

Nash Social Welfare, Logic, and Inductive Game Theory: An Application on the Russian Invasion of Ukraine

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Abstract

In this lecture, I introduce the theory of Nash social welfare (NSW) function, and discuss its implications for peace and justice, particularly, in the context of the Russian invasion of Ukraine. We face many other devastating events in the present world. Because the NSW theory aims to consider such events, the theory consists of extreme postulates from the viewpoint of extant social thoughts. As expected, it involves various conundrums at the foundational levels in the socio-economic and philosophical senses, in particular, the problems of small communities versus those at the level of the whole world. To understand such contrast, here, we discuss the subjects listed in the title of the lecture, that is, the NSW theory, logic and inductive game theory. These help us consider managements of the world, as well as their applications to the Russian invasion of Ukraine. The main text describes how these seemingly independent subjects are connected in my research development and have important implications for devastating events.

Keywords

Nash Social Welfare, Logic, Inductive Game Theory, Russian Invasion of Ukraine

1. Preface

This is an extension of the lecture note, Kaneko (2023), distributed at the Honorary Doctorate Commencement Ceremony, Warsaw School of Economics (SGH), Warsaw, Poland 3rd April, 2023.

2. Beginning of My Research Career

My main research activity started with *Nash social welfare theory*, on which I

worked with Kenjiro Nakamura. The results were published in Kaneko & Nakamura (1979a, 1979b). The NSW function is given as

$$w(u, x) = \sum_{i \in N} \log [u_i(x) - u_i(x_0)], \quad (1)$$

where

- $N = \{1, \dots, n\}$ is the set of people in the whole world;
- $u = (u_1, \dots, u_n)$ is a list of (Neumann-Morgenstern) utility functions;
- x_0 is the worst state of the world, called the *origin*;
- x is a candidate of alternative world states, to be evaluated by the NSW function.

The theory led me to various questions on its applications to practical social problems as well as foundational questions on its ethical/normative status. Kaneko & Nakamura (1979a: p.423) assumed the origin x_0 .

The worst state for all individuals that we may imagine, for example, all the members of the world die.

The total destruction of the whole world became feasible with the use of nuclear bombs during the 1940s-1950s. Einstein's (1968) piece principle, "the objective of avoiding total destruction (of the Earth) must have priority over any other objectives", was derived from observations of the world during the 1940s-1950s. Hobbes' (1991, original 1651) state of nature was the practically worst case in the 17th century; the whole world of that time exceeded the scope of people. But now, the situation of the whole world is narrower to people and much more serious than in Hobbes' time and even Einstein's time.

In the NSW theory, the total destruction corresponds to the worst state x_0 . The origin x_0 is evaluated as $x = x_0$ and

$$w(u, x) = \sum_{i \in N} \log [u_i(x) - u_i(x_0)] = -\infty. \quad (2)$$

If a worldwide alternative x includes the genocide of a group of people, the function $w(u, x)$ takes value $-\infty$, too. This x is the same as the origin x_0 with respect to the NSW theory. I will examine the genocide in Ukraine committed by Russia, later.

When I started research on the NSW theory, my knowledge was limited and I was not able to have a systematic thought on the NSW theory. For example, I did not understand Hobbes' social contractarian theory from his state of nature to the absolutist state. I raised the following naive questions.

Question 1: Because the NSW function is real-valued, its application to a social problem can be formulated as a simple maximization of $w(u, x)$. However, this is too simplistic. What kind of alternatives x should the NSW function be applied to?

Question 2: Formally, the NSW theory is a different but equivalent formulation of the Nash (1950) bargaining solution (cf., Kaneko, 1980). The literal aim of the NSW theory is to study social welfare, rather than bargaining. Why do they have equivalent structures? What are the substantive differences?

In the 1980s, the problem of rationality became popular in the game theory community, due to Selten (1975). The studies in this area, however, examined the stability of an equilibrium and/or representations of beliefs/knowledge. It was developed in the direction of the Bayesian game theory, and did not directly address the problem of rationality. The term “rationality” is often treated as synonymous to “utility maximization”. The concept of rationality must be the appropriateness of our thinking. We need to consider our thinking more directly. Gradually, I made up my mind to study logic, while thinking that it helps me consider questions Q1 and Q2.

Around 1984, I started studying logic seriously. I needed quite some time to start understanding what logic is, but finally I discovered that logic is intimately related to our intellectual activities in the sense that symbolic manipulations express intended meanings.

I have a good example: One day, I observed a small child answered literally to the question “How old are you?” asked by an adult person:

“I am two years and three months old”.

Of course, he was unable to explain the meaning of his answer, but it was correct in that situation. If we ask ourselves whether we differ from that boy, we find that our knowledge is similar only with differences in degree.

The basic postulate of logic begins with the idea of symbolic operations, which is a radical treatment of bounded rationality. Now, Q1 can be answered; the detailed maximization of the NSW function could be meaningful only when the number of people n is small, i.e., 2 or 3. When n is large such as the population of a nation, the NSW function can be applied to a choice of an institution ignoring detailed differences among people. Other than institutions, an application of the NSW function offers only a choice for extreme alternatives such as whether to “stop genocide or not”, neglecting almost all small details.

Additional Note: The NSF theory took the form of the Arrow-type social welfare function with n number of participants (Arrow, 1951). Arrow tried to capture the concept of social welfare as an attribute of a social choice rule such as majority decision, and proved the impossibility theorem to dictate that there are no social welfare functions satisfying some conditions given by him. Instead, we consider social welfare as a purely normative criterion, which aims to evaluate outcomes entailed by a social decision rule, which is the main conceptual difference between the Arrow approach and the NSW theory.

There are some literatures treating subjects similar to the NSW theory, such as utilitarianism due to Harsanyi (1953, 1955), Rawlsian theory of justice (cf., Rawls, 1971), and global justice (cf., Pogge, 2001). This note does not touch them; I will discuss some comparisons between the NSW theory and these theories in a different paper.

The Nash bargaining theory with fixed threats was developed by Nash (1950) for two persons. The NSW theory differs also from the Nash bargaining theory in that the former starts with the common fixed-threat for worldwide, that is, the origin x_0 that the earth meets the entire destruction, but the former allows the

threat depending upon each social situation. This specific choice of the origin x_0 needs more discussions (see Kaneko, 2018).

3. Logic and Inductive Game Theory

Around 1985, I met a proof theorist, Takashi Nagashima, and started working on game logic with him. Proof theory deals with a pure form of logic. The papers with him (Kaneko & Nagashima, 1996, 1997) focused on a critique on logical thinking involved in game theory. Later, Nobuyuki Suzuki, Tai-Wei Hu, and I have developed epistemic logic with emphasis on finiteness (cf., Kaneko & Suzuki, 2003; Hu, Kaneko, & Suzuki, 2019).

Around 1995, I started working on inductive game theory (IGT) with Akihiko Matsui and Jeffrey J. Kline (cf., Kaneko & Matsui, 1999; Kaneko & Kline, 2008). This theory interprets symbolic pieces of information as a source for understanding in social contexts. This shares ideas of patterned behavior with “convention” of von Neumann & Morgenstern (1944) and Lewis (1969).

Question 3: We have appealed the calculation result (2) from the NSW function by referring to our intuition. Then, what is this intuition? Or, does each of us have the source for the intuition such as morality in mind?

The same is asked for the Nash bargaining theory. His theory with two persons is understood as the result calculated by a logically rational player. The first question of Q3 is answered, in that, intuition is based on memory of calculations executed by the logical ability of our mind. This differs from the presumption that some morality exists hiddenly in our mind, which is often implicitly presumed (cf., Harsanyi, 1953, 1955). Our logical ability is bounded, but when the situation is simple, such logical thinking works. Thus, the second question of Q3 was affirmatively answered in this restricted manner but not as the assumption of hidden morality.

Now, what is “morality” in our ordinary lives? This has two sides; one side is the answer to Q3. The other is social morality has been emerging in human relations. IGT starts with this view; Kaneko & Matsui (1999) studied the relation between discrimination as behavioral patterns and prejudices as *ex post* rationalization. Once morality is accepted in society, people follow it as behavioral principle.

Additional note: As stated in Section 1, rationality is often regarded as synonymous to utility (profit) maximization, and then, bounded rationality is an incomplete version of utility maximization. In this lecture, the term “rationality” means an attribute of the cognitive and logical abilities of the mind of a decision maker. In social situations, there are many other types of boundedness of rationality. In the first paragraph of this section, a few boundedness aspects of the logical ability of a decision maker are mentioned, for example, effects of a language in Kaneko & Nagashima (1996) and bounded interpersonal relations in the other papers referred above. Inductive game theory is to explore sources of individual beliefs in experiences. Other types will be mentioned in the additional note of Section 3.

4. Barbarity from Histories, and the Orwellian Denial of Reality

When I heard the news on the Russian invasion of Ukraine and the genocides there, I was shocked. Since then, Russian barbarity has persisted. The question is:

Question 4: How do we understand Russian behavior from the viewpoint of the NSW theory, logic, and IGT?

Many assertions by Putin and his fellows have puzzled me; the words “Russia” and “Ukraine” are reversible in their assertions. These remind me of [George Orwell’s \(1949\) “1984”](#), which describes the authoritative state and its severe surveillance with language control. The following question is raised:

Question 5: How do we understand no respect for truth/reality?

Putin has coherently shown that he wants to change the world state x before 2022 into the new state x so that the Ukrainian people are eliminated and Ukraine becomes a part of Russia. That is, the new utility levels are

$$u_i(y) \leq u_i(x_0) \text{ if } i \text{ is Ukrainian, but } u_i(y) > u_i(x_0) \text{ if } i \text{ is Russian.} \quad (3)$$

One basic postulate of the NSW theory is that all the people in the world want to avoid the origin x_0 , which is an implication of Einstein’s peace principle. This worldwide alternative y gives the same value $-\infty$ as the origin x_0 . Then, the following questions are raised.

Question 6: Do we count the people who show (3) as members of the world? Can such people exist in general? Is Putin exceptional?

The first question could be negatively answered by the constitution of the World federal government (WFG), which I propose as the ultimate worldwide institution and will be mentioned in Section 4. It states that such people should be eliminated from the world, unless they change their behavior and thought to be compatible with the constitution.

The second question is a purely positive scientific question, and it is rephrased as: may such people/cultures be created in some societies with historical backgrounds? Looking at Russian history, we find extremely barbaric events continuously under authoritarian regimes from its beginning. Such historical backgrounds have created moral values for the ruling class to treat other people as objects/animals.

The implication of IGT is that human beliefs and morality are products of historical social interactions. Often, beliefs are derived after observing social patterns of behaviors, which is opposite to the basic postulate of standard of economics and/or game theory.

Finally, we return to Q5, which deals with the lack of respect for truth/reality. In a severe authoritarian regime, the value of survival is more important than truth/reality. Even people’s thoughts may become controlled and it becomes impossible for people to think about truth or reality. This situation is well expressed as language control in Orwell’s 1984; for example, by eliminating the word “revolution” from language, the nation could be free from revolution.

People who have grown up under such circumstances give very little values to truth/reality.

Additional note: An important facet of (bounded) rationality is that people follow some patterns behaviorally as well as mentally, which has been developed in a society. Convention, due to Lewis (1969), is such a concept, and von Neumann & Morgenstern (1944), Chapter 1, emphasized the concept of standards of behavior in a society. Such patterns have been developed as rules of thumb for behavior and thinking. Language and culture are important examples. In a society, each person's behavior may be optimal individually, yet up to the cultures/institutions. Some societies may have developed terrible habits such as no respects to truth/reality. Its symbolic nature is better understood by logic and inductive game theory.

5. Conclusion

I described the development of my research on the NSW theory. To understand the theory more deeply, I have worked on logic and IGT. In this lecture, I mentioned their applications and implications, as well as those of the NSW theory to understand the phenomena caused by the Russian invasion.

For long time, I got stuck on the idea of the “world government”, though I had vague ideas to deny it. A several years ago, I found that this should be replaced by the concept the world federal government (WFG) (Kaneko, 2021). This suggests that it consists of independent nations but is ruled by the constitution of the WFG, which should be coherent with the NSW theory.

The NSW theory together with WFG forms a social contractarian theory for the whole world; its constitution consists of non-detailed statements such as “avoid terrible events like genocide, famine, etc.”. An implication of the Nash bargaining theory and the origin x_0 is that each person has total independence and freedom, because each is given the integrity equally comparable with the entire world. This prohibits treating people as objects or animals.

Democracy and market economy are key institutions and tools for the practical management of the world. In Hobbes' time, either democracy or market economy was little practiced. In the present world, they are indispensable; we should be conscious of each having certain, sometimes serious, drawbacks. Majority rule is the foundation of democracy, but minority people may be often discriminated against. Economics teaches that Pareto optimality is achieved through market economy, but it may lead to inequalities among people as well as worldwide problems such as global warming, cf., Hammond, Kaneko & Wooders (1989), Kaneko & Wooders (1994). We should study carefully constraints on democracy and market economy.

Additional note: The NSW theory has the scope of the whole world. Because each region has its own socio-economic history and culture, the central management of the whole world is simply unpractical. Instead, we need decentralized management. This leads us to the idea of the world federal government (WFG).

The constitution of the WFG should be designed by the NSW theory together with other scientific (social/economic) theories. In such decentralized management, democracy and market economy play basic roles, and social infrastructures for them are designed to keep integrity and independence of each person, each group of people, and each nation. To avoid devastating events such as genocides, famines, and natural disasters, suggestions dictated by the NSW theory have priority to the authority of each nation.

Afterword: I tried to convey my theoretical thought from 1976 to now. Specifically, I talked about integrity of each person and independence of each nation. These are directly applied to Ukraine, but they do not recommend an easy compromise. In the Ukrainian case, a cease-fire without going back to the state of 2014 gives a moratorium period to Russian, and after it, perhaps, Russia will start another invasion.

I heard that the Senate of SGH has started the procedure of awarding honorary doctorate to President Zelensky. I myself entirely support him and Ukraine from my sincere heart and theoretical thinking.

My theories help thinking about those basic problems of the present world, but they need more systematic thoughts from the foundations to practical applications. It is a long way to go. I finish this lecture with the quotation from Hippocrates 460-375 B.C.

Life is short, the art long, opportunity fleeting, experience unreliable, judgement difficult.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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