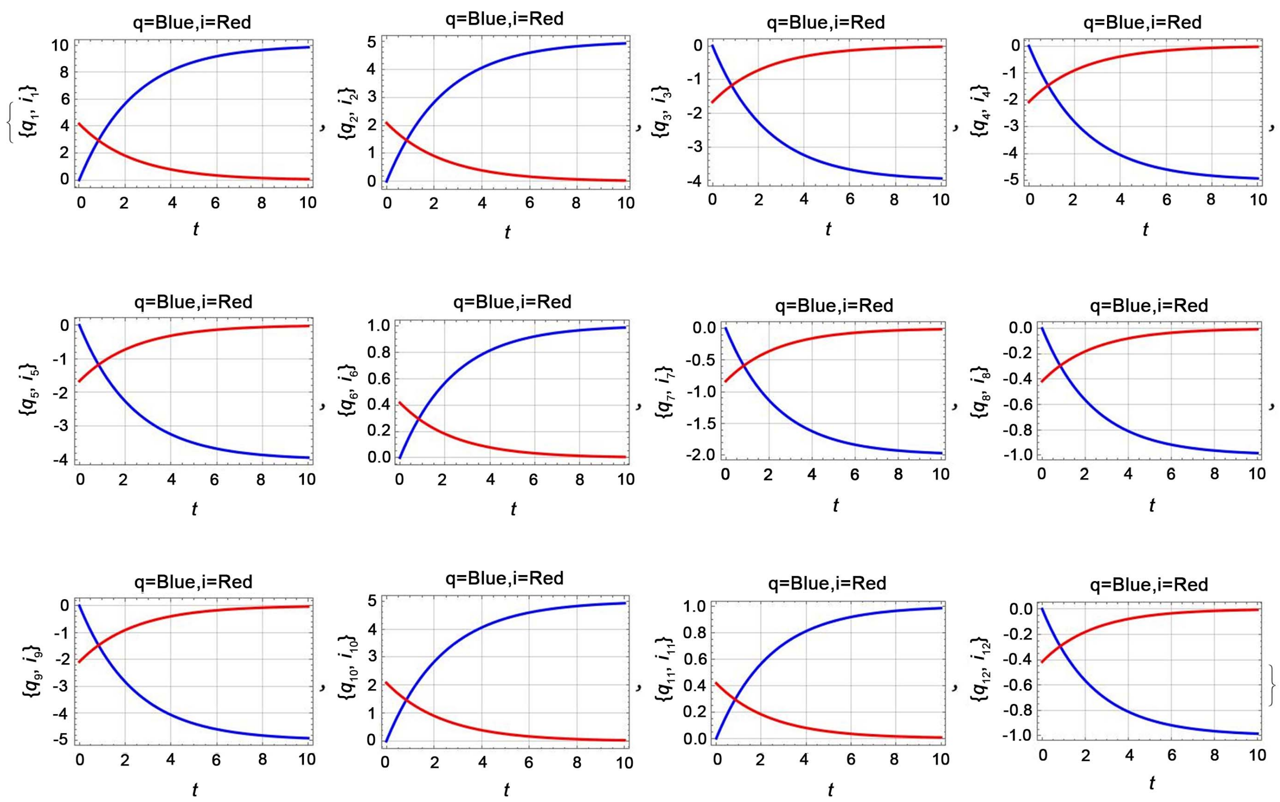

Figure 2. The charge and their associated currents in all 12 sides of the cube in **Figure 1**.

about the "effective" resistance of the resistor in the *RC* circuit of the AB side in **Figure 1**! Although all the twelve resistors are set to unity, $R = 1.0\ \Omega$ the value of that resistor in the *RC* cannot be 1.0 Ω. This is because the other eleven resistors through the cubic network alter its value. To find its effective value we use the red colored current in Plate 1 in **Figure 2**. The abscissa of the intersection of a horizontal line with the ordinate of the 1/eth value of the maximum current is 2.67 s. Because this condition requires $t/RC = 1$ for $C = 1.0$ F this yields $R = 2.67$ Ω. In other words, a cube composed of 12 identical 1.0 Ω resistors makes the resistor that is in series with the 1.0 F capacitor 2.67 Ω.

3. An Alternate 3D Circuit Design

Motivated with the progress made in Section 2, we designed a circuit shown in **Figure 3**.

As shown this circuit is different from the one shown in **Figure 1**. This one includes two capacitors inserted on two different sides of the cube. Notice, there is no specific reason for our choice, the second capacitor may be interested in any of the eleven sides. Here we report the result of the shown configuration. An interested reader may exercise I to place the second capacitor in any of the eleven sides II to insert non-equal resistors and capacitors and III to insert more than one capacitor in the sides of the cube. In other words, many cases yield to different scenarios. Here we report our findings for the circuit shown in **Figure 3**.

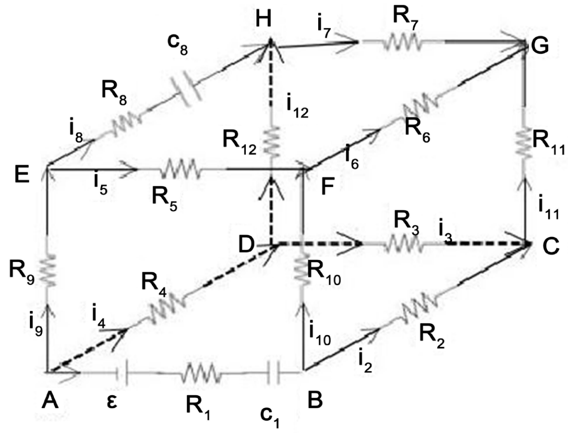

Figure 3. A 3D cubic circuit with two capacitors and twelve resistors.

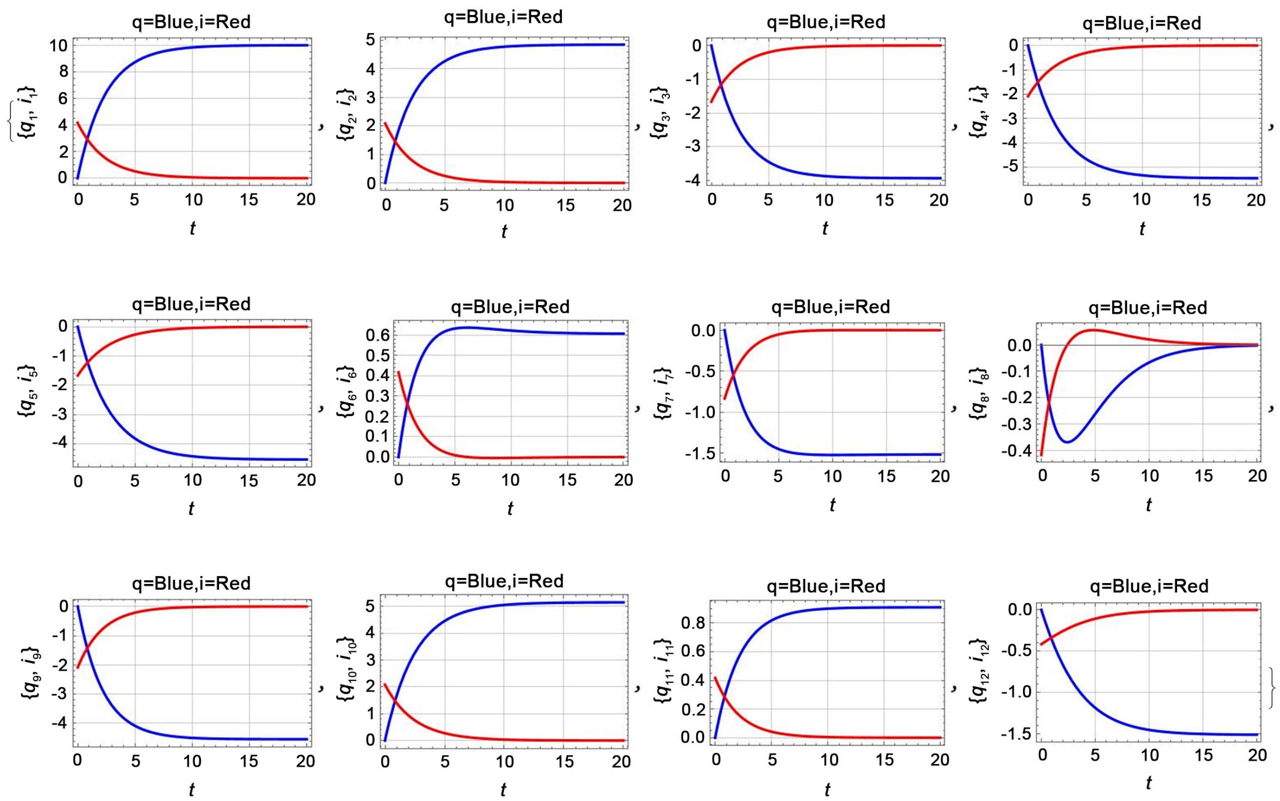

Figure 4. The charge and their associated currents in all 12 sides of the cube in Figure 3.

To so doing, we replace $eq12$ of (2) with $eq12$ of (4). The mentioned replacement with (1) and modified (2) leads to the needed equations. The solutions, $q(t)$, and the associated currents, $i(t) = \mathrm{d}q(t)/\mathrm{d}t$, noticing $c_8 = c_1 = c = 1.0$ F, yield the depicted plates in Figure 3.

$$eq12 = R_4 q_4'(t) + R_{12} q_{12}'(t) - \frac{1}{C_8} q_8(t) - R_8 q_8'(t) - R_9 q_9'(t) = 0 \tag{4}$$

One notices the characters of the charges and the currents in Figure 2 and Figure 4 are different. The time of the latter is extended to 20 s revealing the asymptote feature. Despite their differences, they share a common feature, all the currents as expected over time plunge to zero as they should. Among all twelve cases, plates #6 and #8 are the peculiar ones. Especially, plate #8 shows the capacitor c8 starts uncharged after going through charging it loses the accumulated charge ending up with a discharged status. During this process, since there is more than one feeder current at the end of the shown time, the overall current vanishes.

4. Conclusion and Comments

The motivation for crafting this report stems from thinking about the electric circuits that are different from what is in the literature. We designed a variety of circuits and suggested variations conducive to potential generalization. All these are different from the traditional 2D circuits. In our previous work [1], we considered a cubic circuit with "only" twelve resistors driven by an exterior source. Here, in this report, we considered an actual 3D cubic structure circuit with twelve resistors embodying one and then two capacitors driven by an internal power supply. For the latter, we discovered previously not reported current distributions. As an extension, one may consider a similar but generalized circuit by including ten more capacitors tailed to each of the resistors in **Figure 3**. The analysis becomes more challenging but doable. The current report and its potential suggested generalization add to the body of knowledge. For *Mathematica* codes, the interested reader would find [5] [6] [7] resourceful.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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