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BER of OFDM System with Multiple NBI Rejection Cascade Complex Coefficient Adaptive IIR Notch Filter

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ABSTRACT

In this paper, rejection of multiple narrowband interferers in a binary phase shift keying modulated orthogonal frequency division multiplexing (BPSK-OFDM) system is investigated. The BPSK-OFDM system in consideration operates in an additive white Gaussian noise (AWGN) channel. A cascade complex coefficient adaptive infinite impulse response (IIR) notch filter with gradient-based algorithm is used to reject the interferers. Bit error ratio (BER) performance of the system is studied and a general closed-form expression is derived assuming negligible steady-state leakage NBI and by estimating the decision variable as Gaussian distributed based on Central Limit Theorem (CLT). Dependence of the BER performance on the notch bandwidth coefficient is demonstrated by the analysis. Extensive simulation results are included to substantiate accuracy of the analysis.

Keywords: Multiple Narrowband Interference; Cascade Adaptive Notch Filter; OFDM System; Bit Error Ratio

1. Introduction

Complex coefficient adaptive notch filters (ANF) have been proposed for use in applications that require detection and rejection or enhancement of narrowband signals embedded in broadband signals with in-phase and quadrature-phase components. One such applications is in the rejection of narrowband interference (NBI) in quadriphase shift keying (QPSK) spread-spectrum communication systems [1,2]. It is well known that infinite impulse response (IIR) implementation of adaptive notch filters is more advantageous as compared to finite impulse response (FIR) with much less computation complexity. A complex coefficient adaptive notch filter implemented as a constrained IIR filter with a complex Gauss-Newton adaptation algorithm was proposed in [2]. Furthermore, its application in suppression of NBI in QPSK spread-spectrum communication system showed a substantial improvement in overall system bit error ratio (BER).

A complex coefficient adaptive IIR notch filter with a simplified gradient-based algorithm that does not require any matrix inversion was proposed in [4] and its convergence and steady state behaviors were extensively analyzed in [3,4]. Its application in rejection of a single NBI in a QPSK DS-SS communication system over an additive white Gaussian noise (AWGN) channel was also

discussed in [5]. Cascade extension of the algorithm in [4] for detection of multiple sinusoids was further proposed and analyzed in [6]. Its application in the rejection of multiple NBI in a QPSK direct sequence code-division multiple access (DS-CDMA) system was discussed in [7] with promising improvement in BER. However, the ANF has not been attempted in rejection of multiple NBI in an orthogonal frequency division multiplexing (OFDM) system. OFDM has recently become a preferred transmission technique in emerging broadband wireless communication systems [8,9]. It has been reported that WLAN and WMAN standards based on OFDM that operate in the unlicensed frequency bands are susceptible to NBI from other systems coexisting in the same spectrum. Such systems include Bluetooth, microwave ovens, cordless telephones, etc. The observation has challenged researchers to find NBI rejection techniques that can alleviate effects of NBI in OFDM systems [10-17].

Several NBI suppression techniques for OFDM systems are found in literature. The use of pre-coding, spread spectrum OFDM, post-detection receiver techniques involving equalizers and frequency domain subtractive cancellation using singular value decomposition have been reported [11-17]. Coulson [10] proposed an NBI suppression technique based on excision filtering. It was demonstrated by computer simulation that the technique improved ensemble average BER to about 0.001

for binary phase shift keying (BPSK) modulated OFDM with signal-to-interference ratios (SIR) as low as −30 dB [10]. However, the author did not show how excision filter parameters affect the BER performance of the OFDM system.

This paper employs similar method to that proposed in [10] whereby the multiple NBI excision filter is implemented using a cascade complex coefficient adaptive IIR notch filter with simplified gradient-based algorithm presented in [8]. The OFDM system is considered to be operating in an AWGN channel. Estimating the Fast Fourier Transform (FFT) of the ANF output for each subcarrier as Gaussian distributed based on Central Limit Theorem (CLT), a closed-form BER expression is derived for small step size constant with negligible steady-state leakage NBI. It is observed that besides the received signal-to-noise ratio (SNR), BER performance depends largely on the value of notch bandwidth coefficient. Computer simulation results are included to support analytical findings.

Main contributions of this paper are as follows: 1) It presents a systematic approach to the analysis of BER performance of OFDM systems with cascaded complex coefficient filters for multiple NBI suppression. Such approach has not been presented elsewhere; 2) It presents a simple closed-form BER expression for an OFDM system with multiple NBI suppression filters. The expression relates BER with filter notch bandwidth parameter and received signal-to-noise ratio (SNR).

The organization of this paper is as follows: In Section 2, OFDM system model and the NBI suppression cascade complex coefficient adaptive IIR notch filter are presented. Section 3 presents BER analysis while simulation results and discussions on findings are presented in Section 4. Lastly, concluding remarks and future research areas are included in Section 5.

2. System Model

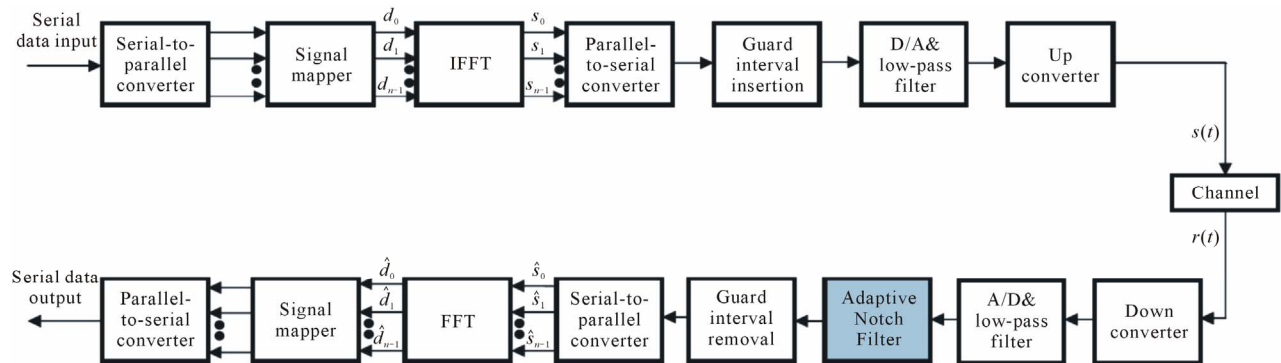**Figure 1** below shows a block diagram of the BPSK-OFDM system used in this paper. Received discrete-time low pass equivalent BPSK-OFDM symbol r_n is modeled as

$$r_n = s_n + i_n + \eta_n \tag{1}$$

s_n is the n-th of L samples in the transmitted BPSK-OFDM symbol. It is expressed as

$$s_n = \omega_n \frac{1}{L} \sum_{k=0}^{L-1} d_k e^{-j2\pi n \frac{k}{L}} \tag{2}$$

d_k is the k-th data symbol from BPSK modulation constellation, *i.e.*, $d_k \in \left\{ +\sqrt{E_b}, -\sqrt{E_b} \right\}$ where E_b is the signal average energy per bit and ω_n is a rectangular windowing function, *i.e.*, $\omega_n = 1$, $\forall n$. It should be noted that s_n is the inverse discrete Fourier transform (IDFT) of L data symbols d_k. η_n is the n-th sample of a zero-mean complex additive white Gaussian noise (AWGN) with variance $N_0/2$ where N_0 is the power spectral density. i_n is the n-th sample of the NBI which is modeled as

$$i_n = \sum_{i=1}^{N} A_i e^{-j(2\pi \xi_i n + \phi_i)} \tag{3}$$

where A_i, ξ_i, and ϕ_i are unknown amplitude, frequency and phase of the i-th of N narrowband interferers. Validity of the interference model in (3) was discussed in detail in [10].

The cascade realization of the complex coefficient adaptive IIR notch filter for detection of multiple sinusoids in noise was proposed in [8]. The equivalent transfer function $H^{eq}(z)$ from the ANF input signal r_n to the output signal y_n is given by

$$H^{eq}(z) = \prod_{i=1}^{N} H_i(z)$$
$$H_i(z) = \frac{1+\beta}{2} \frac{1 - e^{j\alpha_{i,n}} z^{-1}}{1 - \beta e^{j\alpha_i,^n} z^{-1}} \tag{4}$$

Here, β is the notch bandwidth coefficient and $\alpha_{i,n}$ is the notch frequency coefficient for the i-th section of

Figure 1. OFDM system model.

the cascade complex coefficient adaptive IIR notch filter at the n-th iteration, respectively.

An adaptation algorithm for the i-th notch frequency coefficient for tracking and suppressing the i-th NBI is expressed as [6]

$$\alpha_{i,n+1} = \alpha_{i,n} + \mu \operatorname{Re}\left[y_n \varphi_{i,n}^* \right] \tag{5}$$

where Re[.] stands for real part, $\varphi_{i,n}$ is the i-th gradient signal and $\varphi_{i,n}^*$ is its conjugate. Transfer function from r_n to $\varphi_{i,n}$ was obtained in [6] as

$$F_i(z) = G_i(z) \prod_{\substack{j=1 \\ i \neq j}}^{N} H_j(z)$$

$$G_i(z) = \frac{1+\beta}{2} \frac{j e^{j\alpha_{i,n}} z^{-1}}{1 - \beta e^{j\alpha_{i,n}} z^{-1}} \tag{6}$$

Referring to (4), the unknown frequency of the i-th NBI at the n-th iteration can be estimated by $\hat{\xi}_{i,n}$ where $\hat{\xi}_{i,n} = \alpha_{i,n}$. The adaptation algorithm in (5) was used in [7] for suppression of multiple NBI in QPSK DS-CDMA system with significant BER improvement. Convergence behavior of the proposed adaptation algorithm is shown in **Figure 2**.

3. Analysis of Bit Error Ratio

This section presents analysis of BER of the BPSK-OFDM system in the presence of multiple NBI with interference suppression cascaded complex coefficient ANF. The analysis ignores errors that can occur as a result of time drift or phase offset by assuming ideal time and phase synchronization. The analysis further assumes negligible steady-state leakage NBI power. This assumption is valid for small values of step size μ.
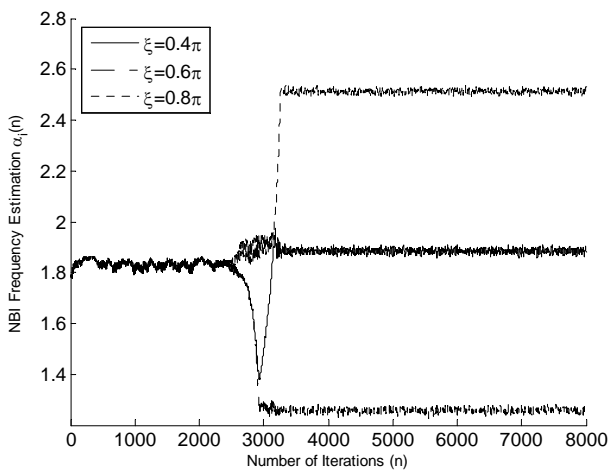

Figure 2. Convergence behavior of the cascade complex coefficient adaptive notch filter for estimating three NBI frequencies in OFDM received signal for $A_1 = A_2 = A_3 = \sqrt{20}$, $\xi_1 = 0.4\pi$ rad./s, $\xi_2 = 0.6\pi$ rad./s, $\xi_2 = 0.8\pi$ rad./s, $\beta = 0.9$, $\mu = 10^{-4}$, $L = 64$, and SNR = 0 dB.

The cascade ANF output signal y_n is applied to an L-point discrete Fourier transform (DFT) at the receiver. The resulting k-th sample $\hat{d}_k$ is therefore given by

$$\hat{d}_k = \sum_{n=0}^{L-1} y_n \exp\left(-j 2\pi n \frac{k}{L} \right) = \tilde{d}_{k,s} + \tilde{d}_{k,\eta} \tag{9}$$

The k-th sample $\hat{d}_k$ in (9) is composed of two components *i.e.* $\tilde{d}_{k,s}$ and $\tilde{d}_{k,\eta}$. The former is the DFT of the cascade ANF output signal due to the OFDM transmitted signal s_n while the latter is due to channel AWGN η_n. The assumption of negligible leakage steady-state NBI justifies the exclusion of the DFT of the ANF output due to NBI i_n.

3.1. Statistical Characteristics of $\tilde{d}_{k,s}$

The DFT of the cascade ANF output signal due to the BPSK-OFDM transmitted signal s_n is given by:

$$
\begin{aligned}
\tilde{d}_{k,s} &= \sum_{n=0}^{L-1} \left(h_n^{eq} \times s_n \right) e^{-j2\pi n \frac{k}{L}} \\
&= \sum_{n=0}^{L-1} h_0^{eq} s_n e^{-j2\pi n \frac{k}{L}} + \sum_{n=0}^{L-1} \left\{ \sum_{l=1}^{\infty} h_l^{eq} s_{n-l} \right\} e^{-j2\pi n \frac{k}{L}} \\
&= h_0^{eq} d_k + \sum_{n=0}^{L-1} \left\{ \sum_{l=1}^{\infty} h_l^{eq} s_{n-l} \right\} e^{-j2\pi n \frac{k}{L}}
\end{aligned} \tag{10}
$$

where * denotes convolution sum and h_l^{eq} is a complex impulse response of $H^{eq}(z)$ in (4) that decays exponentially with l. The right hand side of (10) has two terms; the first is the desired data component while the second is due to intersymbol interference (ISI) induced by the cascade ANF. h_0^{eq} is the value of h_l^{eq} for $l = 0$ and is obtained by

$$h_0^{eq} = \frac{1}{2\pi j} \oint H^{eq}(z) z^{-1} \mathrm{d}z \tag{11}$$

The expectation of $\tilde{d}_{k,s}$, $E\left[\tilde{d}_{k,s} \right]$ is derived by referring to (10) and by assuming statistically independent and identically distributed data symbols $\{d_k\}$. It is expressed as

$$
\begin{aligned}
E\left[\tilde{d}_{k,s} \right] &= h_0^{eq} d_k + E\left[\sum_{n=0}^{L-1} \left\{ \sum_{l=0}^{\infty} h_l^{eq} \sum_{p=0}^{L-1} d_p e^{j2\pi n \frac{p-l}{L}} \right\} e^{-j2\pi n \frac{k}{L}} \right] \\
&= h_0^{eq} d_k
\end{aligned}
$$

$$\tag{12}$$

It can be deduced from (12) that the cascade ANF attenuates the expectation of the k-th data symbol $E\left[\tilde{d}_{k,s} \right]$ by a factor proportional to h_0^{eq} which depends on the notch bandwidth coefficient β. Mean-square value and variance of $\tilde{d}_{k,s}$, $E\left[|d_{k,s}|^2 \right]$ and $\operatorname{var}\left[d_{k,s} \right]$, respectively, can be shown to be expressed as

$$E\left[\left|\tilde{d}_{k,s}\right|^2\right] = \left|h_0^{eq}\right|^2\left|d_k\right|^2 + \left|d_k\right|^2\sum_{l=1}^{\infty}\left|h_l^{eq}\right|^2$$

$$\text{var}\left[\tilde{d}_{k,s}\right] = \left|d_k\right|^2\sum_{l=1}^{\infty}\left|h_l^{eq}\right|^2 \tag{13}$$

3.2. Statistical Characteristics of $\tilde{d}_{k,\eta}$

By considering the AWGN η_n the DFT of the cascade ANF output signal can be shown to be given by:

$$\tilde{d}_{k,\eta} = \sum_{n=0}^{L-1}\left(h_n^{eq}\times\eta_n\right)\mathrm{e}^{-j2\pi n\frac{k}{L}} = \sum_{n=0}^{L-1}\left\{\sum_{l=0}^{\infty}h_l^{eq}\eta_{n-l}\right\}\mathrm{e}^{-j2\pi n\frac{k}{L}} \tag{14}$$

Expectation of $\tilde{d}_{k,\eta}$ is zero since η_n is modeled as zero mean random process. Its variance $\text{var}\left[\tilde{d}_{k,\eta}\right]$ is expressed as

$$\text{var}\left[\tilde{d}_{k,\eta}\right] = \frac{N_0}{2}\sum_{l=0}^{\infty}\left|h_l^{eq}\right|^2 \tag{15}$$

which depends on the notch bandwidth coefficient β.

Noting that s_n and η_n are statistically independent and uncorrelated, variance of $\hat{d}_k$ denoted as $\text{var}\left[\hat{d}_{k,s}\right]$ becomes the total sum of $\text{var}\left[\tilde{d}_{k,s}\right]$ and $\text{var}\left[\tilde{d}_{k,\eta}\right]$ in (13) and (15), respectively, *i.e.*

$$\text{var}\left[\hat{d}_k\right] = \left|d_k\right|^2\sum_{l=1}^{\infty}\left|h_l^{eq}\right|^2 + \frac{N_0}{2}\sum_{l=0}^{\infty}\left|h_l^{eq}\right|^2 \tag{16}$$

$\hat{d}_k$ in (9) can be approximated by Gaussian distribution for large L using Central Limit Theorem (CLT). Average BER for the k-th frequency bin $P_{e,k}$ is obtained by the Gaussian approximation and by referring to (12) and (16). It is given by:

$$P_{e,k} \approx Q\left[\sqrt{\frac{a\left(h_0^{eq}\right)^2 d_k^2}{\text{var}\left[\hat{d}_k\right]}}\right]$$

$$= Q\left[\sqrt{\frac{\left(h_0^{eq}\right)^2}{\sum_{l=1}^{\infty}\left|h_l^{eq}\right|^2 + \frac{N_0}{2E_b}\sum_{l=0}^{\infty}\left|h_l^{eq}\right|^2}}\right] \tag{17}$$

Equation (17) shows the average BER to be the same for all $k = 0,1,2,\cdots,L-1$. The average BER for the OFDM symbol will therefore be the same as in (17). It is observed that the average BER depends on factors that are functions of the notch bandwidth coefficient β.

4. Simulation Results

In this section, simulation and analytical results are presented, discussed and compared to validate accuracy of the BER closed-form expression (17) derived in the previous section. All simulations results were obtained using MATLAB® software. The results presented were ob-

tained for the following OFDM signal parameters:

FFT size L	64
Number of data subcarriers	52
Number of bits per symbol	52
Number of symbols	100

The value of step size adaptation constant μ in (5) was fixed to 10^{-6}. Simulation results were obtained by averaging over ten independent computer runs after the adaptation algorithm in (5) had attained its steady state.

Analytical and simulation results for BER plotted against notch bandwidth coefficient β with signal-to-noise ratio (SNR) as a parameter for number of NBIs $N = 2$ and $N = 3$ are shown in **Figures 3** and **4**, respectively. Frequencies of NBIs ξ_1, ξ_2, ξ_3 in (3) were set to 0.4π, 0.6π and 0.8π, respectively. Interference-to-signal ratio for the i-th narrowband interferer (ISR$_i$) was set to 10 dB,
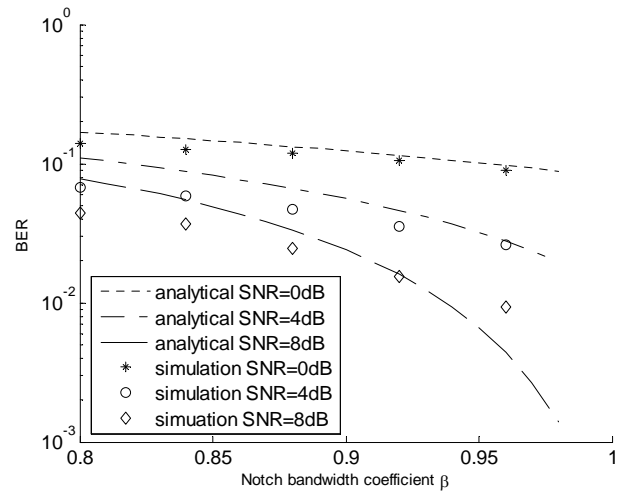

Figure 3. BER analytical and simulation results for $N = 2$, $SNR_i = 10$ dB $\forall i$, $\xi_1 = 0.4\pi$, $\xi_2 = 0.6\pi$, $\mu = 10^{-5}$, and $L = 64$.
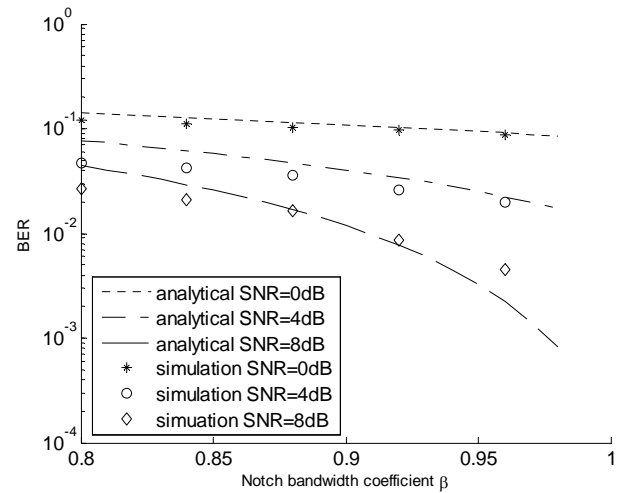

Figure 4. BER analytical and simulation results for $N = 3$, $SNR_i = 10$ dB $\forall i$, $\xi_1 = 0.4\pi$, $\xi_2 = 0.6\pi$, $\xi_3 = 0.8\pi$, $\mu = 10^{-5}$, and $L = 64$.

where

$$\text{ISR}_i = 10\log\left(\frac{A_i^2}{2E_b}\right)(\text{dB}).$$

and $E_b = 1.0$. The figures show that the BER decreases with β in both cases. Moreover, close agreement between analytical and simulation results confirms the accuracy of the analysis carried out in Section 3.

Figures 5 and **6** show analytical and simulation BER results plotted as a function of SNR for number of NBIs $N = 2$ and $N = 3$, respectively. The value of β was likewise set to 0.9 and ISR for each NBI was set to 10 dB. BER for the ideal OFDM system without NBI and that with NBI but without NBI suppression ANF are also shown for comparison purpose. Improvement of BER to about 0.001 and 0.01 for $N = 2$ and $N = 3$, respectively is
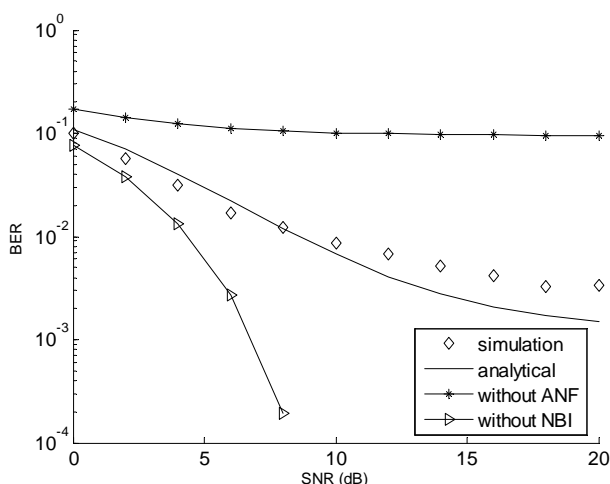

Figure 5. BER analytical and simulation results for $N = 2$, $SNR_i = 10$ dB $\forall$ i, $\xi_1 = 0.4\pi$, $\xi_2 = 0.6\pi$, $\beta = 0.9$, $\mu = 10^{-5}$, and $L = 64$.
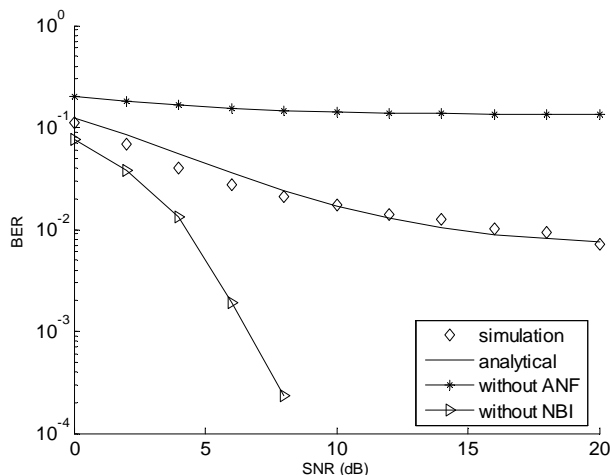

Figure 6. BER analytical and simulation results for $N = 3$, $SNR_i = 10$ dB $\forall$ i, $\xi_1 = 0.4\pi$, $\xi_2 = 0.6\pi$, $\xi_3 = 0.8\pi$, $\beta = 0.9$, $\mu = 10^{-5}$, and $L = 64$.

evident from the figures. Furthermore, the figures show close agreement between analytical and simulation results. Slight discrepancy between simulation and analytical results in all figures can be attributed to the Gaussian distribution assumption made on the decision variable $\hat{d}_k$ in (9) based on CLT. Moreover, leakage NBI becomes significant as the notch bandwidth coefficient β approaches 1.0 that leads to slight difference between analytical and simulation results in **Figures 3** and **4**.

5. Conclusion

Bit error ratio (BER) of a BPSK OFDM system with multiple NBI rejection complex-coefficient cascade ANF was analyzed. A closed-form BER expression was derived using the Central Limit Theorem approximation of the decision variable. Accuracy of the derived BER expression was demonstrated by computer simulation using MATLAB® software with emphasis on its dependence on notch bandwidth coefficient. Moreover, significant BER performance improvement gained by the use of complex coefficient cascade ANF for NBI rejection was also shown. In the future, Rayleigh channel, M-ary modulation techniques and effects of leakage NBI power on the BER performance of OFDM system will be studied.

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