Dark Matter versus MOND

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Abstract

Many physicists believe dark matter accounts for flat velocity curves in spiral galaxies and find further evidence for dark matter in observations of the colliding "bullet cluster" galaxies 1E0657-56. Others claim a modified law of gravity called MOND (MOdified Newtonian Dynamics) explains galactic velocity curves better than dark matter. Merritt recently argued for MOND (ar-Xiv:1703.02389) by claiming dark matter models cannot account for the MOND acceleration threshold $a_0 \approx 1.2 \times 10^{-8}$ cm/sec^2 and the ($V_{\text{observed}}/V_{\text{Newtonian}}$) relation. However, this note shows that the HLSS model involving dark matter accounts for both the MOND acceleration and the ($V_{\text{observed}}/V_{\text{Newtonian}}$) relation. After this paper was accepted for publication, I learned that Man Ho Chan previously reached the same conclusion (ar-Xiv:1310.6801) using a dark matter based analysis independent of the holographic approach used in this paper.

1. Introduction

Dark matter is generally believed to account for the approximately flat velocity curves characteristic of spiral galaxies. Observations of the colliding "bullet cluster" galaxies 1E0657-56 provide further evidence for the existence of dark matter. However, some physicists believe the observed flat velocity curves indicate the law of gravity must be modified at large distances according to MOdified Newtonian Dynamics (MOND). Recently Merritt [1] argued for MOND by claiming dark matter models cannot account for the acceleration threshold $a_0 \approx 1.2 \times 10^{-8}$ cm/sec^2 and the ($V_{\text{observed}}/V_{\text{Newtonian}}$) relation emerging from the MOND approach [2].

2. Purpose

The purpose of this paper is to evaluate the validity of Merritt's claim by

considering a specific model based on dark matter, the holographic large scale structure (HLSS) model [3]. The HLSS model was developed within the ΛCDM paradigm and employs the holographic principle based on thermodynamics and general relativity [4]. This note shows the HLSS model can account for both the MOND acceleration threshold and the ($V_{\text{observed}}/V_{\text{Newtonian}}$) relation.

3. Analysis

In the HLSS model, galaxies with total mass M_g inhabit spherical holographic screens with radius $R_S = \sqrt{\dfrac{M_g}{0.183 \text{ g/cm}^2}}$ if the Hubble constant $H_0 = 67.8 \text{ km} \cdot \text{sec}^{-1} \cdot \text{Mpc}^{-1}$. The HLSS model considers galactic matter density distributions $\rho(r) = \dfrac{M_g}{4\pi R_S r^2}$, where r is the distance from the galactic center. The spherical isothermal halo of dark matter, with radius R_S and mass $M_{DM} = 0.84 M_g$, has density distribution $\rho_{DM}(r) = \dfrac{M_{DM}}{4\pi R_S r^2}$ so the dark matter mass within radius R is $\dfrac{R}{R_S} M_{DM}$. There is no singularity in the galactic matter density distribution $\rho(r) = \dfrac{M_g}{4\pi R_S r^2}$ because mass inside a core volume of radius R_c at the galactic center is concentrated in a central black hole with mass $M_{CBH} = \dfrac{R_c}{R_S} M_g$ [3]. Radial acceleration at radius R due to dark matter is then $a_{DM} = \dfrac{G}{R^2}\left(\dfrac{R}{R_s}\right) M_{DM}$. At radii R sufficiently distant from the galactic center that total baryonic mass of the galaxy $M_B = 0.16 M_g$ can be treated as concentrated at the galactic center, Newtonian radial acceleration resulting from baryonic matter is $a_B = \dfrac{GM_B}{R^2}$. The radius R_γ where $a_{DM} = a_B$ is found from

$$\frac{G}{R_\gamma^2}\left(\frac{R_\gamma}{R_S}\right) M_{DM} = \frac{GM_B}{R_\gamma^2}.$$

Since $M_{DM} = 0.84 M_g$ and $M_B = 0.16 M_g$, $R_\gamma = 0.19 R_S$, and at that radius

$$a_{DM} = a_B = a_0 = 5.4 \times 10^{-8} \text{ cm/sec}^2$$

consistent with the MOND estimate $a_0 \approx 1.2 \times 10^{-8} \text{ cm/sec}^2$.

Another indication that the MOND acceleration $a_0 \approx 1.2 \times 10^{-8} \text{ cm/sec}^2$ is a natural scale in the dark matter based HLSS model involves the situation at the radius R_S of the spherical holographic screen. Then the Newtonian assumption, that total galactic mass can be considered as concentrated at the galactic center, is certainly mathematically justified. There, the sum of radial acceleration from dark matter and radial acceleration from baryonic matter is

$$a_S = \frac{GM_{DM}}{R_S^2} + \frac{GM_B}{R_S^2} = \frac{G}{R_S^2}\left(M_{DM} + M_B\right).$$

Using

$$M_{DM} + M_B = M_g = 0.183R_S^2$$

then yields

$$a_S = 0.183G = 1.2 \times 10^{-8} \, \text{cm}/\text{sec}^2$$

equal to the estimated MOND acceleration $a_0 \approx 1.2 \times 10^{-8} \, \text{cm}/\text{sec}^2$.

The tangential velocity V at radius R is related to radial acceleration a_r by $V^2 = Ra_r$. So, the ratio ($V_{\text{observed}}/V_{\text{Newtonian}}$) is approximately

$$\left(\frac{V_{\text{observed}}}{V_{\text{Newtonian}}} \right)^2 \approx \frac{R(a_{DM} + a_B)}{Ra_B},$$

resulting in

$$\frac{V_{\text{observed}}}{V_{\text{Newtonian}}} = \sqrt{1 + \frac{a_{DM}}{a_B}} = \sqrt{1 + \frac{RM_{DM}}{R_S M_B}}.$$

Then, when $R \ll 0.19R_s$,

$$\frac{V_{\text{observed}}}{V_{\text{Newtonian}}} \approx 1$$

as noted by Merritt [1]. Next, using

$$\frac{a_0}{a_B} = \frac{M_{DM}}{0.19M_B} \left(\frac{R}{R_S} \right)^2$$

and

$$\frac{R}{R_S} = \sqrt{\frac{0.19M_B}{M_{DM}}} \sqrt{\frac{a_0}{a_B}}$$

results in

$$\frac{V_{\text{observed}}}{V_{\text{Newtonian}}} = \sqrt{1 + \frac{RM_{DM}}{R_S M_B}} = \sqrt{1 + \sqrt{\frac{0.19M_{DM}}{M_B}} \sqrt{\frac{a_o}{a_B}}}.$$

When $a_B \ll a_0, \dfrac{a_o}{a_B} \gg 1$ and

$$\frac{V_{\text{observed}}}{V_{\text{Newtonian}}} = \sqrt{1 + 0.998\sqrt{\frac{a_o}{a_B}}} \approx \left(\frac{a_0}{a_B} \right)^{\frac{1}{4}},$$

again as noted by Merritt [1]. Since $\left(\dfrac{V_{\text{observed}}}{V_{\text{Newtonian}}} \right)^4 = \dfrac{a_0}{a_B}$ when $a_B \ll a_0$, using

$a_B = \dfrac{GM_B}{R^2}$ and $V_{\text{Newtonian}} = \sqrt{\dfrac{GM_B}{R^2}}$ gives

$$V_{\text{observed}}^4 = \left(\frac{GM_B}{R^2} \right)^2 \left(\frac{R^2}{GM_B} \right) a_0 = GM_B a_0$$

also known as the baryonic Tully-Fisher relation.

Finally, if the Hubble constant $H_0 = 67.8 \, \text{km} \cdot \text{sec}^{-1} \cdot \text{Mpc}^{-1}$, the cosmological constant $\Lambda = 1.12 \times 10^{-56} \, \text{cm}^{-2}$, and the accelerations $cH_0 = 6.6 \times 10^{-8} \, \text{cm}/\text{sec}^2$

and $c^2\sqrt{\dfrac{\Lambda}{3}} = 5.5 \times 10^{-8}\ \text{cm}\big/\text{sec}^2$ are both consistent with the acceleration $a_0 = 5.4 \times 10^{-8}\ \text{cm}\big/\text{sec}^2$ estimated above.

4. Conclusion

Contrary to Merritt's claim [1], this note demonstrates that the HLSS model [3], based on dark matter, can account for the MOND acceleration threshold, the ($V_{\text{observed}}/V_{\text{Newtonian}}$) relation, and the baryonic Tully-Fisher relation. After this paper was accepted for publication, I learned that Man Ho Chan previously reached the same conclusion [5] using a dark matter based analysis independent of the holographic approach used in this paper.

Acknowledgements

References

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