

Simplified Penman-Monteith Equation Determined by Temperature-Based Global Radiation Data and Its Multilocal Validation under Subhumid Climatic Conditions in Hungary

Zoltán Varga-Haszonits*, Éva Szalka, Dávid Vasas, Zsolt Giczi, Pál Szakál, Tamás Szakál

Albert Casimir Faculty at Mosonmagyaróvár, Széchenyi István University of Győr, Győr, Hungary Email: *vargahz@t-online.hu

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Abstract

The extent to which specific climatic factors influence evapotranspiration under subhumid conditions in Hungary was investigated. The reference evapotranspiration, calculated with the internationally accepted Penman-Monteith equation proposed by FAO, was considered. The results show that the influence of radiation, which provides energy for evaporation, is the strongest factor and that the influence of global radiation alone is very strong. Taking into account that radiation was measured under rather limited conditions in space and time, global radiation was calculated using the Hargreaves method based on temperature. Accordingly, we have defined a formula based on temperaturebased global radiation and verified the data obtained with the Penman-Monteith formula calculated for 14 meteorological stations. The verification gave good results, therefore the method can be used for practical purposes in the subhumid conditions of Hungary based on the data of the nearest meteorological station.

Keywords

Effecting Factors, Sensitivity Analysis, Global Radiation, Multilocal Validation, Estimation Error

1. Introduction

Water balance is a crucial factor for plant life. Its two most important elements are water uptake and water loss. The main source of water uptake is atmospheric

precipitation, which is continuously measured in the network of meteorological stations. The main problem is to determine the level of evaporation from the soil and transpiration through plants. The combination of these two processes is called evapotranspiration. The measurement of evapotranspiration is a complex and expensive task. It is usually only carried out at experimental stations. Therefore, several empirical and semi-empirical methods have been developed to determine evapotranspiration. Due to its semi-empirical nature, the method developed by Penman was the most commonly used method [1] that was further improved by Monteith [2]. A FAO committee recommended this improved method for general use as the Penman-Monteith method by standardizing its calculation procedures [3].

However, the standardised Penman-Monteith equation is data consuming and its use is therefore limited in space and time, mainly due to the lack of data on global radiation and wind speed. It is therefore useful to examine the level of accuracy with which it can be used to determine evapotranspiration in a simplified form.

First, we investigated the extent to which the different climate effecting factors influence evapotranspiration in the subhumid climate of Hungary, and analysed which effecting factors have the strongest effect and which have only a minor effect. The number of effecting factors can be reduced by neglecting the minor ones, and the formula can be greatly simplified by basing it on one of the strongest effecting factors if the calculations are sufficiently accurate for practical use (water demand, irrigation water demand).

In our previous study, we have already made tests with the omission of the minor effects of wind speed under subhumid conditions in Hungary [4], and in this work, we have tested the accuracy of the formula simplified to a single effecting factor that has a strong effect on evapotranspiration and can be calculated easily when compared to the values calculated by the standardized Penman-Monteith equation.

2. Material and Method

2.1. The Penman-Monteith Equation

The standardized FAO Penman-Monteith equation can be calculated as follows [3]:

$$ET_{ref} = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T_k + 273} u_2(e_s - e_a)}{\Delta + \gamma (1 + 0.34u_2)}$$
(1)

where ET_{ref} is the reference evapotranspiration (MJ·m⁻²·day⁻¹), Δ is the slope of the temperature-vapour pressure curve (kPa·°C⁻¹), R_n is the net radiation (MJ·m⁻²·day⁻¹), G is the heat conduction to the soil (MJ·m⁻²·day⁻¹), γ is the psychometric constant (kPa·°C⁻¹), T is the mean daily temperature at 2 m altitude (°C), u_2 is the wind speed at 2 m altitude (m·s⁻¹), e_s is the saturation vapour pressure (kPa) and e_a is the actual vapour pressure (kPa). Equation (1) was verified on lysimeter data from the experimental station in Szarvas and the results showed a close relationship [5].

Equation (1) was calculated for the 14 stations (**Table 1**) of the metrological station network (**Figure 1**) selected for the agroclimatological studies between 1976 and 2000, where the necessary data were available.



Figure 1. Meteorological stations selected for the agroclimatological studies.

	Global radiation*	Net radiation	Mean temperature	Relative humidity	Actual vapour pressure	Vapour pressure deficit	
Békéscsaba	0.97	0.99	0.89	0.81	0.89	0.96	
Bp-Pestszentlőrinc	0.96	0.99	0.87	0.77	0.89	0.95	
Debrecen	0.96	0.99	0.88	0.83	0.89	0.97	
Győr	0.97	0.99	0.87	0.79	0.85	0.97	
Kecskemét	0.97	0.99	0.88	0.86	0.89	0.97	
Miskolc	0.97	0.99	0.87	0.76	0.87	0.97	
Mosonmagyaróvár	0.97	0.98	0.86	0.79	0.85	0.97	
Nyíregyháza	0.98	0.99	0.88	0.77	0.88	0.98	
Pápa	0.97	0.99	0.88	0.79	0.86	0.98	
Pécs	0.97	0.98	0.90	0.81	0.87	0.95	
Szeged	0.97	0.98	0.91	0.86	0.90	0.97	
Szolnok	0.97	0.99	0.90	0.84	0.89	0.97	
Szombathely	0.97	0.99	0.87	0.75	0.84	0.97	
Zalaegerszeg	0.98	0.99	0.88	0.67	0.83	0.98	

Table 1. Determination coefficients for the relationship between meteorological forcing factors and PM ET_{ref} for the period 1976-2000.

*Global radiation calculated with the Hargreaves method.

However, the data in Equation (1) are not available in space and time at all stations. Therefore, in order to be able to use the formula for calculation, it should be investigated (preferably at as many meteorological stations as possible), to what extent the reference evapotranspiration calculated by the formula depends on the individual effecting factors, whether there are any low-impact factors that are negligible because of their small effect, and whether there are any high-impact factors that, because of their strong effect, can alone determine the reference evapotranspiration with acceptable accuracy in practice.

To determine this, we examined the extent to which each effecting factor affects the reference evapotranspiration.

2.2. Sensitivity Analysis

Sensitivity analysis can be performed in several forms [6]. We chose linear regression, in which the determination coefficient in the analysis indicates the extent to which a change in the independent variable (the effecting factor) affects the change in the resulting variable (reference evapotranspiration) [7]. In the calculation, when one effecting factor was associated with the PM- ET_{ref} values, all other effecting factors were considered to be constant. The results obtained are shown in **Table 1**. In a previous study, we have already found that under the climatic conditions of Hungary, the effect of wind speed can be considered negligible or a constant average wind speed of 2 m/s can be used in the calculation of PM- ET_{ref} [4]. Therefore, the relationship with wind speed was not investigated here.

The elements in **Table 1** were calculated from measured relative humidity values, and the other elements from temperature-based relationships.

Table 1 shows that the net radiation has the strongest effect on evapotranspiration under the subhumid climate conditions of Hungary. Net radiation is followed by global radiation, which has the advantage of being easy to calculate, in addition to its strong effect. Therefore, we first analyzed the relationship between global radiation and the Penman-Monteith equation in terms of developing a simple formula.

Global radiation has been calculated using the Hargreaves method [8]:

$$R_g = k_g \sqrt{T_{\rm max} - T_{\rm min}} R_a \tag{2}$$

where R_g is the global radiation (MJ·m⁻²·day⁻¹), $k_g = 0.16$ is the recommended value for the internal areas where the land mass dominates [3], R_a is the extraterrestrial radiation (MJ·m⁻²·day⁻¹) that can be calculated as follows:

$$R_a = \frac{24}{\Pi} G_{sc} d_r \left(\omega_s \sin \varphi \sin \delta + \cos \varphi \cos \delta \sin \omega_s \right)$$
(3)

where R_a is the extraterrestrial radiation from the sun (MJ·m⁻²·day⁻¹), G_{sc} is the solar constant (4.92 MJ·m⁻²·hour⁻¹), d_r is the relative distance earth-sun = 1 + 0.033cos(2\Pi/365)*J*, is the number of days of the year from 1 January (Julian day), ω_s is the sunset hour angle (radians) = $\arccos(-tg\varphi tg\delta)$, φ is the latitude of site (radians), solar declination $\delta = 0.409 \sin(2\Pi/365)J - 1.39$. The values of

FAO-PM ET_{ref} were calculated as given by Allen et al. [3].

Therefore, the idea was raised to use global radiation in the calculation of reference evapotranspiration, since the sensitivity analysis (**Table 1**) shows that even global radiation alone is closely related to evapotranspiration and can be easily calculated from both sunshine duration data and temperature extremes.

3. Results

Relationship between Global Radiation and FAO-PM ETref

Global radiation was calculated using Equation (2). This allows us to use the formula spatially and temporally, since global radiation can also be calculated on the basis of sunshine duration and temperature, and temperature is measured regularly at meteorological stations. The station coefficients, the constant values and coefficients of determination of the equation providing the reference evapotranspiration values determined by Formula (4) are shown in Table 2.

Data in **Table 2** are the results of linear regression relationships (y = bx + a). Considering the linear equations defined for the 14 meteorological stations, we can see that their coefficients and constants fluctuate within a certain range of values. We have therefore determined the average of the coefficients and constants of the equations and used them to create a formula applicable to the 14 stations. However, the point set of the relationship between global radiation and

Stations	Ь	а	r^2
Békéscsaba	0.0516	1.4029	0.9770
Bp-Pestszentlőrinc	0.0817	1.2949	0.9759
Debrecen	0.0749	1.4314	0.9798
Győr	0.0824	1.2551	0.9808
Kecskemét	0.0615	1.3602	0.9779
Miskolc	0.0531	1.3830	0.9786
Mosonmagyaróvár	0.0972	1.1952	0.9779
Nyíregyháza	0.0786	1.3183	0.9852
Pápa	0.1129	1.1973	0.9761
Pécs	0.1016	1.2605	0.9742
Szeged	0.0791	1.3260	0.9756
Szolnok	0.0646	1.3873	0.9744
Szombathely	0.0867	1.2664	0.9736
Zalaegerszeg	0.0653	1.3622	0.9830
Mean value	0.08	1.32	-

Table 2. Coefficients and constants of the linear relationship between global radiation and FAO-PM ET_{ref}

FAO-PM ET_{ref} gave a better result in the form of a power function, and the relationship was therefore defined in the form of a power function:

$$ET_{g} = 0.08 \cdot R_{g}^{1.32} \tag{4}$$

In Equation (4), ET_g is the evapotranspiration (mm/day) determined based on global radiation, and R_g is the global radiation.

The correlation coefficients obtained with function (4) for the 14 selected meteorological stations are shown in Table 3.

It can be seen from **Table 3** that the correlation between global radiation and FAO-PM ET_{ref} is very close for all 14 meteorological stations, and it is higher than 0.98 for the subhumid climate conditions of Hungary. It is therefore possible to use Formula (4) for practical purposes under subhumid conditions. To do this, however, we need to know the magnitude of the estimation errors and the frequency of each estimation error when compared to the FAO-PM ET_{ref} values. We also examined how estimation errors evolve during each year and between years.

<u>Absolute estimation errors and their frequency</u>. Table 4 shows the distribution of estimation errors by absolute value. The errors are below 1.4 mm/day, which can be considered a good result. Some stations have absolute errors below 1 mm/day and only 4 stations have errors above 1.2 mm/day.

<u>Actual estimation errors and their frequency</u>. **Table 5** shows the actual magnitudes of the estimation errors and the frequency of each error magnitude over the period analysed, *i.e.* 1976 to 2000. The table also shows the frequency of underestimation and overestimation in the calculation of daily evapotranspiration values. The table also shows that half of the stations, *i.e.* 7 stations, have errors of less than 1 mm/day. The table also shows that at all but 1 station (Győr) the error is below 0.7 mm/day in 70% of cases. It is also noticeable that the magnitude of the errors most often varies between 0.3 and 0.7 mm/day.

It can be seen from **Table 5** that the estimation errors vary between +1.4 and -0.9, which means that the method can estimate the reference evapotranspiration with an error of 1.5 mm/day. It can be clearly seen from the table that the estimation errors at most of the stations are positive, which means that they overestimate evapotranspiration compared to the FAO-PM ET_{ref} equation. However, there are also several meteorological stations where the errors are rather negative, *i.e.* where Formula (4) underestimates the value of reference evapotranspiration.

The values of reference evapotranspiration determined by Formula (4) thus are overestimated at some stations and underestimated at others, which suggests that this is due to the modifying effect of the environment surrounding the stations. Indeed, the environment at each station may not only differ but also vary from season to season, and where the environment is agricultural, low crops may be grown in one year and high crops in other years. Not to mention other human interventions that may modify the environmental conditions of the weather station in some way.

Station	r ²
Békéscsaba	0.9761
Bp-Pestszentlőrinc	0.9762
Debrecen	0.9774
Győr	0.9815
Kecskemét	0.9772
Miskolc	0.9781
Mosonmagyaróvár	0.9806
Nyíregyháza	0.9852
Pápa	0.9769
Pécs	0.9744
Szeged	0.9756
Szolnok	0.9740
Szombathely	0.9738
Zalaegerszeg	0.9828

Table 3. Correlation coefficients of the stations obtained with Formula (4).

Table 4. Absolute values of estimation errors and their frequency.

Error (mm)	Békéscsaba	Bp-Pestszentlőrinc	Debrecen	Győr	Kecskemét	Miskolc	Mosonmagyaróvár	Nyíregyháza	Pápa	Pécs	Szeged	Szolnok	Szombathely	Zalaegerszeg
0	0.0	0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	11.5	42.1	10.7		23.8	9.0	26.5	45.4	30.9	35.8	32.8	31.4	38.0	32.8
0.2	25.4	62.0	25.4	0.0	41.0	26.2	44.5	70.5	59.8	56.6	51.9	53.0	56.0	54.4
0.3	38.3	72.7	36.9	0.3	55.7	36.3	56.6	84.7	77.0	69.4	68.3	68.0	66.4	68.9
0.4	52.7	83.3	50.0	0.8	66.9	48.4	66.4	93.7	87.2	77.9	82.8	82.0	76.2	77.9
0.5	60.7	92.9	64.5	3.8	74.6	60.4	73.2	97.5	95.4	84.7	90.4	90.4	86.3	89.9
0.6	68.3	98.4	74.3	8.2	80.6	66.7	80.6	99.2	97.8	88.3	96.2	95.4	91.5	95.6
0.7	74.0	99.5	81.1	13.7	86.1	71.0	89.1	99.7	99.7	90.7	98.4	99.5	96.4	98.4
0.8	79.5	100.0	88.8	22.4	93.2	76.5	95.1	99.7	99.7	93.2	99.5	100.0	98.9	100.0
0.9	85.2		95.4	29.0	97.8	83.3	97.8	100.0	99.7	95.6	100.0		99.5	
1	93.4		98.4	36.3	99.5	89.1	99.7		100.0	98.4			100.0	
1.1	97.5		98.9	44.0	100.0	94.3	100.0			100.0				
1.2	98.9		99.7	55.5		98.4								
1.3	99.7		100.0	72.4		99.7								
1.4	100.0			100.0		100.0								
1.5														

Table 5. Magnitude and frequency of estimation errors.

Error (mm)	Békéscsaba	Bp-Pestszentlőrinc	Debrecen	Győr	Kecskemét	Miskolc	Mosonmagyaróvár	Nyíregyháza	Pápa	Pécs	Szeged	Szolnok	Szombathely	Zalaegerszeg
1.5	0.0					0.0								
1.4	0.3		0.0			0.3								
1.3	0.8		0.3	0.0		1.4								
1.2	1.4		0.8	0.3	0.0	4.1	0.0							
1.1	4.1		0.5	0.5	0.5	5.2	0.3						0.0	
1	8.2		3.0	3.0	1.6	5.7	1.9						0.5	
0.9	5.7	0.0	6.6	4.4	4.6	6.8	2.7						0.5	0.0
0.8	5.5	0.5	7.7	5.5	7.1	5.5	6.0	0.0	0.0		0.0	0.0	2.5	1.6
0.7	5.7	1.1	6.8	8.7	5.5	4.4	8.5	0.3	0.3		0.5	1.9	4.6	2.7
0.6	7.7	5.2	9.8	6.6	6.0	6.3	7.4	0.8	0.5		1.6	1.9	4.9	5.7
0.5	7.9	7.7	14.5	7.4	7.7	12.0	6.8	2.5	4.9		3.3	4.6	9.0	11.7
0.4	14.5	6.6	13.1	7.7	11.2	12.0	9.8	6.0	4.1	0.0	6.8	10.1	7.1	8.2
0.3	12.8	7.7	11.5	11.5	14.8	10.1	11.5	8.2	7.4	1.1	7.4	9.6	6.8	12.0
0.2	13.9	9.3	14.5	14.8	16.7	17.2	14.2	16.7	10.7	1.6	11.2	13.9	11.5	18.0
0.1	9.8	22.7	9.6	16.4	18.9	7.7	11.5	22.7	11.2	7.1	15.6	19.7	20.8	22.1
0	1.6	19.4	1.1	11.2	4.9	1.4	15.0	22.7	19.7	11.5	17.2	11.7	17.2	10.7
-0.1	0.0	10.7	0.3	2.2	0.5	0.0	3.8	8.5	18.3	24.3	7.9	7.7	6.6	3.6
-0.2		3.0	0.0	0.0	0.0		0.5	6.0	9.8	13.7	9.0	5.5	3.6	2.5
-0.3		4.1					0.0	3.0	6.0	11.2	7.7	3.8	2.7	0.8
-0.4		1.9						1.4	3.3	7.4	4.4	3.8	1.1	0.3
-0.5		0.3						0.8	3.8	6.8	4.1	3.0	0.3	0.0
-0.6		0.0						0.5	0.0	3.6	3.3	2.2	0.3	
-0.7								0.0		2.5	0.0	0.5	0.0	
-0.8										9.3		0.0		
-0.90										0.00				

It can be seen from Table 5 that the estimation errors vary between +1.4 and -0.9, which means that the method can estimate the reference evapotranspiration with an error of 1.5 mm/day.

<u>Changes in estimation errors within the year.</u> Estimation errors can vary with seasonal variation within the year. Intra-annual variation can be illustrated by plotting the annual diagram of Formula (4) and the annual diagram of FAO-PM

 ET_{ref} on a single graph (**Figure 2**).

From the figure, which shows the averages over 25 years (1976-2000 period), it can be seen that the reference evapotranspiration calculated by Formula (4) shows values a few tenths of mm/day higher from the beginning of the year until the summer maximum. In the second half of the year, starting from the summer maximum, the values calculated with the two formulae are almost identical. It can therefore be said that the values calculated by Formula (4) show negligible differences from a practical point of view (e.g. for the determination of the amount of irrigation water).

<u>Estimation error in annual amounts</u>. Daily estimation errors can accumulate in the annual amounts, so it is useful to examine the errors in the annual amounts and the differences between the annual evapotranspiration amounts determined with Formula (4) and with FAO-PM ET_{ref}

It can be seen that the annual amount values calculated by Formula (4) are higher than the values calculated by the FAO-PM ET_{ref} formula. The average difference between the two formulae was 72 mm/year, the largest difference occurring in 1978 (138 mm) and the smallest difference was in 2000, with the values (26 mm) obtained with the FAO-PM ET_{ref} formula being higher.

In the case of year-to-year changes, the values obtained with the ET_g Formula (**Figure 3**) were higher than the values obtained with the FAO-PM ET_{ref} formula. The difference fluctuated around 50 mm until the 1990s, but after the 1990s the difference became smaller and in 1992 and 2000 the FAO-PM ET_{ref} values slightly (26 mm) exceeded the ET_g values.

Overall, the modified formula based on global radiation can therefore be used for practical purposes, because the average difference between the two formulae of 72 mm/year is only 8.4% of the annual average value of 854 mm/year of FAO-PM ET_{ref} for the period under consideration.



Figure 2. Diagram of annual average values of reference evapotranspiration of 25 years calculated with Formula (4) and with the FAO-PM ET_{ref} formula.



Figure 3. Yearly variation of annual evapotranspiration totals calculated with the ET_g function and with the FAO-PM ET_{ref} function.

4. Conclusions

Water is a crucial effecting factor in the life of plants. The main source of water uptake is precipitation, which is regularly measured at meteorological stations. Soil absorbs and stores the precipitation that falls on it. The water content of soil used to be measured using gravimetric methods, but nowadays it can be measured regularly using more modern instruments. However, measuring evapotranspiration, *i.e.* the combination of evaporation from the soil and transpiration through plants, is a complex and costly task, and is therefore mainly measured at experimental stations. At the same time, data on evapotranspiration are essential from a practical point of view, in order to determine the water requirements of crops and irrigation water needs.

To solve this problem, the standardized Penman-Monteith method proposed by FAO is generally accepted and used internationally. However, this method is limited in space and time due to its data requirements.

Therefore, it was investigated which climate factors strongly influence evapotranspiration under subhumid conditions in Hungary and, taking into account the strong influence of energy on evapotranspiration, we were able to define a formula based on a single element that can be measured or simply calculated from commonly measured temperature data and gives good results from a practical point of view.

In validating the formula, we first determined the frequency of the estimation errors of different magnitudes that varied between -0.9 mm/day and 1.4 m/day,

which is a good result.

On the basis of our investigations, we found that the values obtained with the developed formula during the intra-annual (seasonal) variations in the first half of the year were overestimated compared to the values obtained with the FAO Penman-Monteith formula within the mentioned error interval in the first half of the year, and that the values of the two formulae changed substantially together in the second half of the year.

Examining the relationships between the year-to-year changes also showed that the difference between the annual amounts of reference evapotranspiration calculated with the formula based on global radiation only and the annual totals obtained using the FAO-PM ET_{ref} formula is only about 8%.

It can therefore be said that the formula based on global radiation only is suitable for calculating the reference evapotranspiration in space (always on the basis of the data from the meteorological station nearest to the application site) and in time for analysing the annual variation of the reference evapotranspiration on the basis of the data from the station with the oldest series of measurements close to the application site.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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