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3D_Multi Resistor Electric Circuit

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Abstract

This report addresses the issues concerning the analysis of an electric circuit composed of multiple resistors configured in a 3-Dimension structure. Noting, *all* the standard textbooks of physics and engineering irrespective of the used components are circuits assembled in two dimensions. Here, by deviating from the "norm" we consider a case where the resistors are arranged in a 3D structure; e.g., a cube. Although, independent of the dimension of the design the same physics principles apply, transitioning from a 2D to a 3D makes the corresponding analysis considerably challenging. In general, with no exception, depending on the used components the analysis faces with solving a set of either algebraic or differential-algebraic equations. Practically, this interfaces with a Computer Algebra System (CAS). The main objective is symbolically to identify the current distributions and the equivalent resistor (s) of cubically assembled resistors.

Keywords

3D Electric Circuit, Equivalent Resistor, Computer Algebra System, *Mathematica*

1. Introduction

As noted, the main objective of this report is to learn about the various aspects of electric systems in a 3D structure. For the sake of simplicity, a cube is considered with sides embodying twelve electric resistors. Shortly, the lesson learned is used in circuits embodying elements other than resistors. For the time being, the case on hand is limited to twelve different-valued ohmic resistors. One of the objectives is to seek an equivalent resistor (s), R_{eq}. E.g., to identify one resistor to replace the "cubic resistor" containing all twelve resistors in a circuit, R_{eq}, without altering the feeder and departing currents. See **Figure 1** for a better understanding of the issue. For simple 2D circuits, this is a classic question with a well-known answer [1] [2]. A literature search shows no answer is available for a 3D case. This report answers. The answer is not unique because the R_{eq} has a value depending on the contact points of the feeder and the departing current. Since the cube has eight corners, even by selecting the feeder's vertex the current can depart at any of the other seven vertices; providing seven pairs associated with seven different R_{eq}, see **Figure 1** again. This report with the help of a CAS, specifically *Mathematica* [3] generates all the seven *symbolic* results. Meaning without assigning numeric values to the twelve resistors we provide a symbolic expression for all R_{eq}. But then of course the symbolic results are converted to numeric. While deriving the needed information we also were able to derive symbolic expressions for the currents in each of the twelve resistors. Hence, we generate information about the distribution of these currents. Graphs of these distributions for chosen numeric values of the currents are shown.

With these objectives, we craft a report answering these questions. The report is composed of three sections. In Section 2 the specifics of a 3-Dimensional case are presented. It contains the basic needed physics principles with associated *Mathematica* codes. The solution of the equations on hand symbolically and numerically is given.

The *symbolic* word intentionally is italicized emphasizing the power of *Mathematica codes to* solving the equations and at the end numeric are giving as well; noting *Mathematica* is not an Excel [4].

And finally, the 3rd Sect. is the conclusion and we outlined the lesson learned and the future-plan.

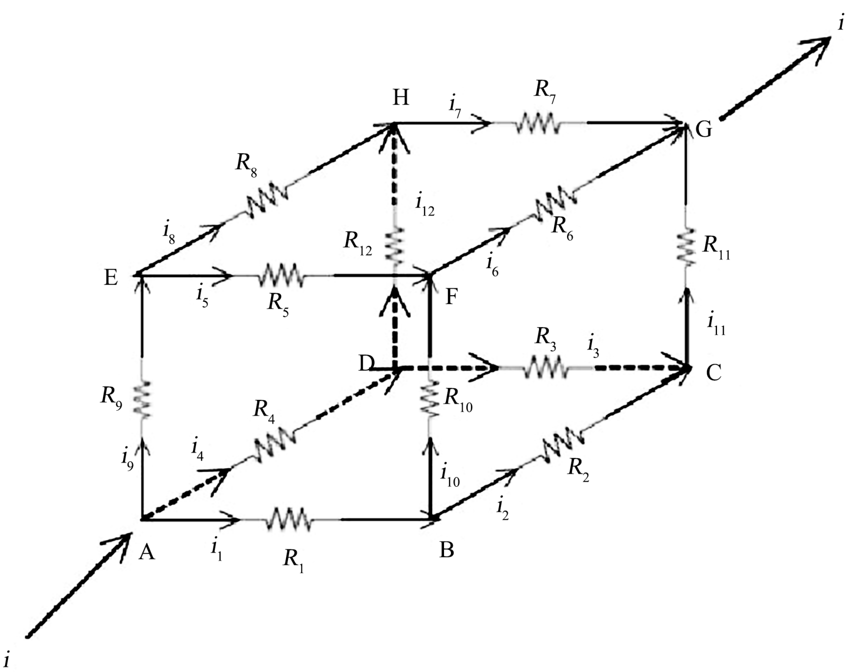

Figure 1. Twelve different-valued resistors formed the cube are labeled R_n with n = 1, 2, …, 12. The cube is fed with an incoming current i at vertex A leaving the structure at vertex G.

2. A 3-Dimensional Cubic Circuit Embodying 12 Different Symbolic Ohmic Resistors

The 3D circuit of interest is shown in **Figure 1**. It is composed of twelve different-valued symbolic ohmic resistors. The resistors are the sides of the cube. Objectively one might ask why a cube and not any other 3D-shaped structure. This is addressed shortly in the upcoming report. As shown in **Figure 1**, the cube is being fed with a current at one of the selected vertices, e.g., vertex **A**, it could depart the structure at any one of the seven leftover vertices, e.g., vertex **G**. As pointed out, the interest is to replace the twelve resistors with one unique resistor, R_{eq}, without altering the incoming and the outgoing currents. Since anyone of the seven vertices leads to a different R_{eq}, seven different R_{eq}s are calculated.

Irrespective of the chosen pair, one needs to apply Kirchhoff's laws; the nodal and the loop laws to generate adequate information addressing the issue of interest. In short, the nodal law states the *net* current at any vertex is null, and the loop law says in any closed loop the net emf equals the net voltage drop across the ohmic resistors. Although these laws are sound, in practice they lead to multiple-coupled linear equations that are cumbersome to solve numerically let alone *symbolically*! Here is where the project is tied to a CAS, specifically *Mathematica*.

By applying the mentioned laws, we form the needed equations.

Analysis

The currents split along the sides of the cube, applying the nodal law this gives eight separates but intertwined linear equations. The currents are labeled i_n as shown in **Figure 1**. Conventionally, the incoming currents are positive, and the outgoing ones are negative. The nodes are labeled and shown in **Figure 1**.

$$\text{node A} = i - i_1 - i_4 - i_9\,;$$

$$\text{node B} = i_1 - i_2 - i_{10}\,;$$

$$\text{node C} = i_2 + i_3 - i_{11}\,;$$

$$\text{node D} = i_4 - i_3 - i_{12}\,;$$

$$\text{node E} = i_9 - i_5 - i_8\,;$$

$$\text{node F} = i_5 + i_{10} - i_6\,;$$

$$\text{node G} = i_6 + i_7 + i_{11}\,;$$

$$\text{node H} = i_8 + i_{12} - i_7\,;$$

Applying the loop law gives another set of needed intertwined equations. The loops are labeled and shown in **Figure 1**.

$$\text{loop ABFEA} = R_1 i_1 + R_{10} i_{10} - R_5 i_5 - R_9 i_9\,;$$

$$\text{loop DCGHD} = R_3 i_3 + R_{11} i_{11} - R_7 i_7 - R_{12} i_{12}\,;$$

$$\text{loop BCGFB} = R_2 i_2 + R_{11} i_{11} - R_6 i_6 - R_{10} i_{10}\,;$$

$$\text{loop ADHEA} = R_4 i_4 + R_{12} i_{12} - R_8 i_8 - R_9 i_9\,;$$

$$\text{loop ABCDA} = R_1 i_1 + R_2 i_2 - R_3 i_3 - R_4 i_4\,;$$

With the mentioned set of twelve equations, one might try solving them symbolically. This requires a non-singular 12 × 12 determinant, its matrix is,

$$\begin{pmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ R_1 & 0 & 0 & 0 & -R_5 & 0 & 0 & 0 & -R_9 & R_{10} & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 & -R_7 & 0 & 0 & 0 & R_{11} & -R_{12} \\ 0 & R_2 & 0 & 0 & 0 & -R_6 & 0 & 0 & 0 & -R_{10} & R_{11} & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 & 0 & -R_8 & -R_9 & 0 & 0 & R_{12} \\ R_1 & R_2 & -R_3 & -R_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Applying *Mathematic* the determinant becomes an expression with 384 symbolic terms, containing all kinds of terms. The expression is long, it fills pages. We avoid its display.

However, if one assumes identical resistors with value *R*, the matrix becomes,

$$\begin{pmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ R & 0 & 0 & 0 & -R & 0 & 0 & 0 & -R & R & 0 & 0 \\ 0 & 0 & R & 0 & 0 & 0 & -R & 0 & 0 & 0 & R & -R \\ 0 & R & 0 & 0 & 0 & -R & 0 & 0 & 0 & -R & R & 0 \\ 0 & 0 & 0 & R & 0 & 0 & 0 & -R & -R & 0 & 0 & R \\ R & R & -R & -R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Its determinant reduces to one term only with a value of $-384R^5$. This ensures the existence of a non-zero solution.

Here we would like to comment on solving this set of intertwined linear equations. Solving these equations symbolically is a monumental task. If one assigned numeric value to the resistors the equations easily may be solved with numeric-based software, e.g. even Excel. Here the issue is sought for the solution symbolically. To our surprise, *Mathematica* does the job. As one may imagine the solution is extremely long filling pages, even *Mathematica* output is abbreviated. We avoid showing the result, but noting the problem is solved. Here is the code,

Solve[{nodeA = = 0, nodeB = = 0, nodeC = = 0, nodeD = = 0, nodeE = = 0, nodeF = = 0, nodeH = = 0, loopABFEA = = 0, loopDCGHD = = 0,

loopBCGFB = = 0, loopADHEA = = 0, loopABCDA = = 0}, {i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}, i_{11}, i_{12}}];

To verify the accuracy of the solutions, *i.e.*, the currents, one may compare the output vs. the intuitive expectation. For identical resistors at any corner of the cube, the incoming currents should split evenly amongst the other two sides. The code **EquaRs** sets the resistors equal to $\boldsymbol{R}$. E.g., for $R = 1.0\ \Omega$ we get,

EquaRs = {$R_1 \to R$, $R_2 \to R$, $R_3 \to R$, $R_4 \to R$, $R_5 \to R$, $R_6 \to R$, $R_7 \to R$, $R_8 \to R$, $R_9 \to R$, $R_{10} \to R$, $R_{11} \to R$, $R_{12} \to R$}

The code is given in the next line. The output is the currents labeled i_n for n = 1, 2, …, 12.

soliEqualR = Solve[{nodeA = = 0, nodeB = = 0, nodeC = = 0, nodeD = = 0, nodeE = = 0, nodeF = = 0, nodeH = = 0, loopABFEA = = 0, loopDCGHD = = 0, loopBCGFB = = 0, loopADHEA = = 0, looBCDA = = 0}/.EquaRs, {i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}, i_{11}, i_{12}}]

{{$i_1 \to i/3$, $i_2 \to i/6$, $i_3 \to i/6$, $i_4 \to i/3$, $i_5 \to i/6$, $i_6 \to i/3$, $i_7 \to i/3$, $i_8 \to i/6$, $i_9 \to i/3$, $i_{10} \to i/6$, $i_{11} \to i/3$, $i_{12} \to i/6$}}

As one notices e.g., at vertex A, the incoming current $\boldsymbol{i}$ splits evenly amongst the connected sides, 1, 4, and 9. This is also true at any of the seven vertices. Assuming the numeric value of the feeder current is 1.0 Amp, the value of all twelve currents is shown in **Figure 2**.

In **Figure 2**, the horizontal axis is the Indices associated with the corresponding currents. e.g., 4 on the horizontal axis corresponds to i_4 with the value of 0.33 Amp.

The shown solution should not be confined to equal resistors. Here we consider a case where the numeric values of all resistors are different. The listing **specialNonEqualR** is one such case. Applying the given code in the text, **Solve** gives the numeric values of all twelve currents. The graph of these currents is shown in **Figure 3**. All the currents are associated with the value of the feeder current $\boldsymbol{i}$, as expected. For instance, by setting the $i = 1$ Amp the numeric values are depicted in **Figure 3**.

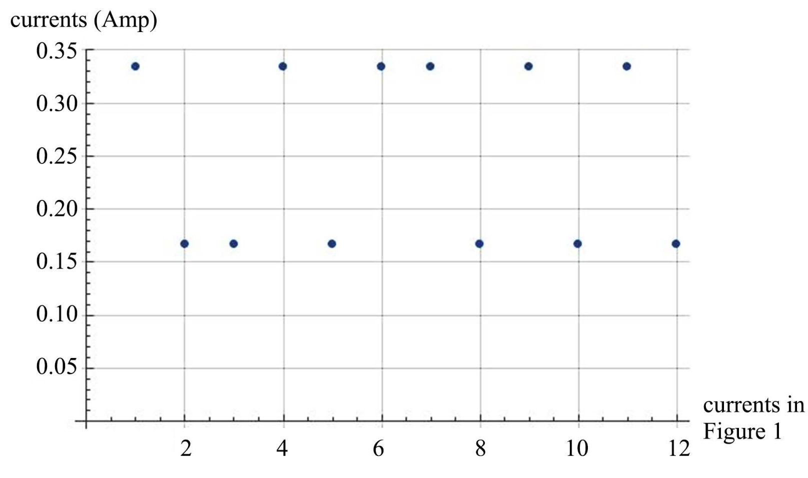

Figure 2. Values of the currents vs. the labeled index of the currents.

specialNonEqualR = {$R_1 \to 1$, $R_2 \to 2$, $R_3 \to 3$, $R_4 \to 4$, $R_5 \to 5$, $R_6 \to 6$, $R_7 \to 7$, $R_8 \to 8$, $R_9 \to 9$, $R_{10} \to 10$, $R_{11} \to 11$, $R_{12} \to 12$}

{{$i_1 \to$ (1,507,727 i)/2,870,812, $i_2 \to$ (8721 i)/29,596, $i_3 \to$ (170,671 i)/2,870,812, $i_4 \to$ (671,897 i)/2,870,812, $i_5 \to$ (380,987 i)/2,870,812, $i_6 \to$ (1,042,777 i)/2,870,812, $i_7 \to$ (811,427 i)/2,870,812, $i_8 \to$ (310,201 i)/2,870,812, $i_9 \to$ (172,797 i)/717,703, $i_{10} \to$ (330,895 i)/1,435,406, $i_{11} \to$ (254,152 i)/717,703, $i_{12} \to$ (250,613 i)/1,435,406}}

The other objective of this report is to replace the twelve different valued resistors shown in **Figure 1** with one, R_{eq}. As mentioned, this depends on the chosen pair. For instance, if the cube is being fed at vertex **A**, this can be paired with seven vertices requiring seven different R_{eq}, we are reporting all seven. Having symbolic expressions for all R_{eq} and because these are super lengthy expressions, we avoid including them in the report. But for special numeric cases, these are included.

Here we give the code calculating symbolic R_{eq} for the $\overline{\text{AH}}$ pair see **Figure 1**. For a special case of identical resistors, this is,

($R_1 i_1 + R_2 i_2 - R_3 i_3 + R_{12} i_{12}$)/$i$/.soliEqualR/.EquaRs (*AH*)

{R/2}

And for $R = 1\ \Omega$ this gives 0.5 Ω. Exercising the same procedure for all seven pairs gives values that are depicted in **Figure 4** and **Figure 5**.

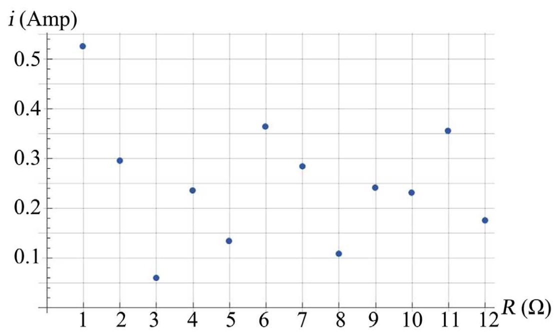

Figure 3. The description is the same as the caption of **Figure 2**. Along the horizontal axis, e.g., 4 is the $R_4 = 4\ \Omega$ and its corresponding ordinate is 0.234.

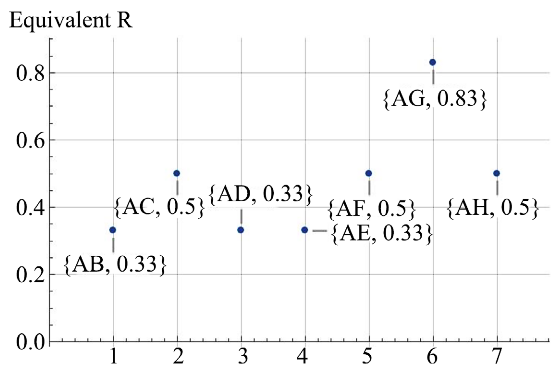

Figure 4. Display of the R_{eq} for the chosen pair e.g., along the horizontal axis 7 means $\overline{\text{AH}}$ that connects A to H via multiple routes described in the text all with the values of 0.5 Ω.

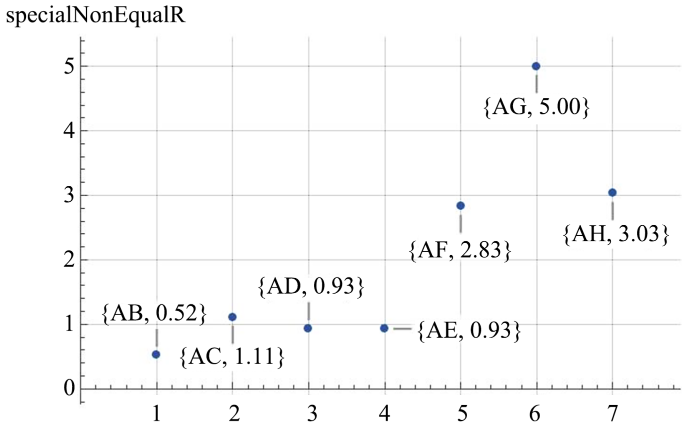

Figure 5. The caption is the same as in **Figure 4**. The difference between the two Figures is the values of the resistors. All the resistors are different given in the listing specialNonEqualR.

3. Conclusion and Comment

We set out crafting this report addressing the issues concerning electric circuits in a 3D structure. One of the issues of interest is the calculation of R_{eq}. That is instead of placing a cube with twelve different-valued resistors in a circuit to place one resistor, equivalent-resistor without altering the input and output currents. Addressing this issue led to the evaluation of not one but seven different possible R_{eq}. This report has identified all seven not only numerically but *symbolically*. The latter has been achieved merely because of the deployed CAS, specifically *Mathematica*. As an unintentional side product, we also generated symbolic expressions for the currents running in all twelve sides of the cube irrespective of the values of the resistors. This project keenly is tied with a CAS, otherwise, it couldn't have been completed. The literature search reveals no similar report whatsoever. Motivated by the outcome the author will report shortly the R_{eq}s for structures other than a cube. We have provided the needed *Mathematica* codes. An interested individual may reproduce our results and potentially generalize it on a need basis. Concerning the embedded codes references [5] [6] [7] are resourceful.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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