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The Prediction of Non-Life Claim Reserves under Inflation

—An Analysis including Diagonal Effects

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Abstract

The extraction of various reserves is one of the most important measures that guarantee insurance companies' solvency. Accurate assessment of non-life insurance claim reserves needs to consider the volatility risks of inflation. This paper presents a stochastic model of claim reserves including inflation factor and diagonal effects. By applying this model, we can predict the values of the claim reserves and evaluate predicting risks. Through analyzing actual data and using the bootstrap method, we can compare Bornhuetter-Ferguson method involving diagonal effects with chain ladder method. It is shown that the former is more efficient and robust than the latter.

Keywords

Claim Reserves, Diagonal Effects, Bornhuetter-Ferguson Method, Chain Ladder Method, Inflation, Bootstrap

1. Introduction

In insurance industries, inflation can be divided into two categories: economic inflation and claim inflation. The latter's impact on claim reserves estimations is more complicated than the former. In actuarial literature, there are few studies on the inflation's impacts on claim reserves estimations. Economic inflation can be quantified by CPI, etc. However, it is difficult to estimate the fluctuations risk on prediction of claim reserves resulting from claims inflation.

It is pointed out in David [1] that in calculation of the loss reserve variance, inflation index should be extracted theoretically from insurance loss data itself, but actual insurance data are not stable enough to provide a credible evaluation; therefore, external factors should be applied to characterize the inflation index.

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A model with diagonal effects depicting the effects of economic inflation was established in Rietdorf [2] of the form,

$$EX_{ij} = \alpha_i \beta_j \delta_{i+j}, \quad VX_{ij} = \varphi \alpha_i \beta_j \delta_{i+j}^2.$$

It should be noted that diagonal effects δ_j come from two aspects: one is economic inflation expressed as a relevant price index which implies that claim payments are related to the calendar time; the other is the claims inflation. This factor, generally speaking, comes from legal issues and the compensation way.

In Kuang [3] [4] the claim inflation is assumed to satisfy,

$$EX_{ij} = \alpha_i \beta_j \delta_{i+j}, \quad VX_{ij} = \varphi \alpha_i \beta_j \delta_{i+j}.$$

We cannot tell whether the economic inflations or the claims inflation lead to the changes along the diagonal, just from diagonal, just from the run-off triangle. To solve this problem, two different models are proposed in Jessen and Rietdorf [5]. Let T_i take places of α_i, where T_i is considered known; we can reduce the number of unknown parameters in above models and derive the unique solutions. Further, through the following models we can determine the value of parameter *c*. When *c* = 0 the change is caused by claim inflation; when *c* = 1 the change is caused by economic inflation; other cases are caused by both.

A Bornhuetter-Ferguson type method including diagonal effects is given by,

$$EX_{ij} = T_i \beta_j \delta_{i+j}, \quad VX_{ij} = \varphi \delta_{i+j}^c EX_{ij}$$

where the exposure parameters $T_i > 0, 1 \le i \le m,$ and $c \in \left\{0, 1, \frac{1}{2}\right\}$ are assumed to be known.

$\varphi, \beta_1, \cdots, \beta_m$ and $\delta_1, \cdots, \delta_{2m-1}$ are positive unknown constants which satisfy

$$\sum_{j=0}^{m-1} \beta_j = 1.$$

A credibility model including diagonal effects is given by,

$$E\Theta_i = T_i, \; V\Theta_i = \xi^{-1} T_i.$$

$$E\left(X_{ij} \mid \Theta_i\right) = \Theta_i \beta_j \delta_{i+j}, \; V\left(X_{ij} \mid \Theta_i\right) = \varphi \delta_{i+j}^c E\left(X_{ij} \mid \Theta_i\right).$$

They choose $c \in \{0, 1, 1/2\}$ for reasons: *c* = 0 corresponds to claims inflation; *c* = 1 corresponds to economic inflation; *c* = 1/2 is chosen in a situation where both effects have an impact on data.

However the specific choice of *c* is based on intuition as well as plots of residuals $\hat{e}_{ij}$ which contain so much randomness and the range of value *c* is not accurate enough.

This paper uses the model structure similar to the one in Jesson and Rietdorf [5] (a Bornhuetter-Ferguson method including diagonal effects). The differences lie in our model which expands the value of *c* on {0; 1/4; 1/2; 3/4; 1} and changes the method of choosing *c*. Instead of checking the residuals plots for each *c*, we take the *c* under which the coefficient of variation is minimum of yearly claims reserving.

The reason why we change the method of choosing *c* is actually the plotted points $\left(j, \hat{e}_{ij}\right), (i, j) \in \mathrm{A}_m$ for each *c* are almost the same. All the residuals can be taken as independent identically distributed. What's more, coefficient of variation and standard deviation of reserve estimators are important indicators to evaluate estimators' accuracy.

This article uses VBA to analyze actual data and simulate estimators' statistical characteristics. The results show that by applying bootstrap method, Bornhuetter-Ferguson method with diagonal effects is more effective and efficient than chain ladder method when predicting claim reserves.

2. Extended Bornhuetter-Ferguson Model including Diagonal Effects

Let X_{ij} be the observable incremental claims, which occurs in accident year *i* and development year *j*. Denote $\Delta_m = \left\{X_{ij} : (i, j) \in \mathrm{A}_m\right\}$, where $\mathrm{A}_m = \left\{(i, j) \in \mathbb{N} \times \mathbb{N}_0 : 1 \le i + j \le m\right\}$.

Let $\mathbb{N}$ be the set of natural numbers, $\mathbb{N}_0$ be the set of positive integers and *m* the calendar year. Write $\mathrm{B}_m = \left\{(i, j) \in \mathbb{N} \times \mathbb{N}_0 : i \le m, j \le m - 1\right\}$.

With data $X_{ij},(i,j)\in \mathrm{A}_m$ we can make predictions of $X_{ij},(i,j)\in \mathrm{B}_m\setminus \mathrm{A}_m$ which are unobservable random variables at time *m*. We can see the detail in **Figure 1**.

As a technical basis for prediction we consider a model for $X_{ij},(i,j)\in \mathrm{B}_m$. The model requires $X_{ij},(i,j)\in \mathrm{B}_m$ are mutually independent and satisfy the condition:

$$\mathrm{E}X_{ij}=T_i\beta_j\delta_{i+j}, \tag{1}$$

$$\mathrm{V}X_{ij}=\varphi\delta_{i+j}^{c}\mathrm{E}X_{ij}, \tag{2}$$

where $T_i>0, 1\le i\le m$ are row effects which will be represented by yearly exposure measures. $\delta_j, 1\le j\le 2m-1$ are the exogenous indexes which related to inflation. $\varphi,\beta_0,\cdots,\beta_{m-1}$ and $\delta_1,\cdots,\delta_{2m-1}$ are positive unknown constants. Satisfy,

$$\sum_{j=0}^{m-1}\beta_j=1 \tag{3}$$

The value of *c* quantifies claims inflation and economic inflation's effect on claims reserving estimation, $c\in\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\}$ is assumed to be known.

c = 0 corresponds to claims inflation; *c* = 1 corresponds to economic inflation; *c* = 1/4, 1/2, 3/4 corresponds to the ratio of the effects of claims inflation and economic inflation on claims reserving estimation.

3. Solving Proportionality Value β_i and Estimating Exogenous Index δ_{i+j}

3.1. Separation Method

We can know from Taylor [6] that if we assume the conditions affecting individual claim sizes remained constant, then the ratios of average claim amount paid in development year *k* per claim with year of origin *i* would have an expected value γ_j which is independent of *i*. With further assumption if claims cost of a particular development year is proportional to some indexes which relate to the year of payment rather than the year of origin, the expected claims cost of development year *j* per claim with year of origin *i* is $\gamma_j\lambda_{i+j}$ where λ_k is exogenous index appropriate to year of payment *k* satisfy $\sum_{j=0}^{k}\gamma_j=1$. These expected values then form the following run-off triangle.

The corresponding value in triangle denoted by observed values $S_{ij}=\frac{X_{ij}}{n_i}$ where n_i = number of claims settled in development year *o* + estimated number of claims outstanding at end of development year *o* (both in respect of year of origin *i*). From **Figure 2** we can derive the following results.

Sum along the diagonal involving λ_k, obtain

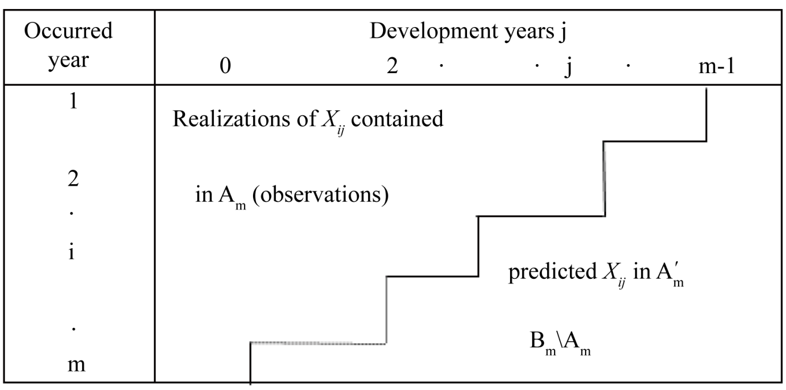


Figure 1. Details to predict $X_{ij},(i,j)\in \mathrm{B}_m\setminus \mathrm{A}_m$.

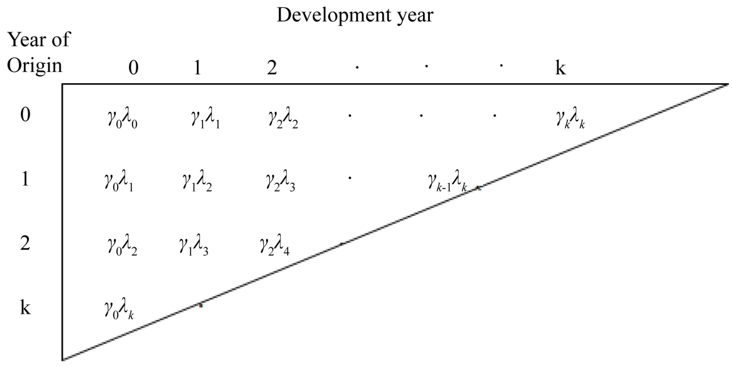


Figure 2. Run-off triangle.

$$d_k = \lambda_k\left(\gamma_0 + \gamma_1 + \cdots + \gamma_k\right) = \lambda_k.$$

Thus estimate of λ_k is $\hat{\lambda}_k = d_k$. Sum along the next diagonal, the result is

$$d_{k-1} = \lambda_{k-1}\left(\gamma_0 + \gamma_1 + \cdots + \gamma_{k-1}\right) = \lambda_{k-1}\left(1-\gamma_k\right).$$

v_k is the sum of the column of the triangle involving γ_k. So

$$v_k = \gamma_k\lambda_k.$$

$$\hat{\gamma}_k = \frac{v_k}{\hat{\lambda}_k}.$$

Now,

$$\hat{\lambda}_{k-1} = \frac{d_{k-1}}{1-\hat{\gamma}_k}.$$

This procedure can be repeated, leading to the general solution:

$$\hat{\lambda}_h = \frac{d_h}{1-\hat{\gamma}_k - \hat{\gamma}_{k-1} - \cdots - \hat{\gamma}_{h+1}}$$

$$\hat{\gamma}_j = \frac{v_j}{\hat{\lambda}_j + \hat{\lambda}_{j+1} + \cdots + \hat{\lambda}_k}$$

where d_h is the sum along the (h + 1)-th diagonal and v_k is the sum down the (k + 1)-th column.

From (1) we have $\frac{EX_{ij}}{T_i} = \beta_j\delta_{i+j}$, let β_j replaces γ_j, δ_{i+j} replaces λ_{i+j}, $\frac{EX_{ij}}{T_i}$ replaces $S_{ij} = \frac{X_{ij}}{n_i}$. We can solve out β_j and δ_{i+j}.

3.2. Total Marginal Principal

In classification system, it requires the sum of pure insurance cost is equal to the sum of the corresponding experience compensation cost under different level of classification variables, *i.e.*, the marginal sum of estimations equals to the marginal sum of observations.

Make a transformation $\tilde{X}_{ij} = \frac{X_{i-j,j}}{T_{i-j}}$. Then $EX_{ij} = T_i\beta_j\delta_{i+j}$ is in the form of,

$$E\tilde{X}_{ij} = \beta_j\delta_j,\ 1 \le i \le m,\ 0 \le j \le i+1.$$

Based on total marginal principal we can derive that,

$$\hat{\delta}_i\sum_{j=0}^{i-1}\hat{\beta}_j = \sum_{j=0}^{i-1}\tilde{X}_{ij},\ 1 \le i \le m,$$

$$\hat{\beta}_j \sum_{i=j+1}^{m} \hat{\delta}_i = \sum_{i=j+1}^{m} \tilde{X}_{ij},\ 0 \le j \le m-1.$$

Put $\sum_{j=0}^{m-1} \hat{\beta}_j = 1$, then $\sum_{i=k+1}^{m} \tilde{C}_{ik} = \sum_{i=k+1}^{m} \sum_{j=0}^{k} \tilde{X}_{ij} = \sum_{j=0}^{k} \sum_{i=k+1}^{m} \tilde{X}_{ij} = \sum_{j=0}^{k} \left[\hat{\beta}_j \sum_{i=k+1}^{m} \hat{\delta}_i \right]$

$$\sum_{i=k+1}^{m} \tilde{C}_{i,k-1} = \sum_{i=k+1}^{m} \sum_{j=0}^{k-1} \tilde{X}_{ij} = \sum_{j=0}^{k-1} \sum_{i=k+1}^{m} \tilde{X}_{ij} = \sum_{j=0}^{k-1} \left[\hat{\beta}_j \sum_{i=k+1}^{m} \hat{\delta}_i \right]$$

$$\frac{\sum_{i=k+1}^{m} \tilde{C}_{ik}}{\sum_{i=k+1}^{m} \tilde{C}_{i,k-1}} = \frac{\sum_{j=0}^{k} \sum_{i=k+1}^{m} \tilde{X}_{ij}}{\sum_{j=0}^{k-1} \sum_{i=k+1}^{m} \tilde{X}_{ij}} = \frac{\sum_{j=0}^{k} \left[\hat{\beta}_j \sum_{i=k+1}^{m} \hat{\delta}_i \right]}{\sum_{j=0}^{k-1} \left[\hat{\beta}_j \sum_{i=k+1}^{m} \delta_i \right]} = \frac{\sum_{j=0}^{k} \hat{\beta}_j}{\sum_{j=0}^{k-1} \hat{\beta}_j}$$

Finally

$$\frac{\sum_{j=0}^{k} \hat{\beta}_j}{\sum_{j=0}^{k-1} \hat{\beta}_j} = \frac{\sum_{j=0}^{k} \sum_{i=k+1}^{m} \tilde{X}_{ij}}{\sum_{j=0}^{k-1} \sum_{i=k+1}^{m} \tilde{X}_{ij}}, 1 \le k \le m-1. \tag{4}$$

$$\hat{\delta}_i = \frac{\sum_{j=0}^{i-1} \tilde{X}_{ij}}{\sum_{j=0}^{i-1} \hat{\beta}_j}, 1 \le i \le m. \tag{5}$$

3.3. Consistency of Parameters' Estimation

In this section we will prove the consistency of β_j, δ_{i+j}. Give the proposition as following

Proposition 3.3. If $T_i \to \infty$ for all $1 \le i \le m$ then $\hat{\delta}_{j+1} \xrightarrow{p} \delta_{j+1},$, $\hat{\beta}_j \xrightarrow{p} \beta_j$, for $0 \le j \le m-1$.

Proof. Make a transformation of $E\left(X_{ij}\right) = T_i \beta_j \delta_{i+j}$ we attain,

$$\frac{EX_{ij}}{T_i} = \beta_j \delta_{i+j}.$$

Apply Chebyshev's inequality, if $T_i \to \infty$ we have,

$$p\left(\left|\frac{X_{ij}}{T_i} - \beta_j \delta_{i+j}\right| \ge \varepsilon\right) \le \frac{1}{\varepsilon^2} \cdot Var\left(\frac{X_{ij}}{T_i}\right) = \frac{1}{\varepsilon^2} \cdot \frac{1}{T_i} \cdot \varphi \delta_{i+j}^{c} \cdot T_i \beta_j \delta_{i+j} = \frac{\varphi \delta_{i+j}^{1+c} \beta_j}{T_i \varepsilon^2} \to 0.$$

Namely $\frac{X_{ij}}{T_i} \xrightarrow{p} \beta_j \delta_{i+j}$. Use recursive schemes (3)-(5) with the continuous mapping theorem we acquire the desired result.

3.4. Prediction of Diagonal Effects

In this subsection we predict diagonal effects $\delta_j, m+1 \le j \le 2m-1$.

We assume $\delta_j, j \ge 1$ obey an AR (1) process, that is, $\hat{\delta}_j \Gamma + \varepsilon_j = \hat{\delta}_{j+1}$.

By means of least-squares method we acquire the least square estimation of Γ,

$$\hat{\Gamma} = \frac{\sum_{j=1}^{m-1} \hat{\delta}_{j+1} \hat{\delta}_j}{\sum_{j=1}^{m-1} \hat{\delta}_j^2} \tag{6}$$

Then the predictors of $\hat{\delta}_j, j \geq m+1$ are in the form of

$$\hat{\delta}_j = \hat{\delta}_m \hat{\Gamma}^{j-m}. \tag{7}$$

As a result $X_{ij}, (i, j) \in \mathrm{B}_m \setminus \mathrm{A}_m$ are predicted by

$$\tilde{\mu}(X_{ij}) = T_i \hat{\beta}_j \hat{\delta}_{i+j}. \tag{8}$$

4. Solving Method of *φ*, *c*

In this section we try to estimate φ and determine parameter c by considering the residuals, $\hat{e}_{ij}, (i, j) \in \mathrm{A}_m$. Define variance structure

$$\hat{e}_{ij} = \frac{X_{ij} - T\hat{\beta}_j \hat{\delta}_{i+j}}{\left(T_i \hat{\beta}_j \hat{\delta}_{i+j}^{1+c}\right)^{1/2}}, (i, j) \in \mathrm{A}_m. \tag{9}$$

Apply the second moment method, $VX_{ij} = E\left(X_{ij} - EX_{ij}\right)^2$ can be estimated by $\frac{\sum_{i=1}^{m}\sum_{j=0}^{m-i}\left(X_{ij} - EX_{ij}\right)^2}{m(m+1)/2}$. Together with (2), we can derive $\frac{\sum_{i=1}^{m}\sum_{j=0}^{m-i}\left(X_{ij} - EX_{ij}\right)^2}{m(m+1)/2} = \varphi \delta_{i+j}^{c} EX_{ij}$.

Combined (8) with (1), we can yield estimation of φ

$$\hat{\varphi} = \frac{2}{m(m+1)} \sum_{i=1}^{m}\sum_{j=0}^{m-i} \hat{e}_{ij}^2. \tag{10}$$

The next step in the estimation procedure is to apply the bootstrap method similar to the one in England and Verrall [7]. It should be noticed the bootstrap method is based on the assumption that the residuals $\hat{e}_{ij}, (i, j) \in \mathrm{A}_m$ are Independent Identically Distributed. By random sampling with replacement we attain $e_{ij}^*(k), 1 \leq k \leq B$.

Finally, we generate Independent Identically Distributed versions of Δ_m by

$$X_{ij}(k) = T_i \hat{\beta}_j \hat{\delta}_{i+j} + e_{ij}^*(k)\left[\hat{\varphi} T_i \hat{\beta}_j \hat{\delta}_{i+j}^{1+c}\right]^{1/2}, 1 \leq k \leq B. \tag{11}$$

To determine parameter c, for each $c \in \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$ we plot points $(j, \hat{e}_{ij}), (i, j) \in \mathrm{A}_m$ to check which residual plots give the best fit to Independent Identically Distributed. Meanwhile, with all parameters being solved we can use (8) and $\hat{R}_i = \sum_{j=m-i+1}^{m-1} \hat{\mu}(X_{ij}), 2 \leq i \leq 13$ to estimate yearly reserve and calculate its standard deviations, variance coefficient. Through above two points we could make the final choice of value c.

5. Empirical Analysis

Our data is from Jesson and Riedorf [5] which contains 13 years run-off for a portfolio of third-party liability for auto insurance. The data is shown in incremental form in **Table 1**.

In the model we assume that row effects $T_i > 0, 1 \leq i \leq m$ are known and represented by yearly exposure measures given in **Table 2**.

The estimators of the parameters $\beta_j, \delta_{j+1}, 0 \leq j \leq 12,$ are shown in **Table 3**.

Now, we should predict $\delta_{13+j}, j \geq 1$.

Firstly, take unit root/stationarity test to $z = \delta_j, j \geq 1$ the result is given in **Table 4**.

Obviously we cannot refuse null hypothesis: z has a unit root.

Secondly, get 1st differences of z and take unit root test. The result is given in **Table 5**.

Table 1. Incremental runs-off triangle.

i\j	0	1	2	3	4	5	6	7	8	9	10	11	12
1	22564	17331	17377	7723	5058	2530	1443	1195	1889	106	33	139	14
2	22901	26734	8974	7089	3116	1911	3284	1591	879	21	575	476	
3	36152	26513	10973	6714	7155	2176	1656	1094	−89	8	115		
4	34722	29642	13593	11496	6256	6404	3900	2157	1133	25			
5	30709	28020	12465	8504	9929	5592	910	3413	1428				
6	33727	32190	13318	9211	8129	5225	2149	773					
7	30727	27677	9251	9221	6169	7492	2952						
8	32498	35446	18532	15110	13990	4986							
9	32228	42937	16231	12942	11078								
10	41947	41634	21056	15442									
11	37247	34135	19061										
12	32891	29719											
13	35993												

Table 2. $T_i, 1 \le i \le 13$ (represent by yearly exposure measures).

i	1	2	3	4	5	6	7	8	9	10	11	12	13
T_i	85047	74409	86077	83082	83427	81557	79495	101564	95482	107062	90091	85413	81995

Table 3. The estimators $\hat{\beta}_j, \hat{\delta}_{j+1}, 0 \le j \le 12$.

j	0	1	2	3	4	5	6	7	8	9	10	11	12
β_j	0.316	0.29	0.132	0.094	0.072	0.043	0.022	0.016	0.01	4e−04	0.002	0.003	1e−04
δ_{j+1}	0.839	0.844	1.333	1.127	1.114	1.123	1.233	1.109	1.181	1.386	1.303	1.419	1.38

Table 4. Unit root test of Z.

Null hypothesis: Z has a unit root			
		t-statistic	Prob.*
Augmented Dickey-Fuller test statistic		−0.0486	0.9308
Test critical values:	1% level	−4.2970	
	5% level	−3.2126	
	10% level	−2.7476	

*MacKinnon (1996) one-sided p-values.

Then we can generate ACF and PACF plots for dz. Autocorrelation and Partial Correlation are shown in **Figure 3**.

Let $\varepsilon_j, 1 \le j \le 2m-1$ be a mean zero white noise process. From **Figure 3** we have $D(z) = \varepsilon_j$.

Finally we take Dicky-Fuller Test of $z = \gamma z(-1) + \varepsilon_j$ given in **Table 6**.

Though R-squared < 0, the value of coefficient approximates to 1. What's more, we need to take estimation error of δ_j into consideration and construct a simple easy-to-implement model. Above all we can assume

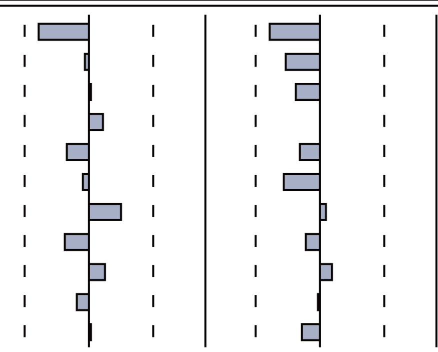
Table 5. Unit root test of D(Z).

Null hypothesis: D(Z) has a unit root			
		t-statistic	Prob.*
Augmented Dickey-Fuller test statistic		−4.9278	0.0034
Test critical values:	1% level	−4.2000	
	5% level	−3.1753	
	10% level	−2.7289	

*MacKinnon (1996) one-sided p-values.

Table 6. Result of Dicky-Fuller test.

Dependent variable: Z				
Method: least squares				
Variable	Coefficient	Std. error	t-statistic	Prob.
Z(−1)	1.024551	0.044718	22.91114	0.0000
R-squared	−0.2423	Mean dependent var		1.21266
Adjusted R-squared	−0.2423	S.D. dependent var		0.16416
S.E. of regression	0.1829	Akaike info criterion		−0.47924
Sum squared resid	0.3682	Schwarz criterion		−0.43883
Log likelihood	3.87546	Hannan-Quinn criter.		−0.49420
Durbin-Watson stat	2.84541			



Autocorrelation / Partial Correlation		AC	PAC	Q-Stat	Prob
	1	-0.453	-0.453	3.1306	0.077
	2	-0.045	-0.315	3.1647	0.205
	3	0.010	-0.222	3.1666	0.367
	4	0.119	0.008	3.4662	0.483
	5	-0.204	-0.185	4.4617	0.485
	6	-0.057	-0.335	4.5539	0.602
	7	0.294	0.045	7.4594	0.383
	8	-0.223	-0.138	9.5403	0.299
	9	0.151	0.107	10.821	0.288
	10	-0.103	-0.024	11.710	0.305
	11	0.010	-0.159	11.725	0.385

Figure 3. ACF, PACF plots.

$\delta_j, j \geq 1$ obey an AR (1) process. Using Equations (6) and (7) in Section 3.4, we can get $\hat{\Gamma} = 1.0245$, $\tilde{\delta}_{13+j} = \hat{\delta}_{13}\hat{\Gamma}_j = 1.38 \cdot 1.0245^j, j \geq 1$.

Further, if we make the assumption that $\varepsilon_j \sim \mathrm{N}(0, \delta^2), j \geq 1$, $\hat{\delta}^2 = (m-1)^{-1} \sum_{j=1}^{m-1} \left(\hat{\delta}_{j+1} - \hat{\Gamma}\hat{\delta}_j\right)^2 = 0.03037$.

For each c = 0, 1/4, 1/2, 3/4, 1, we plot points $(j, \hat{e}_{ij}), (i, j) \in \mathrm{A}_m$, and take runs test to verified stochastic feature by the SPSS. Whether c = 0, 1/4, 1/2, 3/4, 1, the p-value = 0.753 > 0.05, the residual error is mutual independent. What's more, the residual plots seem little difference. Then we compared the statistical characteristic parameters of reserve estimators finding in the case c = 0 the standard deviations, variance coefficient of yearly reserve estimates is minimal.

Naturally apply $\hat{\varphi} = \frac{2}{m(m+1)} \sum_{i=1}^{m} \sum_{j=0}^{m-i} \hat{e}_{ij}^2$, we can derive $c = 0$, $\hat{\varphi} = 345.1$.

Finally we generate identically distributed versions of Δ_m as following:

$$X_{ij}(k) = T_i \hat{\beta}_j \hat{\delta}_{i+j} + e_{ij}^*(k) \left[\hat{\varphi} T_i \hat{\beta}_j \hat{\delta}_{i+j} \right]^{1/2},$$

For each k we can use (7) and the notation $\hat{R}_i^{(1)} = \sum_{j=m-i+1}^{m-1} \tilde{\mu}(X_{ij}), 2 \le i \le 13$ to predict yearly reserve estimators.

Let k = 50000, take the average of 50000 times' claims reserve predictors as each year's claims reserve estimates. We use excel VBA to realize the procedure.

Table 7 and **Table 8** are reserve estimators and its distribution characteristics which are respectively acquired by B-F method including diagonal effects and chain ladder method.

Table 7. Predict reserves distribution characteristics (extended B-F model).

Reserve estimators for year *i*	Average estimators	SD	CV	30% percentile	95% percentile
i = 2	12.4861	62.2340	4.98426	−25.0720	125.0904
i = 3	358.2152	264.7145	0.7389	211.7834	804.4639
i = 4	607.0452	308.6017	0.5083	439.5269	1126.2124
i = 5	677.6418	328.9206	0.4853	500.7691	1233.0226
i = 6	1771.2861	436.2399	0.2462	1536.0599	2501.3376
i = 7	3549.3000	559.5440	0.1576	3247.4517	4492.8169
i = 8	9031.9073	1171.8415	0.1297	8395.3286	11017.5719
i = 9	13242.2765	1254.3915	0.0947	12559.2142	15385.4010
i = 10	26008.1239	2113.7126	0.0812	24844.9622	29632.0981
i = 11	33825.8021	2557.2147	0.0755	32413.2408	38165.5719
i = 12	48971.1816	3642.1307	0.0743	46954.1941	55176.3234
i = 13	81483.1283	5847.1095	0.0717	78283.7712	91463.4789
Total	219538.3945	16231.3945	0.0739	210539.4315	247309.5502

Table 8. Predict reserves distribution characteristics (C-L with bootstrap method).

Reserve estimators for year *i*	Average estimators	SD	CV	30% percentile	95% percentile
i = 2	13.8599	75.4211	5.4416	−22.7766	155.4675
i = 3	386.1435	308.7321	0.7995	217.3350	924.6072
i = 4	779.1104	433.8647	0.5568	541.9171	1526.4023
i = 5	766.3500	408.6431	0.5332	543.6030	1468.2538
i = 6	2025.7001	542.8029	0.2679	1729.3143	2960.2849
i = 7	3598.4837	628.9708	0.1747	3248.8128	4680.8003
i = 8	7917.7464	1019.2950	0.1287	7353.8784	9667.3938
i = 9	13911.9135	1424.9554	0.1024	13112.9060	16367.2016
i = 10	27083.6277	2279.9054	0.0841	25835.3500	31012.4364
i = 11	35325.3129	2825.5943	0.0799	33765.3269	40249.2159
i = 12	44526.3848	3953.9534	0.0888	42337.4765	51501.9350
i = 13	84672.1491	9508.4501	0.1122	79997.1093	102394.1091
Total	221006.7829	13601.9012	0.0615	213612.3944	244420.4966

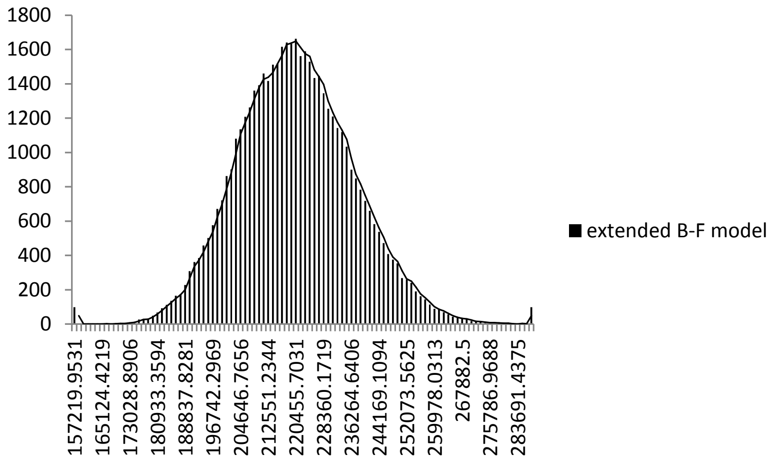


Figure 4. Frequency distribution histogram for the total reserve. (Simulate with the 50000 simulations we are able to approximate the distribution of the reserves.)

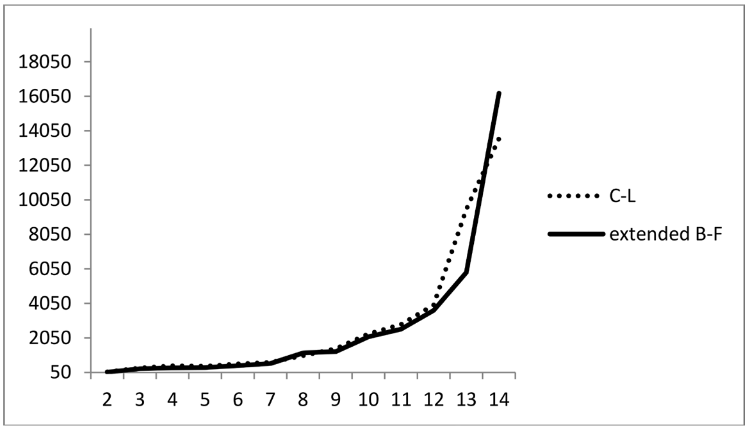


Figure 5. Standard deviation of two methods.

Comparing above results, we could find that applying bootstrap method on extended Bornhuetter-Ferguson model including diagonal effects is more conservative than chain ladder method to predict claims reserve.

We produce a histogram for the total reserve by extended B-F model in **Figure 4**. In **Figure 5** we give reserve estimator's standard deviations with two methods.

We could find that Bornhuetter-Ferguson method including diagonal effects' standard deviation is smaller in general than chain-ladder method except the total reserve estimation. This shows extended Bornhuetter-Ferguson model including diagonal effect could improve the accuracy of the estimation of claims reserve.

6. Conclusion

This paper introduces extended Bornhuetter-Ferguson model which is more accurate on estimating claim reserves than Bornhuetter-Ferguson model when considering inflation. Having comparing with the traditional chain-ladder method, we could conclude that it prefers to the extended Bornhuetter-Ferguson model when the inflation is mainly caused by claims inflation. Lacking of insurance data we cannot verify conclusion by national data. It is necessary to further study the case that the fluctuations risk of claim reserves is caused by economic inflation or the mix of economic and claims inflation. We can also take the Bayes method into consideration in the case which claims that priori estimate is not dependability enough.

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