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Comparison of Two Sample Tests Using Both Relative Efficiency and Power of Test

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Abstract

This paper, comparison of two sample tests, is motivated by the fact that in the test of significant difference between two independent samples, numerous methods can be adopted; each may lead to significant different results; this implies that wrong choice of test statistic could lead to erroneous conclusion. To prevent misleading information, there is a need for proper investigation of some selected methods for test of significant difference between variables/subjects most especially, independent samples. The paper examines the efficiency and sensitivity of four test statistics to ascertain which test performs better. Based on the results, the relative efficiency favours median test as being more efficient than modified median test for both symmetric and asymmetric distributions. In terms of power of test, median test is more sensitive than Modified Median (MMED) test since it has higher power irrespective of the sample sizes for both symmetric and asymmetric distribution. In terms of relative efficiency for asymmetric distribution Modified Mann-Whitney U test is more efficient than Mann-Whitney U test (MMWU), and then for symmetric distribution, Mann-Whitney U test (MMWU) is more efficient than Modified Mann-Whitney in sample size of 5; but for other sample sizes considered Modified Mann-Whitney U test (MMWU) is better than Mann-Whitney. Using power of test for both symmetric and asymmetric distributions, Mann-Whitney is more sensitive than Modified Mann-Whitney U test (MMWU) because it has higher power.

Keywords

Asymmetric, Symmetric, Nonparametric Test, Two Sample Tests, Power of Test, Relative Efficiency

1. Introduction

One of the challenges faced by researchers most especially, statisticians, is to take decisions in the presence of uncertainties. Most often, intelligent guess is made and statistical methods are applied to validate or reject any

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possible assumptions that might have been made to enable the use of such methods. Numerous methods exist for testing statistical hypotheses in various conditions. In some cases, the probability distribution of the population from which samples are drawn is known. For instance, if the population is assumed to be normal; then, the sample size is assumed to be sufficiently large to justify the assumption of normality. In special cases, the sample sizes are very small and the probability distribution of the populations from which samples are drawn is unknown; hence, the sample is said to be distribution free and only non-parametric methods are applied. Thus, in most cases where the assumption of parametric methods is violated or not met, the non-parametric methods are usually preferred. Non-parametric methods that readily suggest themselves include the Median and the Mann-Whitney U test [1]. These methods require that the populations from which the samples are drawn to be continuous so that the probability of obtaining tied observations is at least theoretically zero [2]. Statistical tests could be for either paired or unpaired. In a paired test (Matched sample test), the data are collected from subjects measured at two different points wherein each subject has two measurements which are done before and after the treatment. Unpaired test on the other hand is when data are collected from two different and independent subjects. The size of the two samples may be equal or not, depending on the requirement of the test statistic. Techniques or methods for performing two sample tests abound but the question is "which method(s) perform better and under what conditions do they perform better when dealing with independent samples?" To make an articulate attempt to answer these questions, there is a need for proper and adequate comparative study of similar methods that can be used for the purpose of interest. The methods/techniques are: median test, modified median test intrinsically adjusted for ties, Mann-Whitney U test and modified intrinsically ties adjusted Mann-Whitney U test. All the above listed methods are for test of significant difference between variables/subjects when having independent samples. Wide comparison would expose researchers to conditions under which the methods are used to prevent type I or type II errors. In statistical computation, test statistics sometimes are affected by nature of data; that is, the distribution of the data which could be either symmetric or asymmetric in nature. In the determination of more effective statistical method, not just the null hypothesis should be of paramount interest but also the alternative hypothesis which implies that power of test plays an important role in the determination of effectiveness of statistical methods. The maximum value of power of test is 1 and the least is zero which is non-negativity property. The higher the power of test, the better the method and the lower the value, the less effective the method. In this paper, methods of analyzing two independent samples drawn from independent populations would be considered by subjecting some set of data to different conditions, such as sample size, to determine the condition under which they perform optimally in terms of Relative Efficiency (R.E) and power. The power efficiency of median test decreases as the sample sizes increases reaching an eventual asymptotic efficiency of $2/\pi$ [3]. The modified median test intrinsically adjusted for ties was compared with the existing technique, ordinary median test; and the conclusion was that the modified median test intrinsically adjusted for ties easily enables the isolation of tied observations and estimation of their probability of occurrence [2].

2. Material and Methodology

(1) MEDIAN TEST: Median test is a procedure for testing whether two independent groups (samples) differ in central tendencies represented by the population median [4]. The null hypothesis is

$$H_0 : \pi^+ = \theta_0 \quad \text{Vs} \quad H_1 : \pi^+ \neq \theta_0 \tag{1}$$

where $\theta_0 = 0.5$.

The test statistic is

$$\chi^2 = \frac{\left(W - n_1 n_2 \cdot \theta_0\right)^2}{\operatorname{var}\left(W\right)} = \frac{n_1 n_2 \left(\hat{\pi}^+ - \theta_0\right)^2}{\hat{\pi}^+ \left(1 - \hat{\pi}^+\right)} \tag{2}$$

which under H_0 becomes

$$\chi^2 = \frac{n_1 n_2 \left(\hat{\pi}^+ - 0.5\right)^2}{\hat{\pi}^+ \left(1 - \hat{\pi}^+\right)}. \tag{3}$$

Reject H_0 at α-level of significance if $\chi^2 \geq \chi^2_{1-\alpha;1}$, otherwise, accept.

(2) Modified median test intrinsically adjusted for ties is used for test of equality in population media [2]. The null hypothesis $H_0: M_1 = M_2 = M$ is equivalent to the null hypothesis,

$$H_0: \pi^+ = \pi^- \text{ or } H_0: \pi^+ - \pi^- = 0 \tag{4}$$

$$H_1: \pi^+ - \pi^- \neq 0.$$

The test statistic is

$$\chi^2 = \frac{w^2}{mn\left(\pi^+ + \pi^-\right) - \frac{w^2}{mn}} \tag{5}$$

where

$$w = f^+ - f^-.$$

m is sample size of variable X.

n is sample size of variable Y.

π^+, π^0, π^- are respectively the probabilities that observations or scores by subjects from population X are on the average greater than , equal to, or less than observations or score by subjects from population Y.

f^+, f^0, f^- are respectively the number of 1's, 0's and −1's in the frequency distribution of these mn values of U_{ij}.

Reject H_0 at α-level of significance if $\chi^2 \geq \chi^2_{1-\alpha;1}$, otherwise, accept.

(3) Mann-Whitney U test is used for determination of the likelihood that two samples/groups emanated from the same population/distribution [5].

The test statistic is

$$Z = \frac{U - \mu_u}{\sigma_u}. \tag{6}$$

Then,

$$U = n_1 n_2 + \frac{n_1\left(n_1 + 1\right)}{2} - R_1 \tag{7}$$

where

n_1 is the total number of the first group/observation.

n_2 is the total number of the first group/observation.

R_1 is the sum of the ranks for the first group/observation.

Then

$$\mu_U = \frac{n_1 n_2}{2} \tag{8}$$

is the mean and

$$\sigma_u = \sqrt{\frac{n_1 n_2\left(n_1 + n_2 + 1\right)}{12}} \tag{9}$$

is the standard deviation.

This Z-score is, as usual, compared at a given level of significance with an appropriate critical value obtained from a normal distribution table for a rejection or acceptance of the null hypothesis.

(4) Modified Intrinsically Ties Adjusted Mann-Whitney U test is used to check whether two samples could have been drawn from the same population/distribution [6].

The test statistic is

$$\chi^2 = \frac{w^2}{\operatorname{var}(w)} = \frac{\left(n_{2.}R_{\cdot 1} - n_{1.}R_{\cdot 2}\right)^2}{\left(n_{2.}R_1^{*2} + n_{1.}R_2^{*2} - 2R_{\cdot 1}R_{\cdot 2}\right)\left(\pi^+ + \pi^- - \left(\pi^+ - \pi^-\right)^2\right)} \tag{10}$$

where

n_1 is the sample size of variable x_1.

n_2 is the sample size of variable x_2.

R_1 and R_2 are the respectively sums of the ranks assigned to observations from populations x_1 and x_2 in the combined ranking of these observations from the two populations.

π^+, π^- are respectively the probabilities that observations or scores by subject from population X_1 is on the average greater than or less than observations or scores by subject from population X_2.

The test hypothesis will be

$$H_0 : \pi^+ - \pi^- = 0$$

vs

$$H_1 : \pi^+ - \pi^- \neq 0 .$$

Reject H_0 at α-level of significance if $\chi^2 \geq \chi^2_{1-\alpha;1}$; otherwise, accept.

Relative Efficiency of two test statistics (R.E) is the ratio of the variances of one of the two test statistics to the other (say: T_1 to T_2) for equal sample size n. That is, relative efficiency of test T_2 to T_1 is defined as

$$\text{R.E}\left(T_2;T_1\right) = \frac{\text{var}\left(T_1\right)}{\text{var}\left(T_2\right)} . \tag{11}$$

Between test 1 and test 2, test 2 is relatively more efficient than test 1 if the relative efficiency of the tests, $\text{R.E}\left(T_2;T_1\right)$ is at least unity; that is if $\text{R.E}\left(T_2;T_1\right) \geq 1$ and hence test T_2 is said to be more powerful than test T_1.

Power of a statistical test is the probability of rejecting the null hypothesis when it is in fact false and should be rejected (*i.e.* the probability of not committing a type II error [7]. In other words, the power of test is equal to $1-\beta$, which is also known as the sensitivity [8]; where β is the probability of committing type II error = error rate. Error rate is defined as the ratio of number of erroneous decision to number of replicate. That is;

$$\text{E.R} = \frac{\text{Number of error output}}{\text{No of trials}\left(\text{Replicate}\right)} .$$

In this paper, Monte Carlo's Simulation techniques was used in the generation of data of different distributions and varying sample sizes ranging from 5 to 100 which was repeated 30 times for each sample size. In the simulation, sample size of 5, 10, 50 and 100 were considered to cover both small and large sample sizes. Monte Carlo simulation is defined as a method to generate random sample data based on some known distribution for numerical experiments. Monte Carlo simulation is an algorithm used to determine performance of an estimator or test statistic under various scenarios [9].

3. Algorithm for Monte Carlo Simulation

1) Specify the data generation process.
2) Choose a sample size N for the MC simulation.
3) Choose the number of times to repeat the MC Simulation.
4) Generate a randomsample of size N based on the data generation process.
5) Using random sample generated in 4 above, calculate the test statistic(s).
6) Go backto (4) and (5) until desirable replicate is achieved.
7) Examine parameter estimates, test statistics, etc.

In the paper, for data from a known family of distributions, Gamma (4, 0.3) and Beta (2, 2) were used.

4. Result

From the simulated data using Monte Carlo simulation approach, the following results were obtained: **Tables 1-4** are test statistics value of asymmetric distribution for different sample size while **Tables 5-8** are test statistics value of symmetric distribution for different sample size. **Table 9** is the variance of the test statistic considered. **Table 9** is calculated from **Tables 1-8**.

Variances were computed from **Table 1**, **Table 2**, to **Table 8**.

Table 1. Test statistic value of sample size 5.

Median	Modified median	Mann-Whitney U test	MMWU
0.4	2.1267361	31	5.425347
0.4	0.0400641	27	5.008013
0.4	1.0416667	25	5.208333
0.4	0.0400641	27	5.008013
0.4	3.7224265	32	5.744485
0.4	1.0416667	30	5.208333
3.6	3.42E+01	37	1.18E+01
0.4	3.7224265	23	5.744485
0.4	3.7224265	23	5.744485
0.4	2.1267361	31	5.425347
3.6	6.0019841	33	6.200397
0.4	6.0019841	33	6.200397
3.6	9.265350877	34	6.85307
0.4	1.41E+01	20	7.8125
0.4	0.0400641	27	5.008013
0.4	1.41E+01	20	7.8125
3.6	2.1267361	24	5.425347
0.4	3.7224265	23	5.744485
0.4	0.3652597	29	5.073052
3.6	6.0019841	22	6.200397
3.6	2.15E+01	36	9.300595
0.4	0.3652597	29	5.073052
0.4	0.0400641	27	5.008013
0.4	0.0400641	27	5.008013
3.6	2.1267361	24	5.425347
0.4	2.1267361	24	5.425347
0.4	9.265350877	34	6.85307
0.4	3.72E+00	23	5.744485
3.6	3.42E+01	18	1.18E+01
0.4	1.0416667	30	5.208333

Table 2. Test statistic value of sample size 10.

Median	Modified median	Mann-Whitney U test	MMWU
0	1.461039	111	10.1461
3.2	4.16666667	95	10.41667
0.8	3.34849111	96	10.33485
0.8	9.89010989	90	1.10E+01
3.2	1.69E+01	124	1.17E+01
0.8	0	105	10
0	0.1602564	107	10.01603
0.8	2.68E+01	128	1.27E+01
0.8	1.14E+01	121	1.11E+01
0	6.1120543	93	10.61121
0.8	1.69E+01	86	1.17E+01
0	0	105	10
0	1.010101	100	10.10101
0	0.1602564	103	10.01603
0	0.6441224	109	10.06441
0.8	1.010101	100	10.10101
0	0.3613007	102	10.03613
0	0.3613007	108	10.03613
0	1.461039	99	10.1461
0	1.010101	110	10.10101
0.8	1.010101	115	10.41667
0	4.16666667	101	10.06441
0.8	0.6441224	98	10.19992
0	4.16666667	95	10.41667
0	4.16666667	95	10.41667
0	0.04001601	104	10.004
3.2	0.1602564	107	10.01603
0.8	2.40E+01	127	1.24E+01
0	2.40E+01	127	1.24E+01
0	1.999184	98	10.19992

Table 3. Test statistic value of sample size 50.

Median	Modified median	Mann-Whitney U test	MMWU
1.44	1.58E+01	2624	5.03E+01
0.64	2.29E+01	2406	5.05E+01
1.44	5.77330169	2465	5.01E+01
0.16	3.539404	2572	5.01E+01
0.16	1.58E+01	2426	5.03E+01
1.44	84.48324	2751	5.17E+01
1.44	3.41E+01	2670	5.07E+01
1.71	1.99E+01	2636	5.04E+01
0.64	2.4359713	2564	5.00E+01
1.44	2.3125372	2563	5.00E+01
1.44	5.23E+01	2346	5.10E+01
0	0.9219399	2501	5.00E+01
0.64	4.11E+01	2684	5.08E+01
0	5.77330169	2465	5.01E+01
0.16	2.25E+01	2643	5.04E+01
1.44	98.1891	2768	5.20E+01
0.64	3.22E+01	2666	5.06E+01
1.44	4.49E+01	2359	5.09E+01
0.16	1.78E+01	2420	5.04E+01
0	1.22E+01	2612	5.02E+01
0.49	2.0753214	2489	5.00E+01
0	1.85E+01	2418	5.04E+01
1.44	5.29E+01	2705	5.11E+01
0.64	0.4096671	2509	5.00E+01
0.16	10.28211555	2445	5.02E+01
4	4.49E+01	2691	5.09E+01
2.56	113.9621	2786	5.23E+01
0	3.41E+01	2670	5.07E+01
5.76	318.1589	2105	5.64E+01
1.44	4.54E+01	2692	5.09E+01

Table 4. Test statistic value of sample size 100.

Median	Modified median	Mann-Whitney U test	MMWU
2	146.1039	9450	101.461
2.88	115.816	9515	101.158
0.32	0.2916085	10023	100.003
0.08	3.28E+01	9764	100.328
0.08	1.78E+01	9839	100.178
0.32	1.6902857	9985	100.017
0.32	6.97446092	10182	100.07
2	2.71E+01	10310	100.271
0.72	196.4389	9356	101.964
0	3.16940419	9961	100.032
0.32	4.24E+01	10375	100.424
0	2.41E+01	10295	100.241
0.32	76.26727	10485	100.763
1.28	125.2026	9494	101.252
2.88	124.747	10605	101.248
3.92	213.5569	9327	102.136
0.08	1.87E+01	10266	100.187
2	91.84752	9573	100.919
1.28	4.56E+01	9713	100.456
0	1.28E+01	10229	100.128
0	4.16E+01	9728	100.417
0.72	2.59E+01	9796	100.259
0.72	2.77E+01	10313	100.277
0.32	3.54E+01	9753	100.354
3.92	169.7601	9404	101.698
0.32	82.76295	9597	100.828
5.12	414.0671	11047	104.141
2	244.2155	10822	102.442
0	0.2916085	10077	100.003
0.72	4.35E+01	10379	100.435

Table 5. Test statistic value of sample size 5.

Median	Modified median	Mann-Whitney U test	MMWU
10	2.1267361	15	5.425347
0.4	0.0400641	27	5.008013
0.4	3.7224265	32	5.744485
3.6	5.99E+01	17	1.70E+01
0.4	2.1267361	24	5.425347
0.4	1.0416667	30	5.208333
0.4	2.1267361	31	5.425347
0.4	0.0400641	27	5.008013
0.4	0.0400641	28	5.008013
0.4	0.0400641	27	5.008013
0.4	0.0400641	27	5.008013
0.4	9.265350877	34	6.85307
0.4	2.1267361	24	5.425347
0.4	1.0416667	30	5.208333
0.4	6.0019841	22	6.200397
0.4	1.0416667	30	5.208333
3.6	3.7224265	32	5.744485
0.4	9.265350877	21	6.85307
0.4	0.0400641	28	5.008013
0.4	0.3652597	26	5.073052
0.4	0.0400641	28	5.008013
3.6	137.7604	39	3.26E+01
0.4	1.0416667	30	5.208333
0.4	1.0416667	30	5.208333
0.4	3.7224265	32	5.744485
3.6	5.99E+01	38	1.70E+01
3.6	1.41E+01	20	7.8125
0.4	9.265350877	34	6.85307
0.4	0.3652597	29	5.073052
0.4	6.0019841	22	6.200397

Table 6. Test statistic value of sample size 10.

Median	Modified median	Mann-Whitney U test	MMWU
0	0.04001601	106	10.004
3.2	2.99E+01	129	1.30E+01
3.2	7.25010725	118	10.72501
0	6.1120543	117	10.61121
0	5.08617066	94	10.50862
0.8	4.16666667	115	10.41667
0	3.34849111	96	10.33485
0	4.16666667	95	10.41667
0	2.627258	113	10.26273
0	5.08617066	94	10.50862
3.2	5.08617066	116	10.50862
0	0.04001601	106	10.004
3.2	1.49E+01	87	11.48897
0.8	0.1602564	103	10.01603
0	3.34849111	96	10.33485
0.8	3.34849111	114	10.33485
0.8	5.08617066	116	10.50862
0.8	1.90E+01	125	1.19E+01
0.8	3.34849111	96	10.33485
0.8	8.506944444	119	1.09E+01
0.8	0.04001601	104	10.004
0.8	1.010101	100	10.10101
0	0.6441224	109	10.06441
0.8	5.08617066	116	10.50862
0	2.627258	97	10.26273
0	0.04001601	106	10.004
0	0.3613007	102	10.03613
0.8	9.89010989	90	1.10E+01
0.8	1.14E+01	89	1.11E+01
0.8	1.999184	112	10.19992

Table 7. Test statistic value of sample size 50.

Median	Modified median	Mann-Whitney U test	MMWU
1.96	2.02E+01	2413	5.04E+01
0.04	2.4359713	2457	5.01E+01
0.36	0.77464	2547	5.00E+01
0.36	134.0615	2807	5.27E+01
0.04	3.10144281	2481	5.01E+01
0.36	3.10144281	2719	5.12E+01
1	7.86466359	2498	5.00E+01
0.04	0	2489	5.00E+01
3.24	6.24E+01	2720	5.12E+01
0.36	5.02769078	2469	5.01E+01
3.24	83.71228	2750	5.17E+01
0.36	3.18E+01	2385	5.06E+01
1	6.64E+01	2324	5.13E+01
1.96	3.96E+01	2681	5.08E+01
0.64	1.55E+01	2623	5.51E+01
7.84	2.65E+01	2906	5.51E+01
0.04	3.88E+01	2406	5.05E+01
0.36	0.05760133	2519	5.00E+01
0.36	9.03251706	2450	5.02E+01
0.04	5.39401309	2467	5.01E+01
0.04	0.2704292	2538	5.00E+01
3.24	124.258	2797	5.25E+01
3.24	149.5787	2822	5.30E+01
0.36	1.88E+01	2416.5	5.04E+01
0.04	8.555579317	2598	5.02E+01
0.36	2.99E+01	2661	5.06E+01
0.36	0.1024042	2517	5.00E+01
4.84	2.99E+01	2661	5.06E+01
1	1.4408299	2495	5.00E+01
0.04	2.0753214	2489	5.00E+01

Table 8. Test statistic value of sample size 100.

Median	Modified median	Mann-Whitney U test	MMWU
0	2.23E+01	10286	100.2233
0.32	2.92E+01	10320	100.2925
0.72	5.18E+01	9691	100.5182
0.32	7.846151383	9910	100.0785
1.28	8.88829316	10199	100.0889
2	278.4764	9227	102.7848
5.12	193.564	9361	101.9356
1.28	5.39E+01	9684	100.5387
0	1.7959225	10117	100.018
0.08	5.38529858	9934	100.0539
0.32	4.32827258	9946	100.0433
0.08	2.2505064	10125	100.0225
1.28	0	10357	100.3784
0.32	3.765017	10147	100.0377
2.88	233.3302	9295	102.3333
1.28	2.65E+01	10307	100.2649
0	3.88E+01	9739	100.3884
0.32	1.2101464	10105	100.0121
0.32	4.7546596	10159	100.0475
1.28	104.7176	10559	101.0472
2	294.7239	9204	102.9472
0.72	8.301285418	10194	100.083
3.92	157.6999	9427	101.577
1.28	5.48E+01	10418.5	96.96971
0.32	4.24540158	9947	100.0425
0	5.29279989	10165	100.0529
0.08	5.29279989	9935	100.0529
0.08	4.24540158	9947	100.0425
0.08	2.90E+01	9781	100.2903
0.72	116.2544	9514	101.1625

Table 9. Variances of the test statistic considered.

Type of distr.	Test statistic	5	10	50	100
Symmetric distribution	Median	1.695	1.081	3.219	1.487
	MMED	791.97	41.56	1748.91	7470.9
Asymmetric distribution	Median	2.072	0.924	1.575	1.918
	MMED	82.56	64.37	3605.1	8664.9
Symmetric distribution	Mann-Whitney	27.513	124.97	22424.7	137572.1
	MMWU	31.68	0.416	1.978	1.18
Asymmetric distribution	Mann-Whitney	25.082	125.18	23213.6	200042.0
	MMWU	3.302	0.641	1.442	0.867

As shown in **Table 10**, M_1 to M_4 are the methods considered as M_1 is the Median test and M_2 is Modified Median test (MMED), M_3 is the Mann-Whitney U test and M_4 is Modified Mann-Whitney U (MMWU) test statistic.

All the ratios for the first and second rows are less than 1.0 which implies the method used as numerator is better and more reliable than the method used as denominator for all the sample sizes considered, *i.e*. Median Test is better than Modified Median intrinsically Adjusted for Ties (MMED) using Relative Efficiency (**Table 11**).

Moreover, considering methods 3 and 4 for asymmetric distribution, M_4 (Modified Mann-Whitney U test) is better than M_3 (Mann-Whitney U test) since the values of R.E are all greater than 1.0. Considering symmetric distribution, the efficiency of M_3 (Mann-Whitney U test) is better/stronger for small sample size (5) and as sample size increases, the strength of M_4 (Modified Mann-Whitney U test) increases and outweighs M_3 (Mann-Whitney U test); this implies that the method is inconsistent because its efficiency decreases as sample size increases.

As shown in **Table 12**, Median test has lower error rate which makes it better than MMED. This can be interpreted thus, the usual Median test statistic is a better test statistic when testing relevant hypothesis. Error rate of MMED increases as sample size increases which implies MMED is better used for small sample sizes than the large sample size; but Median test statistic is found to be more adequate for both small and large sample sizes and hence should be preferable.

As shown in **Table 13**, Mann-Whitney U test statistic is more suitable irrespective of the nature of distribution of set of available data as the error rates of MMWU are significantly high. See **Table 13**. It can be deduced that the sensitivity of the test statistics is independent of sample size of the data but the distribution; either symmetric or asymmetric distribution.

Power of test were computed from **Table 12** and **Table 13**.

Power of test is the sensitivity of a test statistic and the greater the value, the more sensitive the test statistic for both symmetric and asymmetric distributions. Median test is more sensitive than MMED because median has higher power than MMED irrespective of the sample sizes.

Considering both Mann-Whitney and MMWU, Mann-Whitney U test is more sensitivity than MMWU for both symmetric and asymmetric distribution. For better understanding of sensitivity of the four test statistics, line chart of power of test is constructed as shown in **Figure 1** and **Figure 2**; Line chart can be used to show position of the strength or power of a test statistic especially in statistical inference. This shows test statistic with higher power with the maximum power of 1.0. See **Figure 1** and **Figure 2**.

5. Graphical Illustration of Power of Test

As shown in **Figure 1**, irrespective of sample size; either large or small, the median test statistic has higher power than the modified median test which makes it more appropriate. The modified median test is sensitive to sample size as its power decreases as sample size increases.

As shown in **Figure 2**, irrespective of sample size; either large or small, the Mann-Whitney U test statistic has higher power than the modified Mann-Whitney U test which makes it more appropriate. The modified method has considerably low power as sample size varies/increases.

Table 10. Relative efficiency of the test statistics.

Distribution	5	10	50	100
Asymmetric (M_1M_2)	0.0251	0.0144	0.0004	0.0002
Symmetric (M_1M_2)	0.0021	0.0260	0.0018	0.0002
Asymmetric (M_3M_4)	7.5960	195.2286	16098.1969	230728.9504
Symmetric (M_3M_4)	0.8685	300.4807	11337.0576	116586.5254

Table 11. Power of tests.

Type of distr.	Test statistic	5	10	50	100
Symmetric distribution	Median	1	1	0.9667	0.9
	MMED	0.6667	0.5	0.3333	0.1667
Asymmetric distribution	Median	1	1	0.9333	0.9
	MMED	0.6667	0.6	0.2	0.1333
Symmetric distribution	Mann-Whitney	0.9	1	0.9333	0.9333
	MMWU	0	0	0	0
Asymmetric distribution	Mann-Whitney	1	1	0.9667	0.9667
	MMWU	0	0	0	0

Table 12. Error rate of the median and modified median test statistics.

Type of distr.	Test statistic	5	10	20	30	50	100
Symmetric distribution	Median	0.0000	0.0000	0.0000	0.0000	0.0333	0.1000
	MMED	0.3333	0.5000	0.7000	0.8000	0.6667	0.8333
Asymmetric distribution	Median	0.0000	0.0000	0.0000	0.0000	0.0667	0.1000
	MMED	0.3333	0.4000	0.6000	0.6667	0.8000	0.8667

Table 13. Error rate of Mann-Whitney and MMWU test statistics.

Type of distr.	Test statistic	5	10	50	100
Symmetric distribution	Mann-Whitney	0.1000	0.0000	0.0667	0.0667
	MMWU	1.0000	1.0000	1.0000	1.0000
Asymmetric distribution	Mann-Whitney	0.0000	0.0000	0.0333	0.0333
	MMWU	1.0000	1.0000	1.0000	1.0000

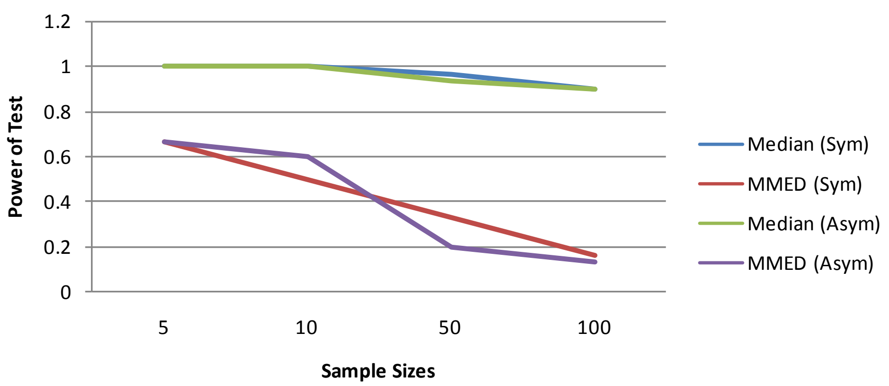

Figure 1. Power of median and modified median test for both symmetric and asymmetric distribution.

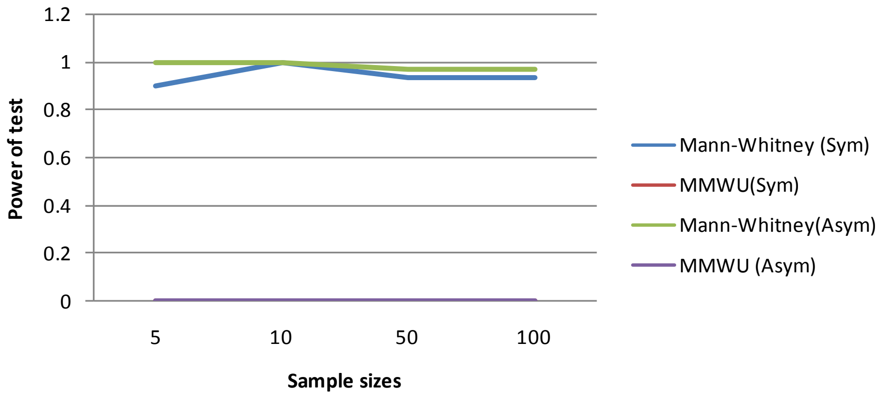

Figure 2. Power of Mann-Whitney U and Modified Mann-Whitney U test for both symmetric and asymmetric distribution.

6. Summary and Conclusion

We have in this paper presented a nonparametric statistical method for the analysis of two sample tests. Based on the result of the analysis used, it is observed that for both symmetric and asymmetric distributions, median test is more efficient than Modified Median (MMED) test using relative efficiency as a measure of the efficiency of test statistic since the relative efficiency values are less than 1.0 while in terms of power of test for both symmetric and asymmetric distributions, median test is more sensitive than Modified Median (MMED) test since it has higher power. For Mann-Whitney U test and Modified Mann-Whitney U test (MMWU) using both relative efficiency and power of test, Mann-Whitney U test is more efficient and more sensitive than Modified Mann-Whitney U test (MMWU) since the relative efficiency values are greater than 1 and also it has higher power. In terms of sample size, efficiency of the method is independent of sample sizes except Modified Median Test which has higher power for small sample sizes.

References

[1] Gibbon, J.D. (1992) Nonparametric Statistics: An Introduction. Quantitative Applications in Social Sciences, Sage Publications, New York.

[2] Afuecheta, E.O., Oyeka, C.A., Ebuh, G.U. and Nnanatu, C.C. (2012) Modified Median Test Intrinsically Adjusted for Ties. *Journal of Basic Physical Research*, **3**, 30-34.

[3] Mood, A.M. (1954) On the Asymptotic Efficiency of Certain Nonparametric Two-Sample Tests. *Annals of Mathematical Statistics*, **25**, 514-522. http://dx.doi.org/10.1214/aoms/1177728719

[4] Siegel, S. (1988) Nonparametric Statistics for the Sciences. McGraw-Hill, Kogakusha Ltd., Tokyo, 399.

[5] Mann, H.B. and Whitney, D.R. (1947) On a Test of Whether One of Two Random Variables Is Stochastically Larger than the Other. *Annals of Mathematical Statistics*, **18**, 50-60. http://dx.doi.org/10.1214/aoms/1177730491

[6] Oyeka, I.C.A. and Okeh, U.M. (2013) Modified Intrinsically Ties Adjusted Mann-Whitney U Test. *IOSR Journal of Mathematics*, **7**, 52-56. http://dx.doi.org/10.9790/5728-0745256

[7] Mumby, P.J. (2002) Statistical Power of Non-Parametric Tests: A Quick Guide for Designing Sampling Strategies. *Marine Pollution Bulletin*, **44**, 85-87. http://www.elsevier.com/locate/marpolbul
http://dx.doi.org/10.1016/S0025-326X(01)00097-2

[8] Gupta, S.C. (2011) Fundamentals of Statistics. 6th Revised and Enlarged Edition, Himilaza Publishing House PVT Ltd., Mumbai, 16.28-16.31.

[9] Schaffer, M. (2010) Procedure for Monte Carlo Simulation. SGPE QM Lab 3, Monte Carlos Mark Version of 4.10.2010.