Archaeological Discovery, 2023, 11, 65-106
https://www.scirp.org/journal/ad
ISSN Online: 2331-1967
ISSN Print: 2331-1959

The Concavity of the Great Pyramid Can Be Derived from Inward Sloping Courses Needed for the Stability

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How to cite this paper: Kato, A. (2023). The Concavity of the Great Pyramid Can Be Derived from Inward Sloping Courses Needed for the Stability. *Archaeological Discovery, 11,* 65-106.
https://doi.org/10.4236/ad.2023.112004

Received: December 26, 2022
Accepted: February 25, 2023
Published: February 28, 2023

Abstract

The Great Pyramid has the character of concavity that each of its four faces is slightly indented along its central line. Applying the geometry on an inclined plane, we show that this concavity could be derived from its inner structure of inward sloping courses gently inclined towards the center of each course, at about 11 degrees to the horizontal, i.e., the slope 1/5 by the ratio of "rise over run". We explain why the inclined layers together with the reinforced base were necessary for the long-term stability of the Pyramid against the severe natural forces like the high gravitational compression, earthquakes and rainstorms, pointing out the feasible fact that the Pyramid has experienced severe rainstorms more than 500 times during the 4500 years. The crucial point about stability is that the effects of such natural forces are quite different between the core of inclined courses and that of truly level courses in the sense that the former can be tightened to become stronger over time, but the latter would be disintegrated to be weaker over time. Scaled-down models of the Pyramid are introduced to understand the large-scale dynamics of the Pyramid. In particular, the small model reduced by 10^{-3} helps us to imagine the transformation of vertical into lateral forces, pointed out by Mendelssohn. On the other hand, the Step Pyramid of Djoser can be identified almost as the half-sized model of the Great Pyramid when the Great Pyramid was assumed to be composed only of truly level courses. And this identification tells the fate of the Great Pyramid only of truly level courses that it would have almost collapsed until now like the Step Pyramid before the recent restoration.

Keywords

Great Pyramid, Concavity, Rheology, Model

1. Introduction

The Great Pyramid at Giza is known to have an amazing character of concavity that each of its four faces is slightly indented along its central line, from base to peak. In other words, the Great Pyramid is a concave octagonal pyramid, rather than the standard square pyramid. This concavity is so subtle to be seen from any ground position, but can be observed from the air. The British Air Force pilot, P. Groves, captured it as in **Figure 1** quite accidently on equinox, 1926, when he was flying over the Pyramid[1] (Groves & McCrindle, 1926). Thanks to the modern technologies, we can nowadays get a picture like **Figure 2** by the Earth observation satellite Quick Bird[2]. Quite recently, a remarkable study using the modern technology of radar measurements has appeared in (Biondi & Malanga, 2022), where the eight-sided nature of all the three pyramids of Khufu, Khafre and Menkaure were rigorously demonstrated. This is a great achievement as it is the first time ever to capture the concavity of the Khafre Pyramid, which was too subtle to be seen compared with that of Khufu's and Menkaure's. Historically,

Figure 1. Photo of the Giza pyramids captured by Brigadier General P. R. C. Groves, British Royal Air Force, at sunset during the Autumnal Equinox. The Great Pyramid, almost in the middle of this picture, shows its bright western face together with the southern face divided into two right triangles. This picture was published in the National Geographic, September 1926.

[1]Natonal Geographic 1926-09: Misquotation prevails in literatures about the year this marvelous photo was taken.
[2]Similar pictures can be seen in
http://philippelopes.free.fr/PyramideDeKheopsHuitFacesRevelationDesPyramides.htm.

Figure 2. Pansharpened image of the Great Pyramid, by Quick Bird on Feb.2, 2002. https://www.advite.com/satellitephotos.htm.

the concavity of the Great Pyramid was observed much earlier by Flinders Petrie (Petrie, 1883) who reported that: "I continually observed that the courses of the core had dips of as much as 1/2 degree to 1 degree so that it is not at all certain that the courses of the casing were truly level··· the faces of the core masonry being very distinctly hollowed. This hollowing is a striking feature; and beside the general curve of the face, each side has a sort of groove especially down the middle of the face, showing that there must have been a sudden increase of the casing thickness down the midline. The whole of the hollowing was estimated at 37 (inches) on the North face···" ("37 inches" is about 0.94 meters, less than one meter.) Here, recall the fact that the original shape of the Great Pyramid was the complete square pyramid covered with the casing stones; the survey (Dash & Paulson, 2015) proved the base (with the casing stones) of the Great Pyramid was a perfect square. It was when the casing stones were lost that the concavity revealed. Therefore, the concavity had been covered by the adjustment of casing stones with "a sudden increase of the casing thickness down the midline" as reported above. Another kind of observation was done on equinox, 1934, by a French mathematician André Pochan, who photographed the southern side of the Great Pyramid using the infrared camera, as in **Figure 3**, to observe that this face was divided into two right triangles with different temperatures. His illustration **Figure 4** in (Pochan, 1971) describes the maximal indent as 0.92 meters.

We show in this article, through the geometric analysis of an inclined plane, that the slight concavity of the Great Pyramid could be derived from the gentle

Figure 3. Infrared photograph of the southern face of the Great Pyramid taken by Pochan, at 6 p.m. on equinox, 1934 (Pochan, 1971).

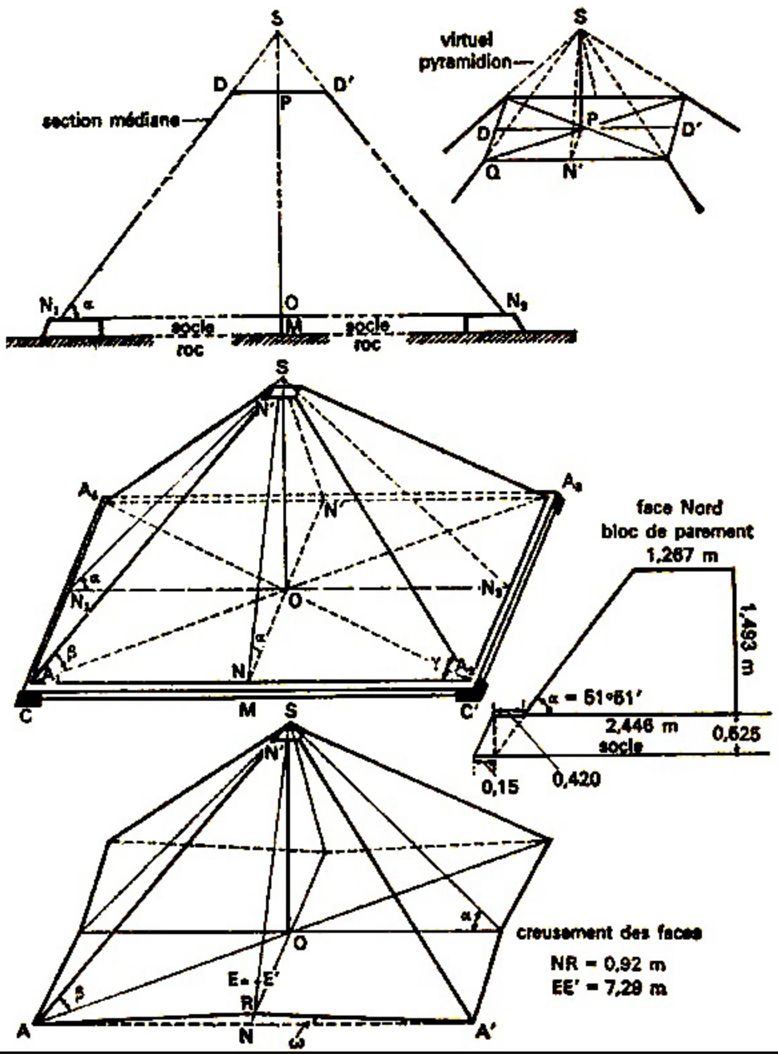


Figure 4. Illustrations of the Great Pyramid in (Pochan, 1971) as a concave octagonal pyramid with the maximal indent 0.92 meters.

slope of inclined courses, which we estimate at about 11 degrees to the horizontal, i.e., the slope 1/5 by the ratio of "rise over run". Due to the perfect masonry we cannot see the core of the Great Pyramid. Contrasting examples whose cores can be seen are the Meidum Pyramid and the three Queens Pyramids of Khufu. The Meidum Pyramid discloses its poor internal masonry through its large hole in the north side, and the inside of the Queens Pyramids is described in Chapter 9 of (Isler, 2001) that "The stones inside the tiers, which form the bulk of pyramid masonry, are small and poorly fitted compared to those on the face, some near the center of the nucleus being placed almost haphazardly". These examples teach us well that we cannot judge the inner structure of any pyramid only by its external appearance even for the small-sized pyramids like the Queens Pyramids, about 30 m high. Therefore, the best possible viewpoint we stand on should be the scientific one based upon geometry, physics and geology.

In Section 2, we will present how to lay blocks on an inclined plane to form inclined courses, and show how the geometry of an inclined plane generates the concavity of the pyramid. The strong structure of the base is definitely needed for the stability, and the Great Pyramid was built on a natural carved outcrop. We show in Section 3 how this outcrop was incorporated to reinforce the structure of the base, and how the concavity tightens the Pyramid consisting of inclined courses. Section 4 remarks a bit unexpected fact that the Great Pyramid has long been exposed to rainstorms "quite frequently" if measured by the timescale of the Pyramid. This fact is very important since most blocks of the Pyramid are limestones, quite vulnerable to the erosion by rainwater. The necessity of the inclined courses for the stability of the Pyramid will be explained in Section 5, based on the idea of (Mendelssohn, 1976) that in a pyramid containing stones of irregular shape and consisting only of truly level courses, the vertically downwards acting force generated by the gravity will develop lateral components, favouring a break-up and flattening of the structure. We show that the core of inclined courses could be tightened by earthquakes and erosion by rainwater, but the core of truly level courses would be loosened by them. In Section 6 we introduce scaled-down models to understand the large-scale (both in space and time) dynamics of the Pyramid from the viewpoint of the rheology. A small model of size 20 cm, reduced by the scale 10^{-3}, helps us to understand the transformation of vertical into lateral forces. Models not so small are also useful, and indeed, the Step Pyramid of Djoser on the Saqqara plateau can be identified almost as a half-sized model of the Great Pyramid if the Great Pyramid was assumed to be made entirely of truly level courses. Therefore, the seriously deteriorated Step Pyramid just before the recent restoration tells the fate of the Great Pyramid if it were consisted entirely of truly level courses.

2. How to Lay Blocks on an Inclined Plane

Figure 5 illustrates the outline of the Great Pyramid as a square pyramid with the top A_0 and the base $A_1A_3A_5A_7$ with its center *O*. Suppose the square $B_1B_3B_5B_7$

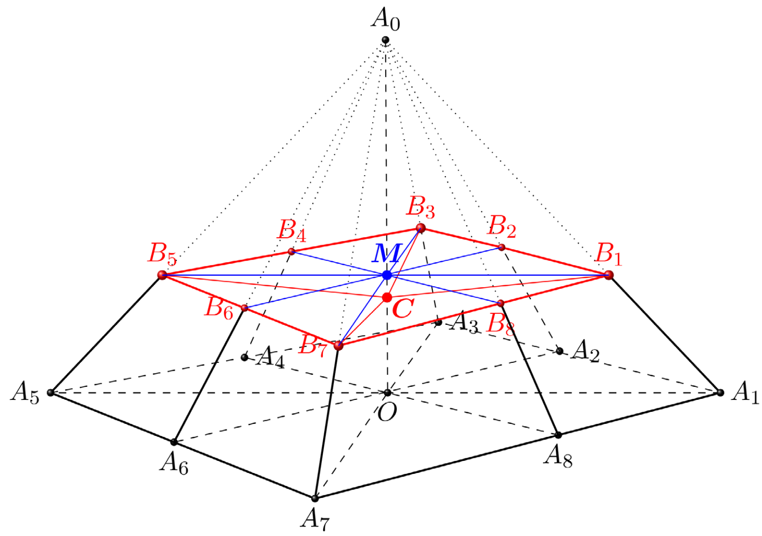


Figure 5. Pyramid $A_0A_1A_3A_5A_7$ illustrates the outline of the Great Pyramid as a square pyramid with the top A_0 and the base $A_1A_3A_5A_7$ with its center O. The square $B_1B_3B_5B_7$ with the center M denotes its horizontal cross-section. The point C is chosen between O and M, to consider the frustum with the sunken rooftop consisting of four inclined triangular planes $\Delta CB_1B_3, \Delta CB_3B_5, \Delta CB_5B_7, \Delta CB_7B_1$.

with the center M is its horizontal cross-section, and choose a point C between O and M. The midpoints of the segments
$A_1A_3, A_3A_5, A_5A_7, A_7A_1, B_1B_3, B_3B_5, B_5B_7, B_7B_1$ are denoted
$A_2, A_4, A_6, A_8, B_2, B_4, B_6, B_8$, respectively.

Let us consider the frustum with the sunken top surface composed of four inclined triangular planes ΔCB_1B_3, ΔCB_3B_5, ΔCB_5B_7, ΔCB_7B_1, and suppose now that we have completed the piling of stones below this sunken surface, and what we do next is to add a new course of blocks on this surface. Most blocks used in the Pyramid are assumed to be cubic, and the height of blocks used for this new course should be uniform. Due to the symmetry, it would suffice to show how we can lay blocks on the inclined triangular plane ΔCB_1B_3. Note that the horizontal ΔMB_1B_3 in **Figure 5** is the right triangle, but the triangle ΔCB_1B_3 is not, since the point C is below M:

$$\overline{CB_1} = \overline{CB_3} > \overline{MB_1} = \overline{MB_3},$$

$$\angle B_1CB_3 < \angle B_1MB_3 = \pi/2,$$

$$\angle B_2B_1C = \angle B_2B_3C > \angle B_2B_1M = \angle B_2B_3M = \pi/4.$$

The most essential idea which motivated this paper is that:

The geometry on the inclined triangle ΔCB_1B_3 *differs slightly from that of the horizontal triangle* ΔMB_1B_3.

Choose the point $B_2^\star$ on the line B_2C such that $\angle B_2^\star B_3C = \angle B_2^\star B_1C = \pi/4$. Then, the edge $B_3B_2^\star B_1$ becomes indented as in **Figure 6**. Put $\angle B_2B_3B_2^\star = \angle B_2B_1B_2^\star = \beta$ and $\angle B_2CB_3 = \angle B_2CB_1 = \alpha$, where $0 < \alpha < \pi/4$. We lay blocks in such a way that the sunken edge $B_3B_2^\star B_1$ becomes the outermost

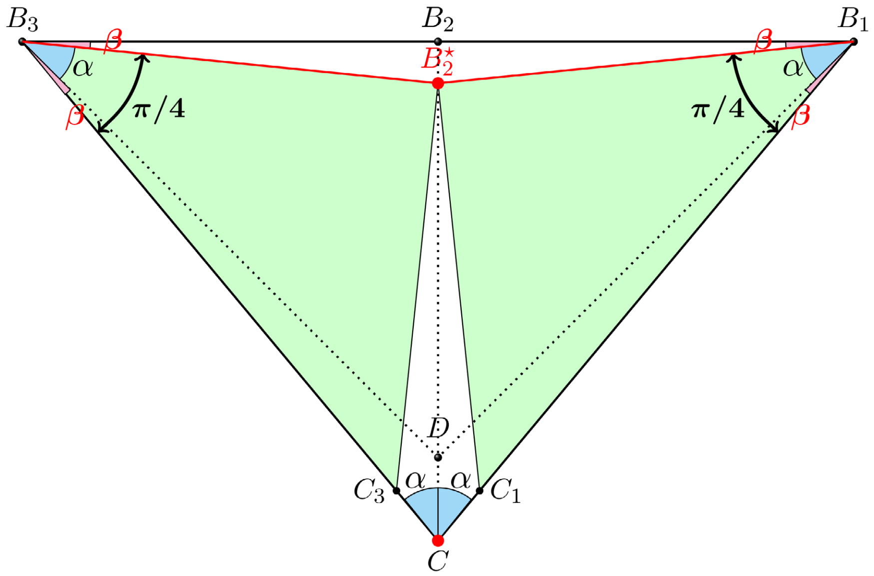


Figure 6. Geometry on the plane ΔCB_1B_3 where the point $B_2^\star$ on the line B_2C is chosen to be $\angle B_2^\star B_1C = \angle B_2^\star B_3C = \pi/4$, and put $\angle B_2CB_1 = \angle B_2CB_3 = \alpha < \pi/4$, $\angle B_2B_1B_2^\star = \angle B_2B_3B_2^\star = \beta$. The point D on the line B_2C is chosen to satisfy $\overline{B_2D} = \overline{B_2B_1} = \overline{B_2B_3}$ so that ΔDB_1B_3 is a right triangle congruent with the horizontal ΔMB_1B_3 in **Figure 5**. The points C_1 and C_3 on the line B_1C and B_3C, respectively, are chosen to be $\angle B_1B_2^\star C_1 = \pi/2$, $\angle B_3B_2^\star C_3 = \pi/2$, so that $\Delta B_1B_2^\star C_1$ and $\Delta B_3B_2^\star C_3$ (green-colored) are congruent isosceles right triangles. Observe that $\alpha + \beta = \pi/4$.

one, so we may call the angle β "the angle of indentation" and the distance $B_2B_2^\star$ "the maximal indent". (Note well that in most of our illustrations the indentations of faces are quite exaggerated, as the actual size of the angle of indentation is less than one degree.) In order to see the difference between the inclined ΔCB_1B_3 and the horizontal ΔMB_1B_3, consider the plane ΔCB_1B_3 as in **Figure 6** and take the point D on the line B_2C such that $\angle B_1DB_3 = \pi/2$. Then ΔDB_1B_3 is congruent with ΔMB_1B_3. Since $\angle B_2B_3D = \angle B_2^\star B_3C = \pi/4$, the definition of the angle β implies that $\angle DB_3C = \beta$. Then we get $\alpha + \beta = \pi/4$ since the external angle $\angle B_3DB_2$ at the vertex D of ΔDB_3C is $\pi/4$. Consequently, $\angle B_2^\star B_3D = \pi/4 - \beta = \alpha$. By symmetry, we have $\angle DB_1C = \beta$ and $\angle B_2^\star B_1D = \alpha$. Next, choose a point C_3 on the line B_3C such that $\angle B_3B_2^\star C_3 = \pi/2$, and similarly, a point C_1 on the line B_1C such that $\angle B_1B_2^\star C_1 = \pi/2$. Then, the green-colored triangles $\Delta B_1B_2^\star C_1$ and $\Delta B_3B_2^\star C_3$ are congruent isosceles right triangles. So, intuitively speaking, we can imagine that a butterfly with the body B_2D and wings ΔB_1B_2D, ΔB_3B_2D slightly moved its wings to $\Delta B_1B_2^\star C_1$ and $\Delta B_3B_2^\star C_3$. Note that to choose the sunken edge $B_3B_2^\star B_1$ as the outermost one is quite a reasonable selection because the angle $\pi/4 = \angle B_2^\star B_3C = \angle B_2^\star B_1C$ can be precisely measured using the bisection of the right angle, while it would be very difficult to measure the precise angle $\angle B_2B_3C = \angle B_2B_1C$, slightly bigger than $\pi/4$. Note also that on the inclined plane it would be very difficult to measure precisely any long distance between two points using streched cord, since mason's line becomes catenary due to its weight; see (Isler, 1983). In short, we should measure "angle" rather than "length" on the inclined plane. How to lay cubic blocks on this inclined plane $CB_1B_2^{\star}B_3$ is not so difficult. In **Figure 7**, first, place stones on the (gray-colored) area $CC_1B_2^{\star}C_3$ along the central line $CB_2^{\star}$. Then, we need to lay stones on the triangles $\Delta B_1B_2^{\star}C_1$ and $\Delta B_3B_2^{\star}C_3$. Here, note that, for instance suppose $\overline{B_3B_2}=100$ meters and $\overline{B_2B_2^{\star}}=1$ meter, then $\overline{B_3B_2^{\star}}=\sqrt{100^2+1}=100.0049\cdots$ meters, so that $\overline{B_3B_2^{\star}}$ is longer than $\overline{B_3B_2}$ just about a half centimeter. Hence, practically, we can assume all of four isosceles right triangles $\Delta B_1B_2^{\star}C_1, \Delta B_3B_2^{\star}C_3, \Delta B_1B_2D, \Delta B_3B_2D$ are congruent. And recall that $\Delta B_1B_2D, \Delta B_3B_2D$ are congruent with the horizontal triangles $\Delta B_1B_2M, \Delta B_3B_2M$ in **Figure 5**. Therefore, what we need to do is to lay blocks on $\Delta B_1B_2^{\star}C_1, \Delta B_3B_2^{\star}C_3$ in the same way as on the horizontal triangles $\Delta B_1B_2M, \Delta B_3B_2M$, respectively. Note also that the gray-colored part $CC_1B_2^{\star}C_3$ in **Figure 7** is actually a very narrow area since the distance between C_1 and C_3 is $2\overline{B_2B_2^{\star}}$, which is just 2 meters in case $\overline{B_2B_2^{\star}}=1$ meter. When this kind of laying was done on each of four inclined triangles of the sunken rooftop of **Figure 5**, the whole arrangement would become like **Figure 8**. Notice that, since this is a top view and the point C is below the horizontal square $B_1B_3B_5B_7$, some apparent angles differ from their actual ones, e.g., the actual angle $\angle B_2^{\star}B_3C$ is $\pi/4$, though it appears to be smaller than $\pi/4$ in **Figure 8**. (Precisely speaking, we should place blocks on an area a bit smaller than $B_1B_2^{\star}B_3B_4^{\star}B_5B_6^{\star}B_7B_8^{\star}$, which can be done easily, for instance, if we do not place blocks on the blue part of **Figure 8**. Note also that it might happen that some central part, as shown in white, is already occupied by other structure.) We made a wooden model **Figure 9** of the sunken surface of **Figure 8**.

Now we want to calculate the maximal indent $\overline{B_2B_2^{\star}}$. So, let us introduce the 3-dimensional coordinate system as in **Figure 10** setting the origin at the center

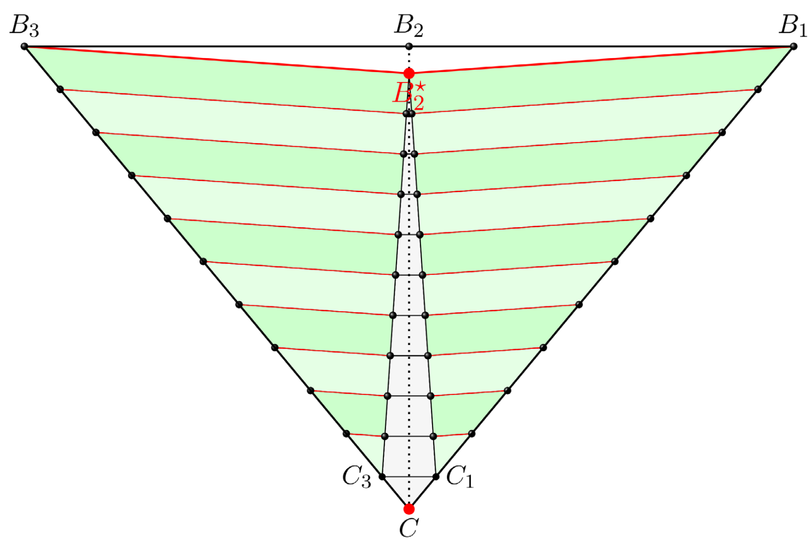


Figure 7. Example of a layment of blocks on the inclined plane $CB_1B_2^{\star}B_3$ in consecutive rows, filling first the (gray-colored) erea $CC_1B_2^{\star}C_3$ along the central line $CB_2^{\star}$.

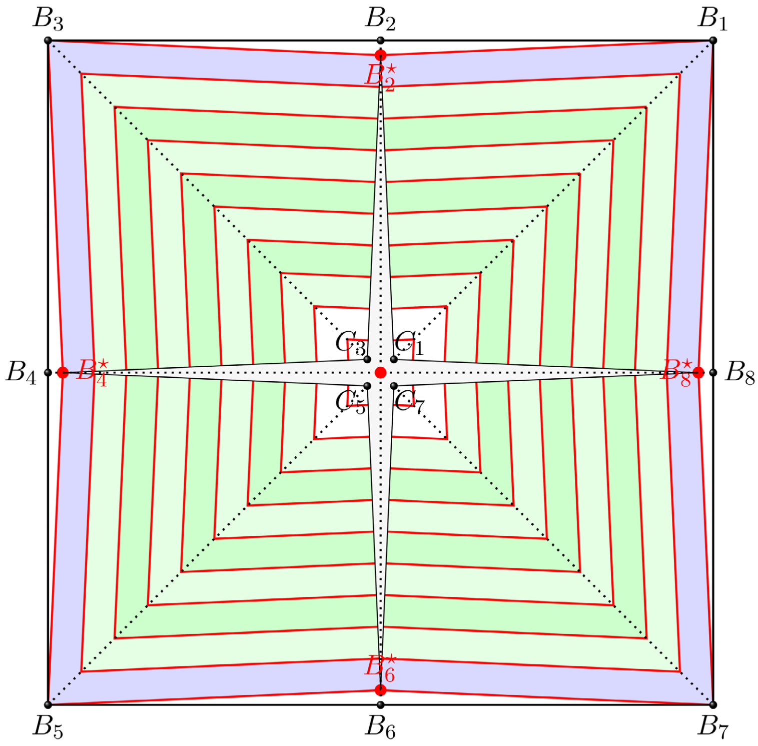


Figure 8. Top View of stone arrangement on the sunken rooftop of **Figure 5**, where the stones are layed on each of the four inclined triangular planes as in **Figure 7**.

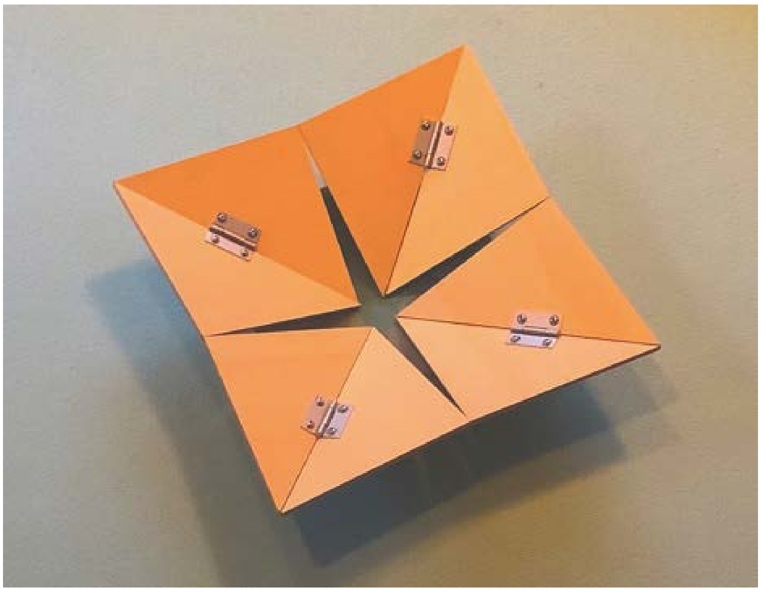


Figure 9. Wooden Model of the sunken surface $B_1B_2^*B_3B_4^*B_5B_6^*B_7B_8^*$ of **Figure 8** consisting of eight congruent isosceles right triangles, whose length of the legs of a right angle is 15 cm.

M of the square $B_1B_3B_5B_7$. Let $2a$ be the side length of the square $B_1B_3B_5B_7$ so that $\overline{B_1B_2} = \overline{B_3B_2} = \overline{MB_2} = a$. Let h be the depth of the sunken surface, i.e., $\overline{MC} = h$, and let θ be the slope of the triangular plane CB_1B_3, i.e., $\tan\theta = \overline{CM} / \overline{B_2M} = h/a$. Then we see that the relation between α and θ is

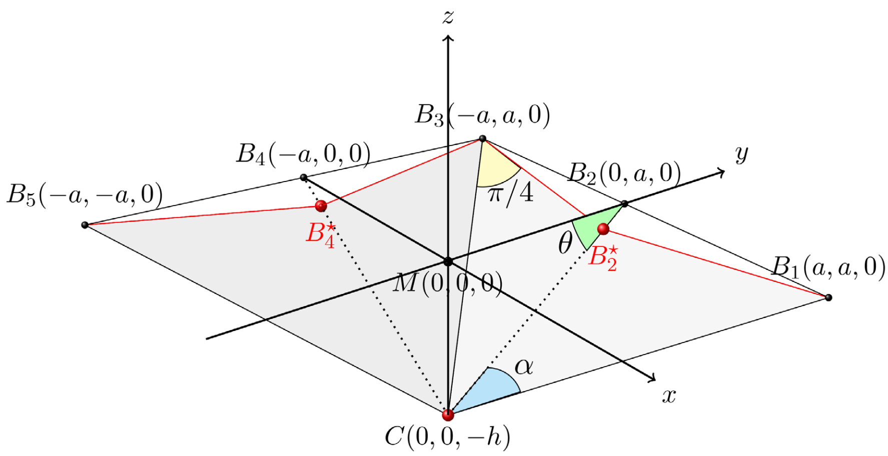


Figure 10. Introduction of a 3-dimensional coordinate system setting the origin at the center M of the square $B_1B_3B_5B_7$ with the side length $2a$ (see **Figure 5**). The depth of the point C is h, and the slope of the triangular plane CB_1B_3 is θ, so that $\tan\theta = h/a$.

$$\tan\alpha = \overline{B_1B_2}\big/\overline{CB_2} = a\Big/\sqrt{a^2+h^2} = 1\Big/\sqrt{1+(h/a)^2} = 1\Big/\sqrt{1+\tan^2\theta}.$$

In order to calculate $\overline{B_2B_2^\star}$ we will evaluate $\overline{B_2B_2^\star}\big/\overline{B_3B_2} = \overline{B_2B_2^\star}\big/a = \tan\beta$. The fact $\alpha+\beta = \pi/4$ implies

$$\tan\beta = \frac{1-\tan\alpha}{1+\tan\alpha}.$$

Putting $\tan\theta = s$ for simplicity, we get

$$\tan\alpha = 1\Big/\sqrt{1+s^2}, \quad \tan\beta = \frac{\sqrt{1+s^2}-1}{\sqrt{1+s^2}+1} = 1+\frac{2}{s^2}\left(1-\sqrt{1+s^2}\right).$$

Now let us utilize the expansion

$$\sqrt{1+X} = 1+X/2 - X^2/8 + X^3/16 - \cdots$$

which is valid for X with $0 \le X \le 1$. We here assume a quite reasonable assumption $0 < \theta \le \pi/4$, implying $0 < \tan\theta = s \le 1$, so that we can apply the above expansion for $X = s^2$, i.e.,

$$\sqrt{1+s^2} = 1+s^2/2 - s^4/8 + s^6/16 - \cdots.$$

Thus we finally obtain the evaluation

$$\tan\beta = 1+\frac{2}{s^2}\left(1-\sqrt{1+s^2}\right) = s^2/4 - s^4/8 + \cdots$$

hence

$$\overline{B_2B_2^\star} = a\tan\beta = a\left(s^2/4 - s^4/8 + \cdots\right).$$

When $s = \tan\theta$ is small, the term $s^4/8$ is quite small compared with $s^2/4$, so, we can roughly estimate that

$$\tan\beta \approx \frac{1}{4}\tan^2\theta, \quad \overline{B_2B_2^\star} = a\tan\beta \approx \frac{a}{4}\tan^2\theta.$$

The side length of the base of the Great Pyramid is about $230 = 115\times 2$ meters, so let us simply assume $a = 100$ meters to get the concrete values of the angle β and the indent $\overline{B_2B_2^*}$ for various candidates of $\tan\theta$. Then we get **Table 1**. As mentioned in Section 1, Petrie reported the angle of indentation was between a half to one degree and the maximal indent was about 0.94 meters, and Pochan got the closer value of indent 0.92 meters as shown in **Figure 4**. These measurements settle us to conclude that the case which well fits to the Great Pyramid is when $\tan\theta = 1/5$ in **Table 1**, that is:

The concavity of the Great Pyramid is derived from its core layers inclined towards the center at about 11 degrees to the horizontal. Note that the angle of the slope of the inclined layers should be as simple as possible since such an angle was needed to be measured precisely and repeatedly many times during the construction, and it is known that the ancient Egyptian measured the slope by the ratio of "rise over run", and their calculation of ratio is based upon the "unit fractions" like $1/2, 1/3, 1/4, 1/5, \cdots$. So, it would be quite natural to assume that the practical value of the slope $\tan\theta$ was one of such unit fractions. We have drawn two pictures **Figure 11** and **Figure 12**, almost to scale in case of $\tan\theta = 1/5$. **Figure 11** corresponds to **Figure 8**. **Figure 12** shows the vertical cross section along the north-south direction A_2A_6, of the piling of inclined layers (blue-colored) on the well-founded, lowest several courses (light-gray-colored), where the slope of inclined layers $\arctan(1/5) \approx 11^\circ$ can be compared with that of the Descending Passage $\arctan(1/2) \approx 26^\circ$. The point Q is chosen to be the point in the Queen's Chamber, at the height 23 m from the base of the Pyramid; then, since $23/115 = 23/(230/2) = 1/5$ (assuming $\overline{A_2A_6} = 230\text{ m}$), the angle $\angle QA_2O = \angle QA_6O$ coincides with $\arctan(1/5)$. (Since the floor of the Queen's Chamber is about 21.5 m heigh from the base of the Pyramid, and the Chamber itself is about 6 m high, the angle $\angle QA_2O = \angle QA_6O$ depends on the choice of the point Q. Creighton & Osborn (2008) chose the "center" of the Chamber and calculated it as 11.73˚. Our choice of the height $23 = 21.5 + 1.5$ m is almost that of the head of a man when he stands on the floor of the Chamber.) Note that if we illustrate the vertical cross section $A_0A_1A_5$ along the

Table 1. Values of the angle of indentation β and the maximal indent $\overline{B_2B_2^*}$ for various candidates of the slope $\tan\theta$ of the inclined courses.

$\tan\theta$	θ (degree)	$\tan\beta$	β (degree)	Max. indent $\overline{B_2B_2^*}$ (meter)
1/2	26.56···	0.0546···	3.13···	5.46···
1/3	18.43···	0.0263···	1.50···	2.63···
1/4	14.03···	0.0151···	0.87···	1.51···
1/5	**11.30···**	**0.0098···**	**0.56···**	**0.98···**
1/6	9.46···	0.0068···	0.39···	0.68···
1/7	8.13···	0.0050···	0.28···	0.50···
1/8	7.12···	0.0039···	0.22···	0.39···

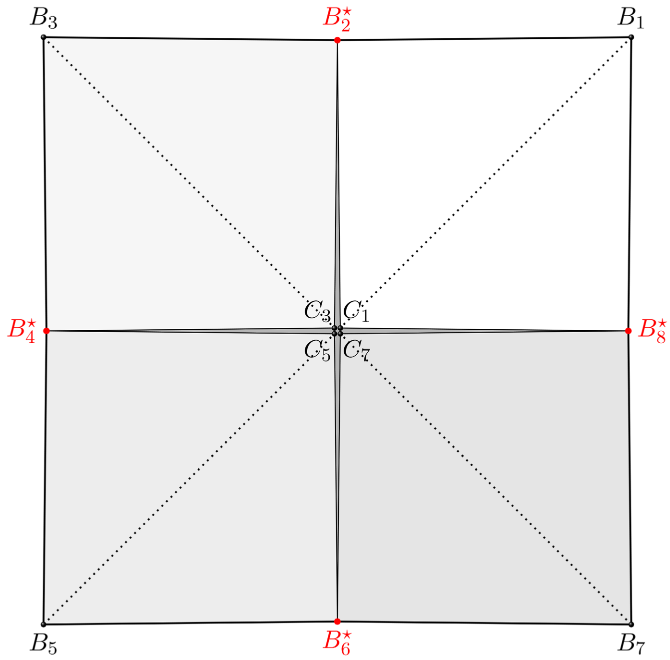


Figure 11. Illustration of **Figure 8**, almost to scale, in case of $\tan\theta = 1/5$ in **Table 1**, providing its alternative view as the union of four square-like inclined surfaces, corresponding to the hinged surfaces in **Figure 9** and each hinged surface becomes a square when it is flattened.

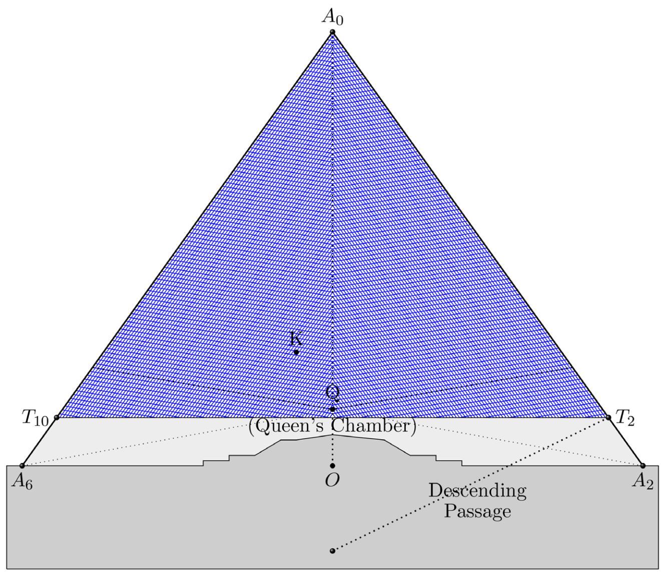


Figure 12. Illustration of the vertical cross section $A_0A_2A_6$ of the Great Pyramid in **Figure 5**, where Q and K denote the positions of the Queen's and the King's Chamber, respectively, supposing A_2A_6 is the north-south direction. Shown in blue color is the piling of inclined layers with the slope 1/5 (measured by the ratio of "rise over run") ≈ 11˚. The gentleness of this angle can be compared with the slope 1/2 ≈ 26˚ of the Descending Passage. The dark gray part is the natural bedrock and its upper part above the ground level A_2OA_6 is an outcrop or inselberg. The light gray part shows the lowest several courses tightly founded by well-squared blocks. This illustration is almost to scale, though we disregarded the indentation of faces and the detailed structure around the central axis A_0O .

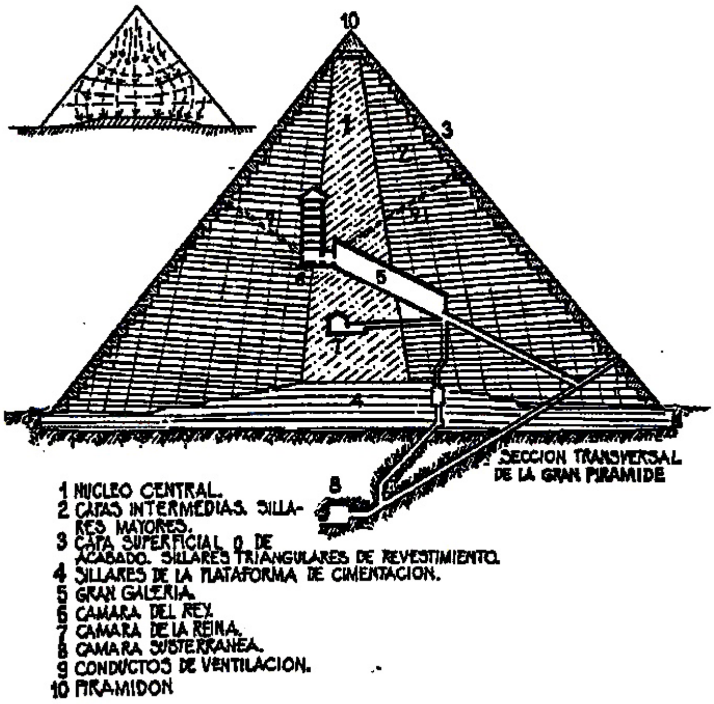


Figure 13. Inner structure of the Great Pyramid described in (Andrade, 1992), well illustrating the gentle slope of the courses.

diagonal A_1A_5 of the base, the slope of inclined layers will appear much gentler, which is $\arctan\left(1/5\times 1/\sqrt{2}\right)\approx 8^\circ$, the slope of CB_1 or CB_5 in **Figure 5** or **Figure 10**. We are a bit surprised to find **Figure 13** in (Andrade, 1992), quite similar to our **Figure 12**, well embodying the "gentle" slope of the inclined layers. The idea of "the base divided into four triangles slightly inclined towards the center" is also suggested in (Yasseen, 2018). While **Figure 8** can be viewed as the union of four inclined triangular planes, **Figure 11** sees **Figure 8** alternatively as the union of four square-like inclined surfaces $C_1B_8^*B_1B_2^*$, $C_3B_2^*B_3B_4^*$, $C_5B_4^*B_5B_6^*$, $C_7B_6^*B_7B_8^*$. This suggests another way of piling stones by dividing the base into four "squares" like **Figure 11**. Note for example that, though the surface $C_1B_2^*B_1B_8^*$ is not flat, it is a union of two flat right triangles $\Delta C_1B_2^*B_1, \Delta C_1B_8^*B_1$ so that its area is the same as that of the square of the side length $\overline{C_1B_8^*}=\overline{B_8^*B_1}=\overline{B_1B_2^*}=\overline{B_2^*C_1}$, and it would be easier to count the number of stones needed to fill a square rather than a triangle. Additionally, it would be a wise way to convey stones from the four corners to the center since the slope of $B_1C_1, B_3C_3, B_5C_5, B_7C_7$ is only 8 degrees, as mentioned before.

3. Tightening the Pyramid by Reinforced Base and Concavity

Needless to say, the inclined courses mentioned in Section 2 should be placed on a firm foundation. Here we show how the base of the Pyramid was reinforced by the incorporation of the outcrop, and that the concavity strengthens the structure of the whole pyramid including the base. The Great Pyramid was built on a natural carved outcrop whose volume is estimated to be about 20 percent of the monument (Raynaud et al., 2008). Let us present our idea that how they incorporated this outcrop into the monument for its stability. Quite recently, a great discovery was done by (Zalewski, 2017) that each triangular face of the Great Pyramid includes at its bottom a triangular light-colored area consisting of the special type of strong limestone, of the type "grainstone", different from the other parts. This part, which Zalewski called the "Alpha triangle", has the height 16.65 meters and the base about 150 meters, and is composed of the blocks of the uniform size and shape fitted together precisely, and the spaces between them are filled with homogeneous mortar[3]. From the position of the four Alpha triangles it would be natural to assume that what they made inside was a solid substructure like the gray one in **Figure 14** in order to incorporate the outcrop. This gray structure is a union of two solid triangular prisms, one with the bases of the Alpha triangles $\Delta T_1T_2T_3$ and $\Delta T_9T_{10}T_{11}$, and another with the bases of the Alpha triangles $\Delta T_5T_6T_7$ and $\Delta T_{13}T_{14}T_{15}$. The intersection of these two prisms forms a pyramid with the top T and the square base $T_4T_8T_{12}T_{16}$. Though there exist various estimations about the dimensions of the outcrop, we believe that this gray substructure is big enough to include almost all of the outcrop. And we may assume that this substructure was constructed very carefully in the same way as the Alpha triangles, i.e., it is made of uniform blocks fitted very precisely and well connected by mortar not only each other but also to the outcrop. Consequently, this cross-shaped substructure reinforces the base of the Great Pyramid in the sense that it protects the base against the tensions in lateral directions.

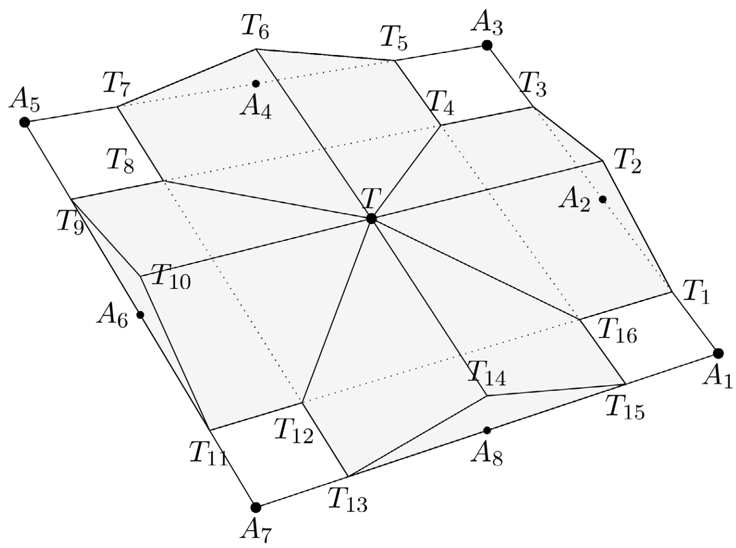


Figure 14. Reinforcement of the base of the Pyramid by the (gray-colored) substructure which incorporates the carved outcrop (see **Figure 12** or **Figure 13**) and bonds it together with the four Alpha triangles $\Delta T_1T_2T_3$, $\Delta T_9T_{10}T_{11}$, $\Delta T_5T_6T_7$, $\Delta T_{13}T_{14}T_{15}$.

[3]The "Alpha triangle" on the western face of the Great Pyramid can be seen very clearly in the picture No.10 in http://chamorrobible.org/gpw/gpw-20040823-English.htm.

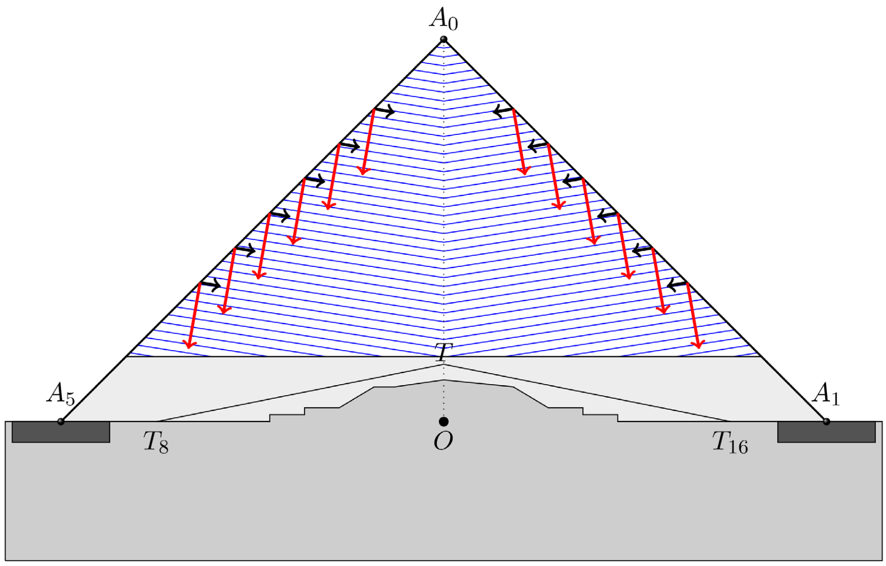


Figure 15. How gravity acts on stones. $A_0A_1A_5$ is the vertical cross section along the diagonal A_1A_5 of the base in case all courses are inclined, and the black parts at A_1, A_5 show the corner sockets. The gravity pushes blocks on the inclined course towards the center of the course (in the direction of black arrows), and also presses them in the direction perpendicular to the course (red arrows). These red-arrowed forces are a bit inclined outward from the vertical (to the same degree as the slope of the inclined course) so that they would cause some extension of the base (the light-gray part). To prevent such extension, the corner sockets would be needed.

Recall the fact that a stone is very weak against the tensile stress, though quite strong against compression. Precisely speaking, this substructure surely protects the base well from the tensions in the directions A_2A_6 and A_4A_8, but it would not be sufficient for the strong tensions in the diagonal directions A_1A_5 and A_3A_7. As illustrated in **Figure 15**, the gravity acting on each block can be decomposed into the force along the inclined course and the one perpendicular to the course, and the latter is a bit inclined outward from the vertical (to the same degree as the slope of the inclined course) to cause some extension of the base. Therefore, we believe, in order to prevent the extension of the diagonals, the corner sockets were additionally set at the four corner points A_1, A_5, A_3, A_7.

Relating with such reinforcement of the base, we want to explain the advantage of the concavity of the four faces, together with the inclined layers, that it helps to tighten the whole Pyramid. **Figure 16** illustrates the force diagram on the base $A_1A_2^*A_3A_4^*A_5A_6^*A_7A_8^*$, where each A_i^* is indented from A_i for $i = 2,4,6,8$. (Practically, these indents are quite small, so **Figure 16** is not to scale.) The reaction forces exerted from the corners against the diagonal tensions are exhibited as the four black-colored vectors of the same length, starting from A_1, A_3, A_5, A_7 and directing to the center O of the base. Denote in particular the reaction forces at the corners A_1, A_7 as $\mathbf{a}, \mathbf{b}$, respectively, and decompose them as

$$\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_2, \quad \mathbf{b} = \mathbf{b}_1 + \mathbf{b}_2$$

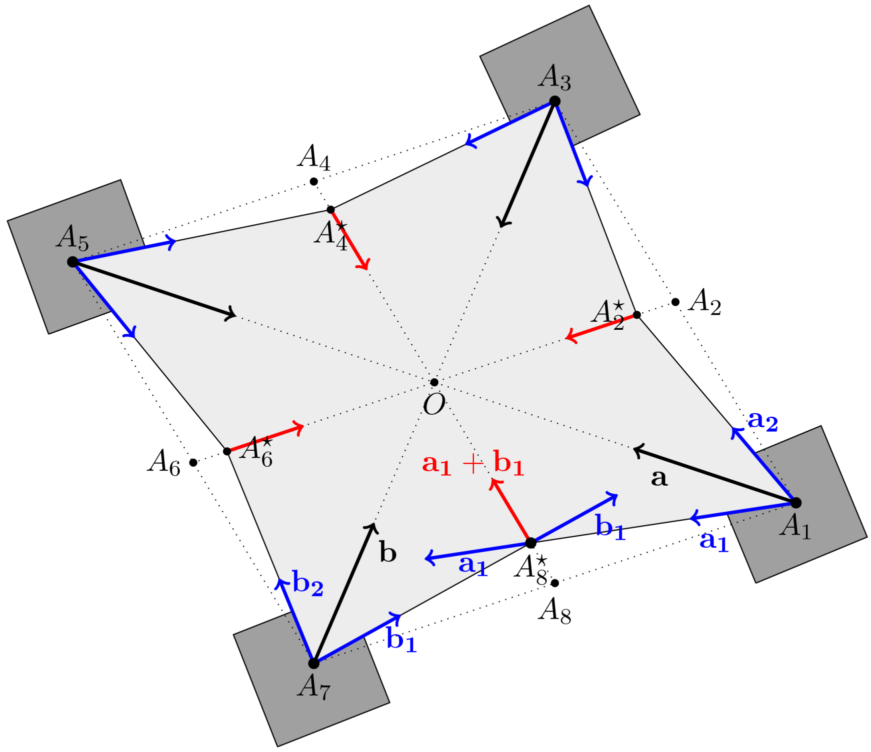


Figure 16. Force diagram on the (light-gray) base with the corners A_1, A_3, A_5, A_7 and the indented spots $A_2^\star, A_4^\star, A_6^\star, A_8^\star$. The dark-gray corner sockets prevent the extension of the diagonals A_3A_7 and A_1A_5 to produce the four black-colored reaction forces, which then generate the four red-colored forces at the indented spots towards the center O. Not to scale.

where both $\mathbf{a}_1$ and $\mathbf{b}_1$ direct to the indented $A_8^\star$, and $\mathbf{a}_2, \mathbf{b}_2$ direct to the indented $A_2^\star$ and $A_6^\star$, respectively. Then we get the (red-colored) vector sum $\mathbf{a}_1 + \mathbf{b}_1$ directing to the center O. Hence the reaction forces at the corners A_1, A_7 generate the new (red-colored) force at $A_8^\star$ towards the center O. By symmetry, we can observe the similar effects of the reaction forces at the other corners, hence concluding that:

The four black-colored reaction forces at the corners A_1, A_3, A_5, A_7 towards the center O produce the four red-colored forces at the indented spots $A_2^\star, A_4^\star, A_6^\star, A_8^\star$ towards the center O.

A similar mechanism works also on every inclined course as shown in the force diagram **Figure 17**, which is quite similar to **Figure 16** except that the dark-gray parts show some stones at the corners and the black-colored forces are due to their weight and the inclination of the course. Also, the effects of the black-colored forces are similar:

The four black-colored forces at the corners B_1, B_3, B_5, B_7 towards the center C produce the four red-colored forces at the indented spots $B_2^\star, B_4^\star, B_6^\star, B_8^\star$ towards the center C.

Thus, the form of concavity contributes to tighten and stabilize the whole Pyramid consisting of inclined courses!

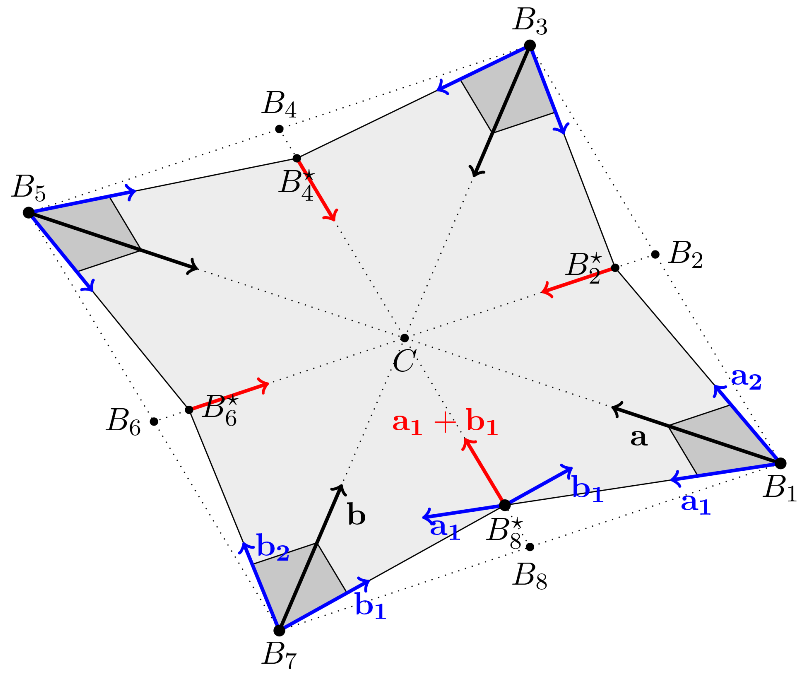


Figure 17. Force diagram on the inclined course with the corners B_1, B_3, B_5, B_7 and the indented spots $B_2^\star, B_4^\star, B_6^\star, B_8^\star$. The dark-gray parts show some stones at the corners, which produce the four black-colored forces due to the gravitation and the inclination of the course. These black forces generate the red ones at the indented spots, in the same way as **Figure 16**. Note that the light-gray surface in this figure is sunken, but the one in **Figure 16** is horizontal.

4. Heavy Rainstorms in Egypt and Measures against Them

We here want to remark a bit unexpected fact that the Great Pyramid has long been exposed to rainstorms, which were "infrequent" by the timescale of our daily life but "quite frequent" by the timescale of the Pyramid. Note though that the Pyramid was not affected by the annual Nile flooding since it lies on the Giza plateau about 60 meters above sea level. A dominant weather pattern in East Africa is called the Red Sea Trough, which is "hot and dry". But quite abruptly, only a few times a year (mainly in October or November) this pattern changes into the "Active" Red Sea Trough that is accompanied by heavy rainstorms, flash floods, and severe societal impacts in the Middle East. De Vries et al. (2013) and Alharbi (2018) explain the atmospheric dynamics leading to extreme precipitation, and twelve many cases caused by the "Active" Red Sea Trough are listed in (De Vries et al., 2013), which affected the Levant during the 25 years, Oct.1979 ~ Oct.2004, and four cases among which brought terrible damage in Egypt. For example, the case of the November 1994, one of the worst disasters with 600 casualities in Upper Egypt, affected Egypt from Luxor all the way to Cairo, and its torrential rains brought terrible damage in the Valley of the Kings (Weeks, 1995)[4]. The Tempest Stela of Ahmose describes a great storm which struck Egypt

[4]Los Angeles Times (Nov.5, 1994) "New Flooding in Egypt Threatens Historic Tombs" (https://www.latimes.com/archives/la-xpm-1994-11-05-mn-58761-story.html).

about 1550 BC and destroyed tombs, temples and pyramids in the Theban region. This severe weather is suspected to be due to the climate change triggered by a massive volcano explosion at Thera, the island of Santorini in the southern Aegean Sea (Ritner & Moeller, 2014). Heavy rainstorms around the Great Pyramid during the Old Kingdom were evidenced by the excavations of the town Heit el-Ghurab, or the Lost City of the Pyramid, as stated in (Ogilvie-Herald, 2020) that: "Excavations have shown the town was repeatedly destroyed by flash flooding and rebuilt during the reigns of the pharaohs Khafre and Menkaure, both kings of the 4th Dynasty. During Khafre's reign Heit-el-Ghurab was struck by three floods in twenty-six years, the first 'destroyed the town, while the others caused widespread damage.' However, during the later reign of Menkaure evidence from the excavations have shown that the flooding was far worse." From this record, it would be quite natural to assume that the Pyramid was actually hit by a few heavy rainstorms even during its construction over 20 years. So we believe, expecting such severe rainstorms and knowing that the limestone is vulnerable to the erosion by rainwater, the Great Pyramid incorporated some measures against the rainstorms. First, it was quite needed that the base of the Pyramid be protected from the erosion, and this would be one of the reasons that they made the specific fine structure like **Figure 14**, in which stones were fitted together precisely and the spaces between them were filled with homogeneous mortar. Second, the whole Pyramid was covered with a smooth roof of casing stones. These outer casing stones were fitted together with extremely high precision, and we believe this precision was not only for the beauty of the appearance but also for the practical reason to protect the Pyramid from the erosion by rainwater. But unfortunately, most of the outer casing stones of the Great Pyramid fell down or loosened due to the massive earthquake in 1303 AD (of magnitude 6.5 on the Richter scale, with the epicenter Fayum). So, after this earthquake the remained inner structure has been exposed directly to rainstorms until now. Even before this destructive earthquake, we suspect, the Pyramid would have been hit by severe earthquakes and violent rainstorms so that some of its outer casing stones could be damaged or loosened to let the rainwater seep through them to bring high humidity inside the Pyramid. According to (Butzer et al., 2013), the Old Kingdom (the 4th Dynasty) paleoclimatic anomaly, accompanied with heavy rainstorms, was repeated on a subdued scale during the Early Middle Ages. And, in the study of the climate during the Middle Kingdom, (Bell, 1975) asserts that "Review of textual and architectural evidence bearing on rainfall suggests that the Middle Kingdom had conditions similar to those of the A.D. 1800s, with heavy rainfalls somewhat less rare than in the present century." Thus, it seems that this pattern of anomaly has continued ever since (perhaps, a long before) the Old Kingdom, and we may conclude that the Great Pyramid has been long exposed to rainstorms quite frequently by the timescale of the Pyramid: Indeed, "a few times in 20 years" would accumulate during the 4500 years into more than 500 times! (An example of photo of a recent rainstorm which passed near the Bent Pyramid can be seen in the report by the photographer T. Badal: "A Refreshing Look at Egypt's Ancient Pyramids"[5]) Nevertheless, the Great Pyramid has survived through this long-term exposure to rainstorms even after the loss of the casing stones, and this surprising fact strongly infers the existence of some system of drainage inside the Pyramid. What system? We proposed in (Kato, 2020) that almost all stones of nucleus were chamfered and the chamfered edges of stones were utilized to make vertical holes and wells ("well" means here an empty column surrounded by walls of stones, like a chimney), in particular, the "Central Well" around the axis of the Great Pyramid from the center of the base to the apex. If all edges of cubic stones were chamfered, their chamfered parts can make not only vertical holes but also horizontal vents, and such vents and holes would form a three-dimensional grid throughout the whole nucleus, with the main vent, the Central Well. Therefore, this grid could serve well as a drainage and ventilation system of water and air. **Figure 18** and **Figure 19** show the general flow of rainwater inside the Pyramid in case all courses are horizontal or inclined, respectively. Noteworthy is that a slight inclination of courses changes the pattern of flow drastically, from **Figure 18** to **Figure 19**. The lateral flow in **Figure 18** is based upon the nature of water as a liquid that any amount of water placed on a horizontal plane will soon be spread laterally, but such flow would be quite slow so that the rainwater would not be drained easily in case of **Figure 18**.

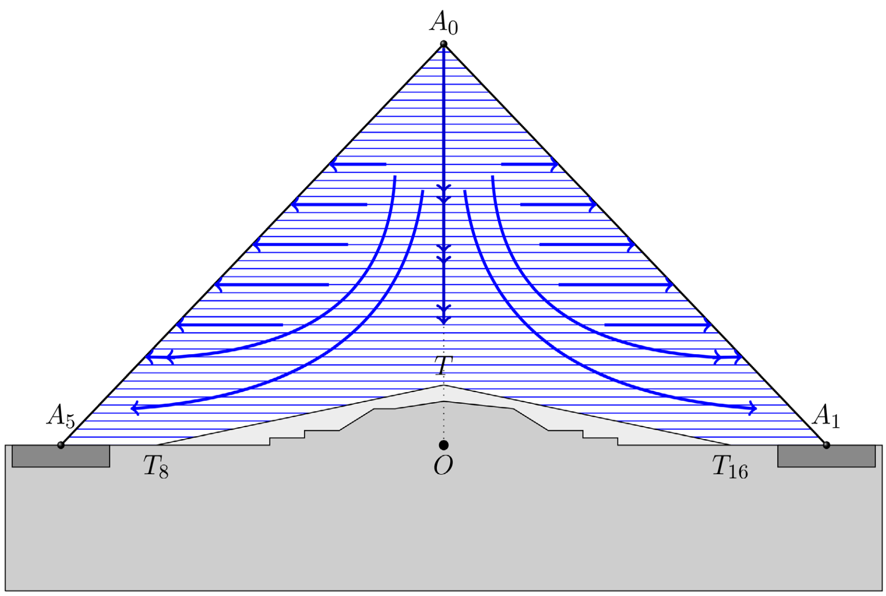


Figure 18. Vertical cross section $A_0A_1A_5$, where A_1A_5 is the diagonal of the base, showing the flow of rainwater (by blue arrows) in case all courses are truly level. The double arrows along the axis A_0T show the flow through the Central Well. About this "Central Well" see (Kato, 2020). We note that the existence of the Central Well does not contradict **Figure 12**, an illustration viewed from the east to the west, since the Queen's Chamber is away from the central axis about seven meters eastwards.

[5] https://www.nytimes.com/2022/07/04/travel/egypt-pyramids.html.

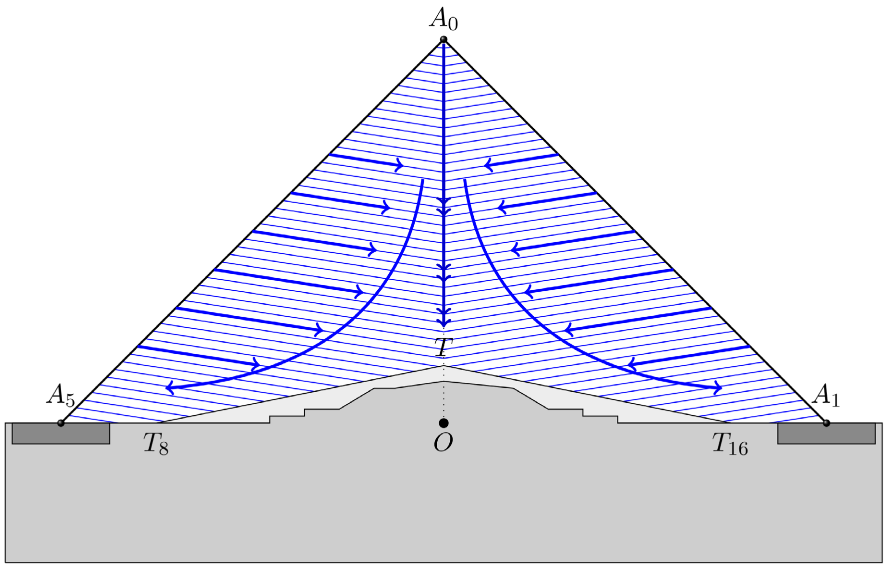


Figure 19. Vertical cross section $A_1A_0A_5$ showing the flow of rainwater in case all courses are inclined. The double arrows along the axis A_0T show the flow through the Central Well.

5. Inward Sloping Structure to Eliminate the Action of Lateral Forces

We have already shown that the concavity could be derived from the system of inward sloping courses. So, what we need to show next is the necessity of such system for the stability of the Great Pyramid. The stability we want to argue is the "long-term" stability against the high gravitational compression as well as against the natural disasters like severe earthquakes and heavy rainstorms (as mentioned in Section 4) experienced by the Great Pyramid during 4500 years. See (Morsy & Halim, 2015; Badawy, 1999; Hemeda et al., 2020) for historical earthquakes in Egypt. Though the actual notable one was the aforementioned case of 1303 AD which shook off almost all of the casing stones, "4500 years" is long enough to experience such "once-in-a-millennium" severe earthquakes at least several times. If we assume that the Pyramid was made only of horizontal courses, it is quite hard to imagine that such a strong ground shaking shook off only the casing stones without disturbing its inner structure. We will explain why? **Figure 20** and **Figure 21** illustrate how the Pyramid would behave under the ground shaking caused by an earthquake. The former is the case when we assume the Pyramid consists only of level courses, while the latter is when the Pyramid consists only of inclined courses. It would be obvious that the casing stones were shaken off because they were pushed outwards by the backing stones just behind them, inferring that the inner stones were also moved by the earthquake. Note that damage to buildings due to earthquake is related more closely to ground motion, rather than the energy of earthquake (the Richter scale), and an appropriate measure commonly used in earthquake engineering is "Peak Ground Acceleration (PGA)" which is equal to the maximum ground acceleration

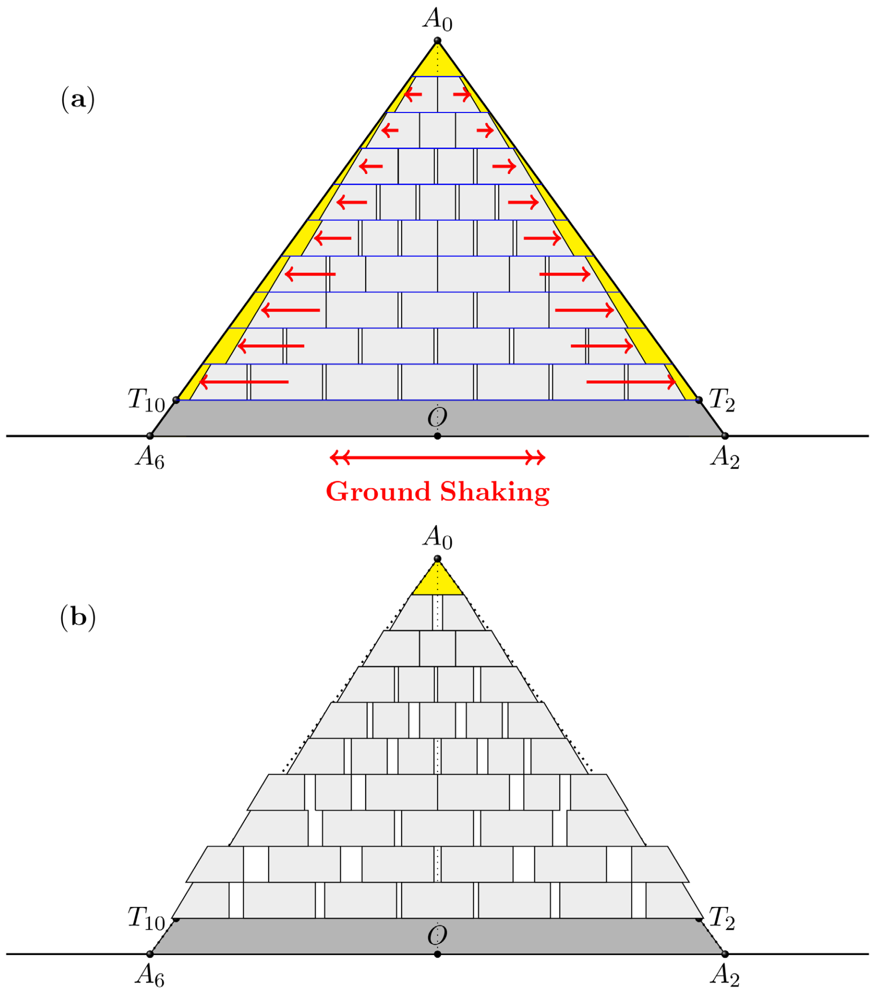


Figure 20. Ground Shaking (a) by an earthquake (strong as the case of the 1303 AD), and its aftermath (b), in case all courses were supposed to be horizontal. The ground shaking generates the lateral shaking of the horizontal courses, and the (yellow-colored) casing stones would be pushed outwards by the backing stones just behind them as shown by the red arrows. Note that the backing stones just behind the casing stones can be moved rather easily (compared with those in the center) since they are not so strongly compressed from the above.

that occurred during earthquake shaking at a location. Of course, each earthquake is an assembly of various kinds of seismic waves, and the main thrusts including PGA are always accompanied by many tremors. In either case of horizontal or inclined courses, such main thrusts would move the casing stones outwards (or upwards in case of inclined courses). But, the aftermath would be different. In case of horizontal courses some inner stones would remain to be separated as in **Figure 20**, but in case of inclined courses, the tremors after PGA would do some job to settle the inner stones down to their original positions as in **Figure 21**, since the courses are inclined. Hence the present non-disturbed, symmetric posture of the Great Pyramid would almost deny the structure only of truly level courses, taking account of all the hitherto earthquakes including the one of 1303 AD.

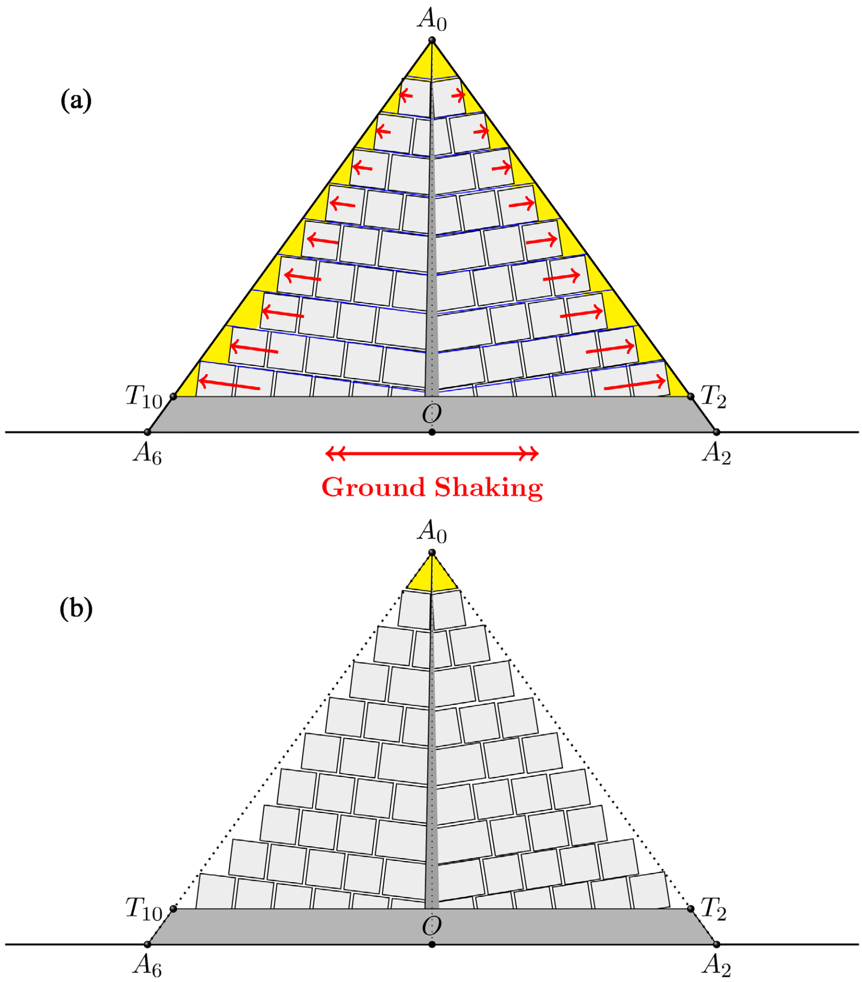


Figure 21. Ground Shaking (a) by an earthquake (strong as the case of the 1303 AD), and its aftermath (b), in case all courses are inclined. The ground shaking generates the shaking of the inclined courses, and the (yellow-colored) casing stones would be pushed outwards (or tossed in the air) by the backing stones just behind them as shown by the red arrows. Though the backing stones may be moved somewhat outwards or upwards by the "PGA (Peak Ground Acceleration)" of the earthquake, they would be settled down to their original positions by the tremors after the PGA, since the courses are inclined.

Recall the simple physical fact that the Great Pyramid stands upright only by the force of gravity. The physicist Mendelssohn pointed out in (Mendelssohn, 1976) that "in a pyramid containing stones of irregular shape, the vertically downwards acting force generated by the gravity will develop lateral components as in (b) of **Figure 22**, favouring a break-up and flattening of the structure", where he assumes the pyramid consisted only of truly level courses. Note that this picture (b) of **Figure 22** is similar to **Figure 18** because the flow of rainwater is governed also by the gravity. Mendelssohn (1973) or Mendelssohn (1976) explains that, when the blocks of irregular shape touch only in a few places, the pressure can rise locally to several hundred atmospheres, crumbling sets in at the affected regions of the stones and a shift in position of the individual blocks might take place. Then this shift causes the transformation of vertical into lateral

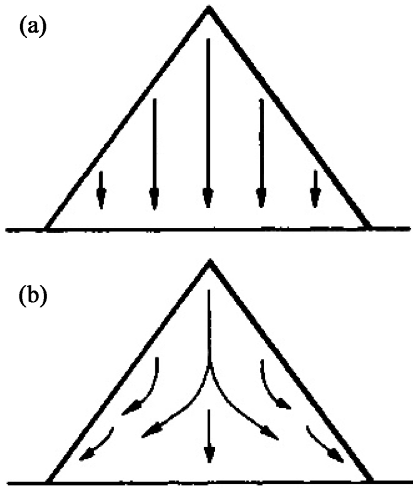


Figure 22. Illustrations in (Mendelssohn, 1973) with the caption: "The distribution of weight forces in (a) a pyramid built of well-squared blocks and (b) a pyramid with poor internal masonry."

forces. In general, as noted in Section 1, internal masonry of a pyramid is quite poor compared with its outside smooth masonry, and in case of the Great Pyramid we can further take account of the energy management in raising the vast quantity of the stones, so that we may naturally assume that blocks inside the Pyramid are relatively small and roughly shaped. Then recall our daily experience that a four legged stool on a floor will often wobble but a three legged one will not. Two roughly shaped stones would contact in a similar way that two faces do not adhere completely, rather touch only at three spots; it is even possible that two stones touch only at one tiny spot when they are placed horizontally. Then the aggregate of stones of irregular shape would be exerted by various kinds of stress like compressive, tensile and shear stresses. A stone is usually very strong against compression, but very weak against tension as the tensile strength is known to be only about one-tenth of the compressive strength. The maximum pressure at the base of the Great Pyramid is about

$$2.5\ \mathrm{t}/\mathrm{m}^2 \times 140 = 350\ \mathrm{t}/\mathrm{m}^2 = 3.5\ \mathrm{MPa}$$

assuming that a cubic limestone of volume 1 m^3 weighs about 2.5 tonnes and the height of the Pyramid is about 140 m. A limestone usually can endure the pressure about $100\ \mathrm{MPa} = 10000\ \mathrm{t}/\mathrm{m}^2$, but this high value is meaningful only if the stone is well squared, and the story would be different if the stone is not well squared and has to support a heavy weight only using some three spots of the stone, as illustrated in **Figure 23**. There would be various mechanisms of how some vertical force, caused by the heavy load or the earthquake, would transform into lateral forces. An instance is shown in **Figure 24** when the stones are laid horizontally. Any original rock would contain some invisible tiny cracks so that it might happen that those cracks would be enlarged little by little by the stress even below the yield strength of the rock, and a long-term accumulation of such strains might deform the rock visibly. This kind of consideration is known to

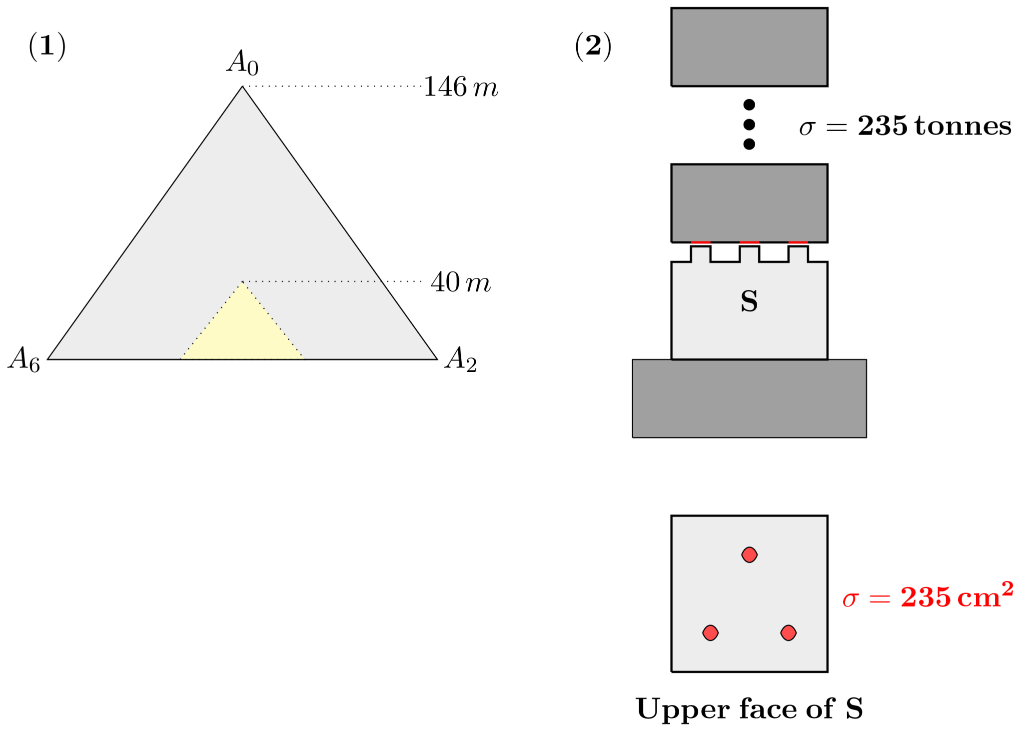


Figure 23. Assuming the compressive strength of a limestone is $100\ \text{MPa} = 1\ \text{t/cm}^2$, any well squared limestone with the upper face of 1 m^2 can support 10,000 tonnes, but a stone with an irregular upper face can support only σ tonnes if it touches the upper stone only with the area σ cm^2. The illustration (2) shows the case of a stone S such that the touching area is $\sigma = 3\pi(10/2)^2 = 235.6\cdots \approx 235\ \text{cm}^2$ consisting of three (red) discs, each of diameter 10 cm. The cross section (1) of the Pyramid indicates the part yellow-colored (just below the King's chamber) consisting of stones each of which has to support more than $2.5\ \text{t} \times 100(\text{m}) = 250\ \text{t} > 235\ \text{t}$. So, if the stone S were placed horizontally inside the yellow part, its upper face would have been crushed when the construction of the Pyramid was completed.

be the "rheological" viewpoint, where the term "rheology" is based upon the idea of Heraclitus *panta rhei* "everything flows". For example, suppose in (b) of Figure 22 blocks had "flowed" in the arrowed direction at the average rate of one millimeter a year; then such "flow" would amount to 4 or 5 meters during 4500 years, an observable distortion much bigger than the concavity of one meter we argued in Section 1. But, no such distortion can be seen in the present symmetric Great Pyramid, leading to the denial of the structure of the Pyramid consisting only of truly level courses.

What we want to remark further is the effect of erosion by the heavy rainstorms as explained in Section 4. The important aspect we want to point out is that the effect of erosion would be quite different between the horizontal course and the inclined one in case both admit somewhat irregular gaps between blocks. The erosion on the horizontal courses would separate stones laterally as illustrated in Figure 25, while the erosion on the inclined courses would tighten the arrangement of blocks, incorporating the gravitational force, as shown in Figure 26. Observe then that each block separated laterally from other blocks as

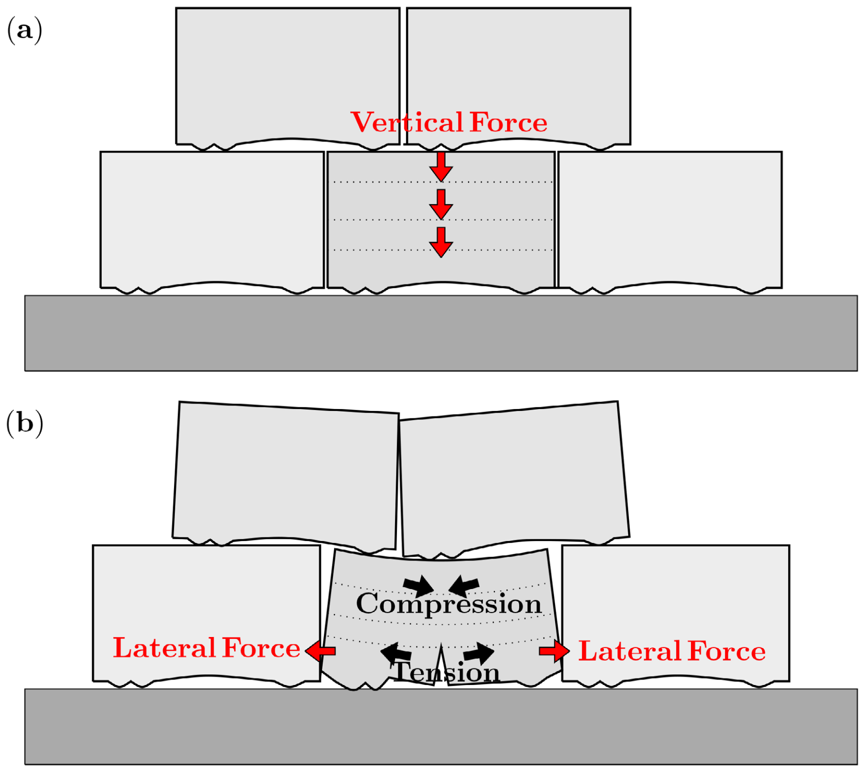


Figure 24. Instance of the mechanism of "the transformation of vertical into lateral forces" by a sag in a stone, due to the fact that a stone is weak against tension though strong against compression. This mechanism works when the stones of irregular shape are laid horizontally. The actual crack as in the middle stone of (b) might be very small in the order of millimeters.

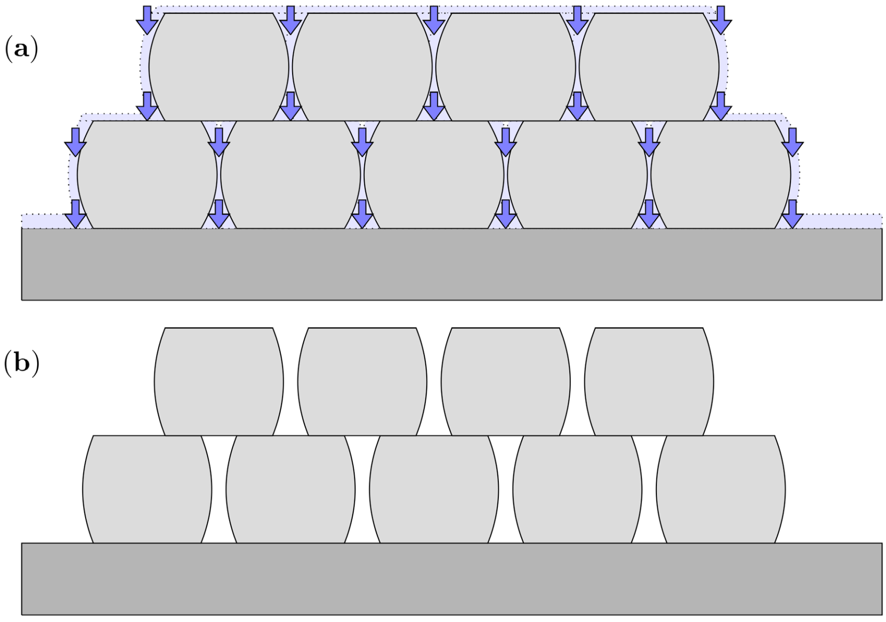
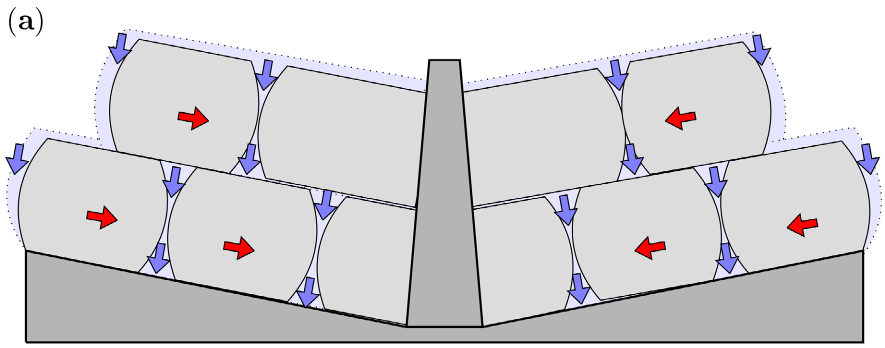


Figure 25. Effect of Erosion of blocks on the Horizontal courses by Rainfall: The rainwater flows down through the gaps between the blocks, not well-squared, as indicated by the blue arrows in (a), eroding the vertical faces of blocks to separate them laterally like (b), a bit exaggerated.



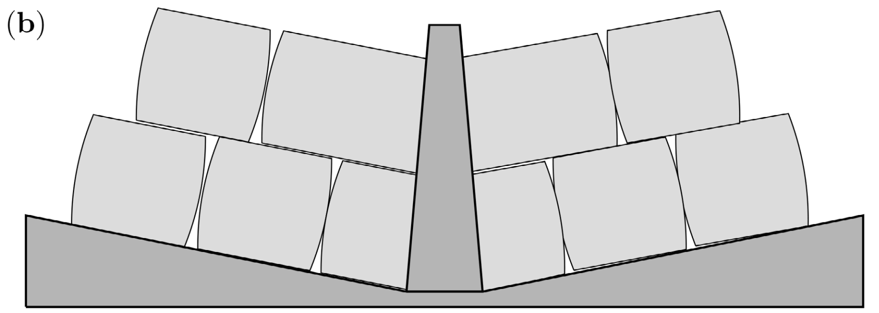


Figure 26. Effect of Erosion of blocks on the Inclined courses by Rainfall: The rainwater flows down the gaps between the blocks, not well-squared, as indicated by the blue arrows in (a). The erosion would incorporate the gravitational force to move blocks inwards (as the red arrows of (a)) so that the arrangement of blocks would be tightened further like (b).

in (b) of Figure 25 would be compressed only in the vertical direction, but each inclined block as in (b) of Figure 26 would be compressed in all of the three directions since it is completely surrounded and confined by other stones. It is well known that a stone is much stronger against the confined compression than against the uniaxial unconfined compression. Hence, the erosion makes the stones on the horizontal course vulnerable to the gravitational compression, but in contrast, the erosion strengthens the stones on the inclined course against compression. Effect of ground shaking due to earthquake is similar to that of erosion as already observed in Figure 20 and Figure 21. Thus, both erosion and earthquake are in favor of inclined layers, but against horizontal ones. The only stable case for a horizontal course would be that it consists of uniform stones fitted very precisely or well connected by mortar each other, since such unified course would behave like a bedrock. Note that granite is much stronger than limestone against earthquake and erosion. The first course of the Great Pyramid is such a stable horizontal course where many large granite blocks are used. But, it would be almost impossible to make the whole Pyramid only of such stable horizontal courses, as such precise placement would consume too vast energy to complete the construction of the Pyramid in two decades or so, taking account of its immense quantity of stones, over two millions.

Note that the above argument is about the inner part of the Pyramid, and we can see some evidence of the erosion like **Figure 25** in some outer stones laid horizontally. For example, see **Figure 27** which is the drone view of the top of the Great Pyramid by the famous photographer A. Ladanivskyy. Here we can see wide vertical gaps between blocks as in **Figure 25**, which would be mainly due to the erosion, not due to the eathquake since the top of the Pyramid is rather stable against ground shaking. We believe the original arrangement of blocks on the top was much tighter. Though not in the Pyramid, another evidence is **Figure 28** of the Moria Roman aqueduct[6,7]. This aqueduct is made of limestone called travertine, and the erosion by rainwater as well as the weathering by wind widened the gaps between stones so that the key stone of this arch is now almost dropping. We note, though, that it is not easy to compare the aqueduct with the Great Pyramid. Indeed, the Pyramid is 2.5 times older than the aqueduct since the aqueduct dates back only to the end of the 2nd century AD, but Moria is much wetter than Giza. How erosion or weathering or high compression has

Figure 27. Drone View of the Top of the Great Pyramid by the photographer A. Ladanivskyy. (https://www.thisiscolossal.com/2021/07/drone-view-of-giza/)

[6] https://www.archaeology.wiki/blog/2021/03/19/the-roman-aqueduct-at-moria/ (19 Mar 2021) "The Roman Aqueduct at Moria".

[7] https://www.archaeology.wiki/blog/2021/03/04/danger-of-the-moria-roman-aqueduct-collapsing/ (04 Mar 2021) "Danger of the Moria Roman aqueduct collapsing".

Figure 28. The Roman Aqueduct at Moria (https://www.archaeology.wiki/blog/2021/03/19/the-roman-aqueduct-at-moria/).

changed outer stones of the Great Pyramid can be seen in (Hemeda & Sonbol, 2020).

6. Scaled-Down Models

Mendelssohn (1971) demonstrated an experiment **Figure 29** about the action of lateral forces in a small model of a pyramid, made of a highly viscous, homogeneous material (the exact size and the name of material of this model are unclear). One may wonder why did he choose a "small" and "plastic" model? In this section we explain the appropriateness of his choice by introducing the idea of the "scaled-down dynamical" model. Compared with the large scale in space and time of the Great Pyramid, our size is small and our lifespan is short so that it is not easy for us to understand properly how the Pyramid behaves in the long run. In order to overcome such difficulties, geologists devised the method of creating "scaled-down" models (Hubbert, 1937; Deus et al., 2010; Schellart & Strack, 2016). So, let us consider some scaled-down models applying the theory of scale model construction as described in pp.143-150 of (Pollard and Fletcher, 2005), or (Hubbert, 1937). See also (Merle, 2015) about the basic principles of the scaling procedure. Note that we scale down everything, space and time, not only the Pyramid but also its environment including the bedrock, earthquakes and rainstorms. The Great Pyramid is the prototype, and let us reduce this prototype by the scale of 10^{-3}, to get a small model **M** (10^{-3}) of size about 20 cm. Precisely stating, the model ratio L_r for the length is

$$L_r = L_m / L_p = 10^{-3}$$

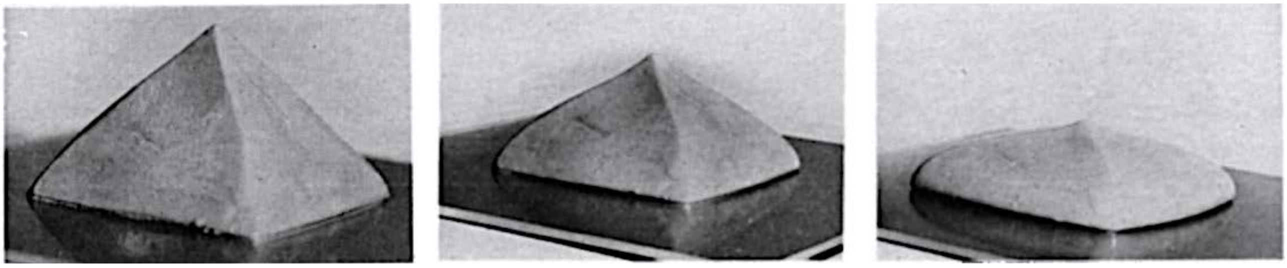


Figure 29. Picture in (Mendelssohn, 1971) with the caption "Three successive stages in the plastic flow of a pyramid under its own weight. Material and structure of the model were homogeneous, showing merely the action of lateral forces." The same picture appears in his book (Mendelssohn, 1976) with the caption "Plastic flow under gravity. The small pyramid model, made of a highly viscous material, collapses under its own weight".

where L_p stands for the length of any part of the prototype, and L_m is the corresponding length of the model (the subscripts *m* and *p* refer to the model and prototype, respectively). Since the dimension of the volume is Volume = (Length)^3, the model ratio V_r for the volume should be

$$V_r = \frac{V_m}{V_p} = \frac{L_m^3}{L_p^3} = L_r^3 = 10^{-9}.$$

For simplicity let us choose the material of the model in such a way that the model ratio ρ_r for the mass density is one, that is, the density of any part in the model is equal to the density of the corresponding part in the prototype: $\rho_m = \rho_p$. Hence the ratio M_r for the mass is the same as that of the volume:

$$M_r = \frac{M_m}{M_p} = \frac{\rho_m V_m}{\rho_p V_p} = V_r = 10^{-9}.$$

Next, we need to determine the ratio T_r for the time

$$T_r = \frac{T_m}{T_p} \text{ where } T_p = 4500 \text{ years.}$$

But, this ratio can not be chosen independently since the time relates to the acceleration, and in order to make the model *dynamically similar* (i.e., holding the similarity of driving forces) to the prototype, we need to require that the gravitational acceleration around the model in the laboratory is the same as that around the Pyramid on the Giza plateau. Therefore, the ratio a_r for the acceleration should be one:

$$a_r = \frac{a_m}{a_p} = \frac{L_m T_m^{-2}}{L_p T_p^{-2}} = L_r T_r^{-2} = 1, \text{ i.e., } L_r = T_r^2.$$

Hence

$$T_r = L_r^{1/2} = 10^{-3/2} = 0.0316\cdots(\approx 1/30)$$

and

$$T_m = T_p T_r = 4500 \text{ years} \times 0.0316\cdots = 142.3\cdots \approx 140 \text{ years.}$$

This means we need to observe the small model **M** (10^{-3}) for $T_m = 140$ years in a laboratory, about which we discuss later. Using the dimensional formulae (Force) = (Mass) × (Acceleration), (Stress) = (Force)/(Area) = (Force)/(Length)^2, we can get the ratios F_r, S_r for the force and stress, respectively, as follows:

$$F_r = M_r \cdot a_r = M_r = L_r^3, \quad S_r = F_r \cdot L_r^{-2} = L_r^3 \cdot L_r^{-2} = L_r.$$

Summarizing, we have the model ratios

$$T_r = L_r^{1/2} = 10^{-3/2} > L_r = S_r = 10^{-3} > V_r = M_r = F_r = L_r^3 = 10^{-9}$$

along with our convention $\rho_r = a_r = 1$. Let us assume for simplicity that the prototype is the square pyramid with base length 200 meters and height 140 meters, consisting of cubic blocks, each with the side length 1 meter and the weight 2.5 tonnes. Let us call the compressive strength simply as "strength", and assume also that the strength of blocks is uniform and is equal to that of the bedrock, so that we can call this same value S_p as "the strength of the Pyramid". So, all blocks are uniform in size, weight and strength. Thence, the model **M** (10^{-3}) has the base length 20 cm and height 14 cm, and consists of very small cubic blocks, each of which has the side length $L_m = 1\,\text{m} \times 10^{-3} = 1\,\text{mm}$, the mass $M_m = 2.5\,\text{tonnes} \times 10^{-9} = 2.5\,\text{milligrams}$, and the strength $S_m = S_p \times 10^{-3}$. Let us note here about the strength of blocks of the Great Pyramid. According to Arnold (1991), "porous" limestone can endure the pressure 20 - 90 MPa and "dense" limestone can endure 80 - 180 MPa, so, a typical limestone would be able to endure the pressure about 100 MPa, as we assumed in Section 5. On the other hand, some backing limestone samples were recently collected from the Great Pyramid (Hemeda & Sonbol, 2020) to examine their strength. The result was about 15 MPa, very low value, probably due to the weathering over four millennia. So, precisely speaking, the strength S_p is a decreasing function of the time. In case $S_p = 100\,\text{MPa}$, we have $S_m = 0.1\,\text{MPa} = 1\,\text{atm}$, which means each block of size 1 mm in the model can support the weight of only 10 grams, and in case $S_p = 15\,\text{MPa}$, only 1.5 grams. So, we may be able to make such blocks using some soft granular material. Then recall the rheological fact in our daily life that granular materials, like sands, grains of wheat, rice or corn, can flow like a liquid under the gravitational force. Hence, our model **M** (10^{-3}) is quite similar to the Mendelssohn's model **Figure 29**, not only geometrically but also dynamically. The flow of grains is generated by the gravitational force conducting stress by "force chain" formed by grains resting on one another. Inside the Pyramid this "force chain", formed by stones (of irregular shape) resting on one another, generates a flow of stones which would be quite slow, maybe a millimeter a year. Direction of this flow would be essentially the same as that of rainwater illustrated in **Figure 18** and **Figure 19**, since both force chain and rainwater are governed by the gravity. Note that the flow of force along the blue curved arrows can be sustained by the reinforced corner structure like the corner sockets (see **Figure 15**), so that the stones would be moved by the force chain, only along the blue "straight" arrows in either figure. Note also that such movement of stones along the straight arrows in **Figure 19** is essentially a compression and would eventually cease since all of them direct to the terminal near the center of the Pyramid. Thus, we may conclude, via the model **M** (10^{-3}), that stones would move laterally to flatten the Pyramid if we assume it were made only of horizontal courses, while stones would be concentrated around the center of the Pyramid to tighten its whole structure if we assume it were made only of inclined courses. Both **Figure 12** and **Figure 30** are almost to scale so that, assuming $\overline{A_2A_6} = 20\text{ cm}$, they can be viewed as illustrations of the cross section of the model **M** (10^{-3}) in case of inclined courses and in case of truly level courses, respectively. These figures show well that the blocks are very small compared with the whole Pyramid so that they can be viewed naturaly as "fine grains" in the model **M** (10^{-3}).

As mentioned before, after we made an appropriate pyramid for the model **M** (10^{-3}), there remains a difficult task to observe it for $T_m = 140$ long years in a laboratory. But this would be almost impossible to accomplish since "140 years" is much longer than our lifespan. So, we should be satisfied with a thought experiment, or observe the model during the first few years to imagine the rest of the experiment. In spite of this defect, still we can say that the term of 140 years is much easier to imagine than the long term of 4500 years. For example, it is not easy for us to imagine the rheological accumulation over four millennia of the very slow flow of stones by "force chain" inside the Pyramid, but we can expect by intuition that such a flow in the small model **M** (10^{-3}) of fine grains of low strength might happen during the observation of 140 long! years. Deriving such proper intuition is the crucial role of the scaled-down model. Note further that in this thought experiment of "140 years", the model should experience the

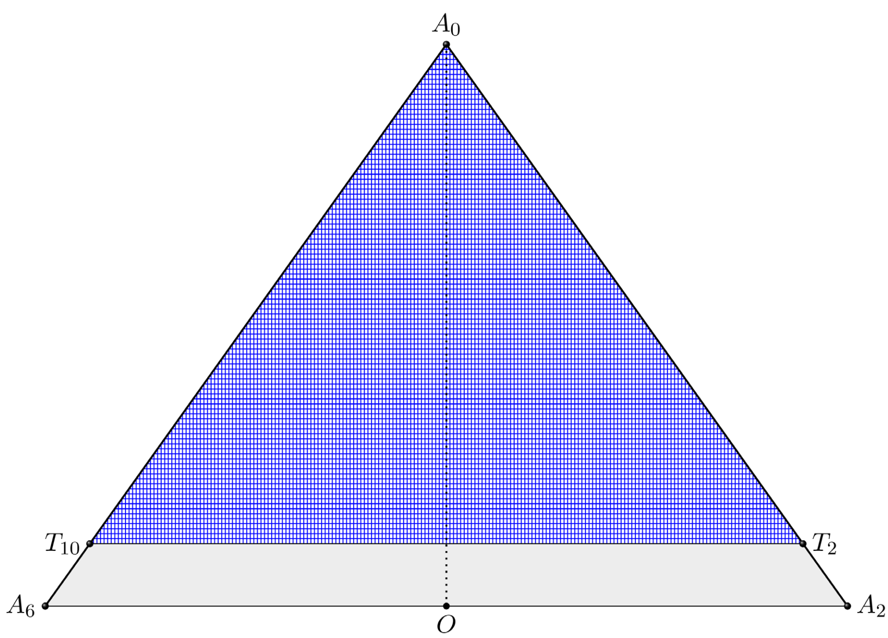


Figure 30. Vertical cross section $A_0A_2A_6$ of the Great Pyramid in case the Pyramid consisted only of truly level courses. This illustration is almost to scale, hence can be the cross section of the model **M** (10^{-3}) if we suppose $\overline{A_2A_6} = 20\text{ cm}$.

downsized versions of all the ground shakings (caused by the earthquakes) and rainstorms that the Great Pyramid has experienced, in a historically similar timing. As mentioned before, damage to buildings due to earthquake can be measured by the maximum ground acceleration called "Peak Ground Acceleration (PGA)", rather than the energy of earthquake (the Richter scale). Then we can take the advantage of our convention that the ratio of the acceleration is one, that is, PGA in the model should be the same as PGA in the prototype. An additional remark in transferring an earthquake into the model is that we have to scale down everything around the Pyramid. So, not only the blocks of the Pyramid, but also its bedrock should be replaced by the one with the low strength reduced by the scale $S_r = 10^{-3}$. Thence we can let the model, together with its bedrock, experience the same (w.r.t. PGA) ground shakings that the Great Pyramid have experienced, in a historically similar timing.

As for rainstorms, by the ratio $T_r = 0.0316\cdots \approx 1/30$ for the time, "a few times in 20 years during the 4500 years" in the prototype would be converted into "several times in every year during the 140 years" in the model. Since (Velocity) = (Acceleration) × (Time), the ratio v_r for the velocity is the same as that of the time: $v_r = a_r T_r = T_r \approx 1/30$. Hence, a storm at the speed of 30 meters/sec in the prototype should be converted into light winds of 1 meter/sec in the model, so that each severe rainstorm in the prototype would turn into a gentle spray of water in the model. So, for instance, if the rainstorm lasted two days, then water should be sprayed onto the model for about one and a half hours: $48\ \text{hours} \times 0.0316\cdots \approx 1.5\ \text{hours}$. Here again, we point out that "several times in every year during 140 years", or equivalently, "once in a few months during 140 years" is much easier to understand than "a few times in 20 years during 4500 years". We note here that it is not an easy task to maintain strict dynamical similarity between the proptotype and the model when we need to evaluate multiple kinds of force including non-mechanical one. For example, "the erosion of limestone by rainwater" is not only the mechanical detachment of tiny particles from the surface, but it also includes a chemical reaction, i.e., a dissolution along micron-scale grain boundaries of limestone. So, we must be very careful about the choice of material for the model if we want strict similarity of erosion between the proptotype and the model. We have disregarded such an intricate problem in the above argument.

Models not so small are also useful in understanding how the long-term dynamical behavior of the Pyramid depends upon its inner structure. For example, let us consider **M** (10^{-2}), **M** (10^{-1}), **M** (1/2) and **M** (5/8).

1) The model **M** (10^{-2}): This model is of size about 2 meters and consists of blocks of size about 1 cm, which would be quite an appropriate size to to be handled with; for example, we can lay blocks of size 1 cm to make horizontal or inclined courses, which we can not do in the model **M** (10^{-3}) since blocks of size 1 mm are too small. The strength of blocks in this model should be $1\ \text{MPa} = 10\ \text{atm}$, which is about the half of the strength of "sun dried brick". (It is known that the strength of "sun dried brick" is about 2 MPa, while that of "first class brick" is

about 10 MPa.)

2) The model **M** (10^{-1}): This model is of base length 20 m and height 14 m, and consists of blocks of size about 10 cm, so its external appearance is almost the same as that of the satellite pyramid (G1-d) within the Great Pyramid complex, whose original dimensions are of base length 21.75 m, and of height 13.8 m. But, a notable difference is that the model **M** (10^{-1}) is made of small blocks of size about 10 cm, while the blocks in (G1-d) would be large as one meter. Further, the strength of each block in the model should be reduced to one-tenth of the strength of the corresponding block in the Great Pyramid, but the blocks in (G1-d) would have the same strength as those of the Great Pyramid. Some core of (G1-d), consisting of horizontal courses, can be seen in its remains (Hawass, 1996). So, to make the model **M** (10^{-1}) of horizontal courses we need to replace the stones of the satellite pyramid (G1-d) by "first class bricks" (with the strength 10 MPa) of size about 10 cm. We here want to remark that the original satellite pyramid (G1-d) could be quite strong and stable since its size is small, which means for example that each cubic block of size one meter at the base of (G1-d) needs only to endure the low gravitational pressure at most 0.35 MPa since (G1-d) is only 14 m heigh, compared with the case 3.5 MPa of the Great Pyramid of 140 m heigh (as mentioned in Section 5). The formulation of the scaling theory was first introduced by (Hubbert, 1937), where Hubbert notes conclusively that "quite generally, for a body of material having a given specific strength, the over-all strength of the body taken as a whole decreases with increase of size. Thus small bodies of a given material are strong; large bodies of the same material are weak, and larger the body the greater its weakness." This applies well to our case: The small (G1-d) could be strong but the large Great Pyramid could be weak, as long as we assume in both the same structure of horizontal courses made of the same material, i.e., a cubic limestone of size one meter. So, we can believe that in spite of the structure of horizontal courses, the original satellite pyramid (G1-d) could have remained intact until now if its blocks were not looted.

3) The model **M** (1/2): A bit surprisingly, we could find the candidate of this model. We next show that the Step Pyramid of Djoser (the Third Dynasty) on the Saqqara plateau can be identified essentially as the model **M**(1/2) in case the Great Pyramid was assumed to be made entirely of truly level courses. The Step Pyramid was built as the first monumental structure made of stone, about 4600 years ago, i.e., about a century before the Great Pyramid. Originally the building was 62 meters high with a base of 109 × 125 meters, and its blocks are of limestones of size 30 - 50 cm. Its core consists of six mastabas of horizontal courses, built on top of each other, and is surrounded by inward-leaning accretion layers (Isler, 2001). On the other hand, the pyramid of the model **M**(1/2) is 70 meters high with a base of 100 × 100 meters, and consists of blocks, each of which is a cube of size 50 cm, weight 2.5/8 tonnes ≈ 300 kg, and of strength $S_p/2$. Since the ratio for the time is $T_r = L_r^{1/2} = 1/\sqrt{2} = 0.707\cdots$, as a model for the Great Py-

ramid we need to observe it during $4500 \times 0.707 \cdots \approx 3200$ years. Now compare the Step Pyramid with the model **M** (1/2). Their appearance is almost the same, including the size of blocks. Their inner structure is also similar as long as we assume the Great Pyramid consisited of truly level courses. A slight difference is that the strength of the model **M** (1/2) is about a half of the strength of the Step Pyramid. Thence, we may roughly conclude that (i) observing the model **M** (1/2) of "lower" strength during "shorter" period (3200 years), and (ii) observing the Step Pyramid of "higher" strength during "longer" period (4600 years), would reach almost the same result. And the result of (iii) is the present status of the Step Pyramid. As reported in (Kukela & Seglins, 2013) and (Ewais et al., 2016), the Step Pyramid has been deteriorated seriously by weathering, and heavily by the earthquakes. (It seems the outer faces of the Step Pyramid were deteriorated especially by the erosion of wind carrying sands, but what we concern is the deterioration of its core due to the structure of level courses.) In particular, the 1992 Cairo earthquake caused severe damage to this pyramid on the Saqqara plateau so that, due to the risk of collapse, the pyramid had been closed to visitors for nearly 14 years, and reopened in March, 2020. (This 1992 earthquake was "moderate", of magnitude 5.8, but unfortunately, the focus was quite near, about 14 km from the Step Pyramid (Khalil et al., 2017).) **Figure 31** shows the photo, taken in 2019, of the Step Pyramid under the restoration work. In short, the result of the observation (ii) is that the Step Pyramid has almost collapsed. Consequently the model **M** (1/2) would collapse similarly, after the observation (i). Therefore, we can conclude that the Great Pyramid would have almost collapsed until now if its core were made only of truly level courses.

4) The model **M** (5/8): The Meidum Pyramid (mentioned in Section 1), the second pyramid after the Step Pyramid, has its unique appearance due to its partial collapse before its completion. It has a base with the side length 144 m and its present height is 65 m, but 91.65 m if finally fully cased, so its original shape

Figure 31. The Djoser Step Pyramid at Saqqara under the restoration work, in April, 2019. A photo in BBC News: https://www.bbc.com/news/uk-wales-47828999.

is proportional to the Great Pyramid. Seyfzadeh (2017) describes that the exterior of the Meidum Pyramid was a scaled-down version of the Great Pyramid by a factor of 5/8. So, we may say that the model **M** (5/8) has the shape of the (intended) Meidum Pyramid, but we can not identify it with the Meidum Pyramid because of the difference of their inner structures. The inner structure of the Meidum Pyramid is exceptionally well known: The Meidum Pyramid has a central core of 60 cubits width, against which 9 accretion layers lean, and such entity was further surrounded with thick outer mantle (about 7 m) of packing stones laid horizontally. Roughly speaking, it is made of the nucleus of "accretion layers" surrounded by the mantle of "horizontal courses". One of the reasons of its collapse was suspected to be due to the fact that the outer mantle was built on sand instead of solid rock, unlike the Great Pyramid (Mendelssohn, 1973). Though the Meidum Pyramid can not be the dynamical model **M** (5/8), we can learn from its partial collapse that the "inclined" core of accretion layers is stronger than the mantle of "horizontal courses" since the most of the core "remains as a tower of three great steps" (Isler, 2001).

Isn't it amazing that some conceptional ties emerge among isolated pyramids through "models", as we have seen in the above (2), (3) and (4)?

In Cacciola et al. (2022) we can see a "gelatin" model of the Step Pyramid, down-sized by the scale of 1/500. This model was made for the experimental test of the Vibrating Barriers (device for the seismic protection of the Step Pyramid), and built with "gelatin", precisely: "The model has been made of a mix of gelatine/glycerine/cold water/hot water with weight proportions 1/3/2/3 and cured for 3 days." Since the Step Pyramid itself can be seen as a model of the Great Pyramid reduced by 1/2, this gelatin model actually reduces the Great Pyramid by $1/2 \times 1/500 = 10^{-3}$, so that it can be regarded as the model **M** (10^{-3}) of the Great Pyramid consisting of truly level courses. Hence, this exemplifies that it is possible to make our model **M** (10^{-3}) using the gelatin, which would be quite similar to the Mendelssohn's model **Figure 29**. See Merle (2015) about gelatin as a material for experiment.

7. Concluding Remarks

We have shown that the concavity of the Great Pyramid could be derived from the geometry of its core of inclined courses gently sloping inwards, and such inclined structure was quite needed for the long-term stability of the Pyramid against high gravitational pressure, earthquakes and rainstorms. It was often asserted only from the external appearance of pyramids that "in the major shift from step to true pyramid, the earlier pyramids were built with slanting accretion layers, but the later ones were built with horizontal courses." To turn this assertion into a true statement in our sense, the term "horizontal" should be replaced by "almost horizontal, but slightly inclined". We believe the "immortal" stability was the most important aspect of the Great Pyramid for the pharaoh Khufu and his people, and they were smart enough to introduce some structure in order to stabilize this huge monument, respecting the traditional one of accretion layers. What we have actually shown in this article is that the inner structure of the inclined courses can incorporate the natural forces to compact the core of the Pyramid towards the center, thus to strengthen it over time, so that we can believe that the Great Pyramid will keep standing upright almost forever though its outer faces will be weathered further. This is similar to the case of "the Roman concrete" which tightens over time by thriving in open chemical exchange with seawater, interlocking crystals. If someone will ask us why the Great Pyramid is so stable, we can answer simply, "because it gets stronger over time".

Remark 1. *Possible modified inner structure of the Great Pyramid:*

There would be various modified cores of inclined courses. **Figure 32** shows an example of modified inner structure of the Great Pyramid including a thin pyramid (with the cross-section $S_2A_0S_{10}$) around the central axis, consisting of horizontal courses. Such horizontal courses would be very helpful in constructing internal structure like the King's and Queen's Chamber, and for such construction the top angle $\angle S_2A_0S_{10} = 2\theta \approx 23^\circ$ where $\tan\theta = 1/5$ would be wide enough. We note that **Figure 32** is a two-dimensional figure of cross-section so that this modified part looks relatively large, but its actual three-dimensional size, i.e., volume, is very small: Let V_0 be the volume of the thin inner pyramid with the cross-section $S_2A_0S_{10}$, and V_1 be the volume of the pyramid with the

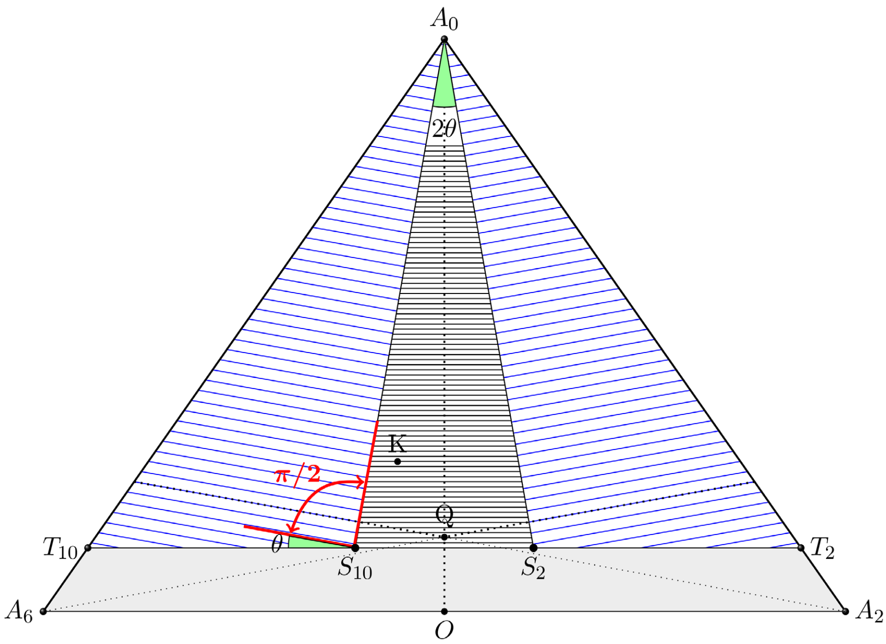


Figure 32. Cross-section of the Great Pyramid with the possible modified inner structure, including a central core of thin pyramid consisting of horizontal courses, with the vertical cross-section $S_2A_0S_{10}$ (its horizontal cross-section is like the white part of **Figure 8**). This thin inner pyramid has the top angle $\angle S_2A_0S_{10} = 2\theta \approx 23^\circ$ and the face angle $\angle A_0S_2S_{10} = A_0S_{10}S_2 = \pi/2 - \theta \approx 79^\circ$, where $\theta = \arctan(1/5) = 11.3\cdots^\circ$. K and Q show the positions of the King's and the Queen's Chamber, respectively, as in **Figure 12**; see (Creighton & Osborn, 2008) about their positions.

cross-section $T_2A_0T_{10}$ (i.e., above the light-gray part). Put $\angle OA_0A_2 = \angle OA_0A_6 = \phi$. Then, since the face angle of the Pyramid is "seked" 5 palms 2 digits, we have $\tan\phi = 5.5/7$. Hence

$$\frac{V_0}{V_1} = \frac{\overline{S_2S_{10}}^2}{\overline{T_2T_{10}}^2} = \frac{(2\tan\theta)^2}{(2\tan\phi)^2} = \left(\frac{\tan\theta}{\tan\phi}\right)^2 = \left(\frac{1/5}{5.5/7}\right)^2 = 0.064\cdots.$$

Thus, the volume of the inner thin pyramid is only about 6.5 percent of V_1, so, surely less than 6 percent of the total volume of the Great Pyramid.

Remark 2. *About the concavity in the Khafre and the Menkaure Pyramid:*

The three big pyramids of Khufu, Khafre and Menkaure were built in this order during the same Fourth Dynasty period around 2500 BC, so it would be quite natural to expect that all of their fundamental structure would be the same, i.e., all of them have the core of inclined courses. As mentioned in Section 1, Biondi & Malanga (2022) revealed the concavity of all the three pyramids, and it seems their result shows that the degree of the concavity of the Menkaure is almost the same as that of the Khufu's, but the concavity of the Khafre's is a bit shallower than the two neighboring pyramids. So, we may infer from **Table 1** that the slope of courses of the Menkaure pyramid would be the same as that of the Great Pyramid, $\tan\theta = 1/5, \theta \approx 11^\circ$, but that of the Khafre's would be about $\tan\theta = 1/8, \theta \approx 7^\circ$; then the maximal indent of the Khafre pyramid is less than 40 cm, and this small amount would be the reason why the concavity of the Khafre pyramid was not captured until now even by any Earth observation satellites, like the Quick Bird. Compared with the Great Pyramid, it is known that the internal structure of Khafre's is very simple. So, if we draw an illustration like **Figure 32**, of the Khafre's Pyramid, we do not need a wide top angle 23˚ for the inside pyramid; instead, 14˚ would be enough. Therefore, the simple structure around the central axis may be one of the reasons for the Khafre's Pyramid to have the very gentle slope of courses, about 7˚. Further we want to note that the slopes 1/5 and 1/8 would be very favoured by ancient Egyptians since, according to (Seyfzadeh, 2018b: p. 320), "the numbers five and eight were likely of special significance to the ancient Egyptians of Khufu's time originating from astronomic periods, converted to theological teaching, and possibly architecturally expressed in pyramids and mastabas".

Remark 3. *Other explanations in literatures:*

Various explanations, different from ours, have appeared about the concavity of the Great Pyramid. Let us mention six of those.

1) Mendelssohn (1973) asserts, with **Figure 33**, that "In each horizontal row of blocks a gentle grading was carried out by which the blocks at the edges were very slightly higher than those in the middle of the face. In this way the corners of each layer of packing blocks was somewhat lifted, making the whole layer slightly concave towards the apex. This method provided an addtional inward thrust which further counteracted any tendency of lateral forces to develop". This mechanism would be quite true as it is almost the same as **Figure 17**, but he believes this delicate construction was carried out throughout the whole structure,

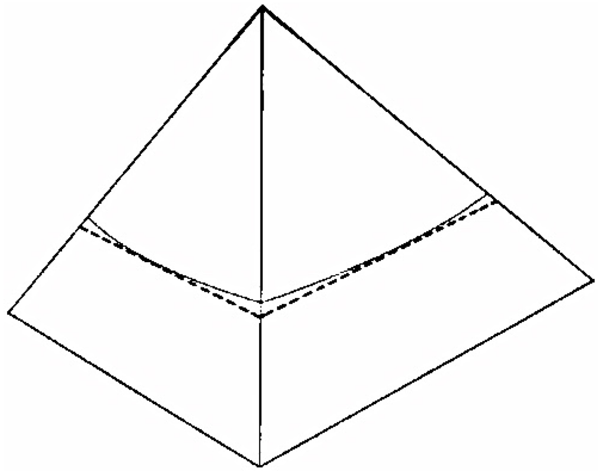


Figure 33. Illustration in (Mendelssohn, 1973) with the caption: "In order to provide additional stability, the masonry courses of the Khufu Pyramid were made slightly concave towards the apex".

as he continues that "This last mentioned safeguard was clearly a laborious and time-consuming device, requiring selection and grading of the blocks before they could be laid. It seems to have been regarded as an unnecessary precaution and was not employed at the next Giza pyramid, that of Khafre." We cannot believe such "laborious and time-consuming" efforts were made from the viewpoint of energy efficiency. It seems he also assumes that the most of blocks were "well-squared" to prevent the lateral forces, as illustrated in (a) of **Figure 22**, which we deny also from the viewpoint of energy efficiency.

2) Isler (1983) explains that the concavity is due to the efforts by the builders to control the alignment of the sides by using long cords, which led to slight variations in the levels of layers and a slight error in the alignment of the planes on each side of the faces. Our theory excludes this idea because we believe the builders of the Pyramid measured "angle" rather than "distance" knowing well that such long, hence heavy, cords cause errors in measurement.

3) Monnier (2022) insists that "the indentation of the faces of the Great Pyramid is very probably the consequence of relatively recent activities affecting the monument", which means that the cumulative effect of the repeated falls of casing blocks due to the dismantling of the casing by the Arab quarrymen, from the end of the 12th century until the 16th century, had made each surface look concave, together with the additional damage resulting from the tourists. We reject this idea because of the following reason. The dismantling of the casing was to reuse the large fine white limestones, intricately cut and beautifully polished, in order to build mosques and fortress in nearby Cairo. Then, the dismantling of the casing by quarrymen must be carried out very carefully, using some device like ropes, not to cause any damage to the polished surface of stones. Therefore, we do not think they fell down the casing stones, hitting and bumping many blocks on the way to the ground. The debris of casing stones around the base would be due to the earthquake, not due to the intentional dismantlement by the

quarrymen. Note also that quarrymen used backing stones as steps of a "ladder" to climb up or down; then it would be quite a foolish act to destroy the "ladder". We do not exclude the case that the casing was dismantled from the top to the bottom rather than from the bottom to the top, since the former would be much safer than the latter. The 1303 AD earthquake fell down or loosened almost all casing blocks of the Great Pyramid so that it was possible for quarrymen to climb up to the apex, which means they could do the dismantlement from the apex to the bottom. It seems the Khafre's Pyramid was not so disturbed by the same earthquake, but we suspect that some casing stones fell down or loosened so that the quarrymen could climb up near to the apex, from where they could start the the dismantlement towards the bottom. The dismantlement from the top to the bottom needs further care not to cause any damage to the remained casing blocks.

4) Edwards (2016) proposed that workers used the angled faces of the outer casing blocks as surfaces on which to transport block-and-sledge assemblies, and the concavity was a side effect of the construction that the external lateral forces induced by hauling block-and-sledge assemblies up the angled faces of the Pyramid. We can not believe such side effect generated the concavity with clear division of each face into two right triangular planes both of which look quite "flat" in **Figure 1** and **Figure 2**. We proposed in (Kato, 2020) another simple way to lift stones using a very simple lift (made of poles, posts and ropes) and a well (an empty column surrounded by walls of stones, like a chimney).

5) Bauval (2016)[8] proposes that the concavity encodes a "virtual space" at the top of the monument on which might have been placed a spherical object. We need more theory behind the coincidence of numbers to understand such a proposal.

6) A spiritual reason was proposed in (Seyfzadeh, 2017, 2018a: p.165) that the concavity was one of features of the Great Pyramid intentionally designed by the architect "Hemiunu" to embedd theological meaning into the Pyramid, and that the maximal indent "0.92 meters" estimated by Pochan (**Figure 4**) can be interpreted as 8 ("Khemenu") ×1/1000 ("Re-Kha"; Egyptian unit fractions were expressed with the mouth symbol Re over some denominator; kha also means "thousand") ×220 cubits (the half-base). As mentioned in the above Remark 2, or in the model **M** (5/8), it is believed (Seyfzadeh, 2018b: p. 320) that the numbers 5 and 8 were deeply embedded in Egyptian religious thought. Though we do not know if this Seyfzadeh's interpretation is more than the coincidence of numbers, we can believe the idea that the 8 sided core structure was accepted by the ancient Egyptian in favor of the number 8.

In the above six ideas, only the Mendelssohn's idea (1) treats the concavity as an intentional design for the structural stability of the Pyramid, and is closest to our's.

Remark 4. *Can we test our theory?*

How our theory of "gently inclined courses in the Great Pyramid" can be

[8]This paper misquotes the year the photo Figure 1 was taken, as noted in Footnote 1.

tested? This would be quite a difficult task. Or, we can rather say that, this difficulty made us to write down this article in order to guess the possible inner structure of the Pyramid scientifically. The perfect masonry keeps us away from seeing the internal structure of the Great Pyramid. Even though there exits some visible inner structure like passages and chambers, various adjustments or modifications would be probably done in setting blocks in order to construct such fine inner structures, so that any "local" seams on the walls of passages or chambers would not be easily identified with the "global" ones for the inclined courses. For example, in **Figure 32**, any horizontal seams on the walls of the King's and the Queen's Chamber cannot be the evidence that the Great Pyramid was built with horizontal courses. The only test we can imagine presently is to detect somehow the flow of rainwater through the Pyramid to confirm the flow like **Figure 19**.

Acknowledgements

We would like to thank the referee for pointing out the recent paper (Biondi & Malanga, 2022).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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