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An Algorithm for Improving Throughput Guarantee of Topology-Transparent MAC Scheduling Strategy*

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Abstract

Topology-transparent MAC scheduling strategies nowadays are all based on combinatorial design. To get throughput guarantee, a cover-free set is output as scheduling strategy of network. In this paper, we aim to modify the cover-free set so that better throughput can be guaranteed. At the first step, the redundant slot of the cover-free set is proposed and found to have negative influence on the minimal guaranteed throughput. Second, we prove that any subset of a cover-free set is still a cover-free set after its redundant slots were squashed out. Our algorithm chooses the subset which has the maximal number of redundant slots, squashes all of its redundant slots, and then designates it as the network scheduling strategy. Therefore, better throughput can be guaranteed if the squashed subset is adopted as network scheduling strategy. For any topology-transparent node scheduling strategy, both the increased minimal throughput and decreased maximal transmission delay can be gotten by just using our algorithm as an extra accessory.

Keywords: MAC Scheduling, Qos, Combinatorial Design, Superimposed Code, Cover-Free Set

1. Introduction

Recently, many multimedia applications, such as video monitoring and voice recording, are suggested to deploy on Mobile Ad hoc Networks (MANET). Their feasibilities have great relationship with the guaranteed transmission throughput and transmission delay. In other word, guaranteed QoS on MANET is required by these applications.

Since MAC layer is directly above the physical layer and is the basis of all other protocols, for a MANET with guaranteed QoS, guaranteed QoS on MAC layer is indispensible [1].

MAC protocols in wireless network are of two types. One is the contention-based MAC protocol, with the well-known Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) protocol as its representative. Due to its possibility of infinite transmission delay caused by wireless collisions, it cannot provide QoS guarantee. The other one is the scheduling-based MAC protocol, where a time frame is divided into multiple time slots, and each node is assigned the transmitting right at some given time slots [2]. We take **Figure 1** for example, for a network whose topology is known, by arranging colliding transmissions into different time slots, wireless collision can be eliminated efficiently.

Scheduling-based MAC protocols can be further categorized into two subcategories, *i.e.*, the topology-dependent and the topology-transparent MAC protocols. For the topology-dependent MAC protocols, to minimize wireless collisions, a uniform topology graph has to be set up either in every node in distributed manner, or in the sink node in centralized manner [3]. Obviously, the topology-dependent MAC protocols are unfit for MANET because of constantly changed topology which is caused by the mobility of node [4]. On the contrary, the topology-transparent MAC scheduling protocols are fully independent of network topology and may be promising choices for applications deployed on MANET.

The scheduled object of the topology-transparent MAC protocols can be either wireless link or node. In this paper, we focus on node scheduling.

Nowadays, all topology-transparent node scheduling protocols are all based on combinatorial designs [5], including the multinomial theory in Galois field [6,7], orthogonal array [8], latin squares [9], balanced incomplete

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block [10,11] and superimposed code[12-14]. The aim of them is same, that is, to give birth to a cover-free set as MAC scheduling codes, which is the key to QoS guarantee. Of course, the sets generated by different strategies are distinct even if they are feed with same parameters.

Due to the existence of combinatorial designs, the cardinal of the cover-free set is no less than the node number of network [5]. Therefore, the problem of how to select an appropriate subset comes into being, for every node has to be uniquely associated with one element of the cover-free set. In other word, if the node number of a network is N, and the cardinal of the set is M, then there are $\binom{M}{N}$ optional subsets which can be assigned as the MAC scheduling tragedy of network. Obviously, random selection is not a considerate policy, although it is adopted by all of the topology-transparent MAC scheduling protocols nowadays. Which subset can provide the best throughput guarantee? Further, can the selected subset be optimized further? This paper tries to answer the questions.

We propose the definition of the redundant slot of the cover-free set, and prove that the redundant slot has negative influence on throughput guarantee. Further, we prove that any subset of a cover-free set is still a cover-free set after any of its redundant slots is squashed out. Therefore, the more minimal throughput and the less maximal delay can be guaranteed. Our algorithm therefore picks the subset which has the maximal number of redundant slots, squashes all redundant slots of the subset, and then designates it as node scheduling strategy of network. Therefore, for any topology-transparent node scheduling strategy, both the increased minimal throughput and decreased maximal transmission delay can be gotten by just using our algorithm as an extra accessory.

2. Network Model

Wireless network is modeled by a directed graph $G = (V, E)$, where V is the set of nodes and E is the set of directed links. If node w is within the transmission range of node u, then a directed edge connecting these two nodes is denoted by $(u, w) \in E$, with u being a neighbor of w. The degree of a node w, *i.e.*, $D(w) = |\{u \mid (u,w) \in E, u, w \in V\}|$ is defined as the number of its neighbors. We assume that the maximum nodal degree D_{max}, *i.e.*, $\max_{w \in V} D(w)$, remains constant when network operates. Of course, $D_{max} > 0$ is necessary for keeping connectivity.

In this paper, we assume that the transmission channel is error-free and a reception failure is caused only by packet collisions. A packet transmitted from a neighbor of a node, is successfully received by the node only if no packet is transmitted from other neighbor nodes simultaneously. All nodes are homogeneous. We also assume that the transceiver at each node is half-duplex. As a result, a node cannot transmit and receive concurrently.

Time is assumed to be synchronized over the network. Furthermore, time is slotted and slots are grouped into time frame. For example, a time frame consists of four time slots in **Figure 1**. In other word, a frame $F = \{S_1, S_2, \ldots, S_b\}$ consists of b consecutive slots. A slot assignment is given by a set $S(w) \subseteq F$ for every node w, where $S(w)$ consists of time slots in which node w has the transmitting right in a frame.

3. Redundant Slot and QoS Guarantee

3.1. Redundant Slot

Definition 1. Assume $[k] = \{0, 1, \ldots, k\text{-}1\}$, a set $A = \{A_0, A_1, \ldots, A_{M-1}\}$ of subsets of the $[T]$ is a (s, M, T) cover-free set if for any proper subset I of $[M]$ such that $|I| = s$ ($|I|$ is the cardinal of set I), and any integer $j \in [M] - I$, we have $\{A_j\} - \bigcup\{A_i\} \neq \Phi$. s is called the intensity of the cover-free set.

Cover-free set is equivalent with the d-disjunct matrix [15] and the superimposed code.

A (s, M, T) cover-free set A can be represented by a $T \times M$ matrix A where

$$A_{ij} = \begin{cases} 1 & if\ i \in A_j \\ 0 & if\ i \notin A_j \end{cases} \quad 0 \le i \le T-1, 0 \le j \le M-1$$

The matrix is referred as the scheduling matrix. For convenience, the *jth* column vector and the *ith* row vector of the scheduling matrix A are denoted as A_{*j} and A_{i*} respectively. Besides, for a $T \times M$ scheduling matrix, A_{Tj} is the MSB (Most Significant Bit) of A_{*j} and A_{0j} is the LSB (Least Significant Bit) of A_{*j}.

Definition 2. For two vectors $X = (x_1, x_2, \ldots, x_m)^T$ and $Y = (y_1, y_2, \ldots, y_m)^T$, the sum of X and Y is $X + Y = (x_1 \vee y_1, x_2 \vee y_2, \ldots, x_m \vee y_m)^T$. If $X + Y = X$, then Y is covered by X.

Obviously, for the scheduling matrix of a (s, M, T) cover-free set, any column vector is not covered by any other s column vectors.

Lemma 1. Any L ($s < L \le M$) elements in a (s, M, T)

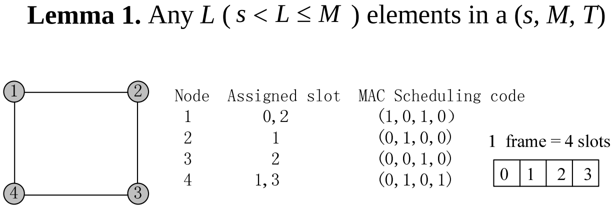

Figure 1. An example of MAC scheduling code.

cover-free set form a (*s*, *L*, *T*) cover-free set.

Proof: For the scheduling matrix of a (*s*, *M*, *T*) cover-free set, any one column vector will not be covered by any other *s* column vectors. So it will not be covered by any other *L* column vectors since $L < s$. Therefore, it is a (*s*, *L*, *T*) cover-free family.

Definition 3. Assume the scheduling matrix of a (*s*, *M*, *T*) cover-free set is denoted as *A*, for some integer *i* ($0 \le i \le T-1$) and any integer *j* ($0 \le j \le M-1$), if $A_{ij} = 0$ or $A_{ij} = 1$, then *i* is a redundant slot of *A*.

Theorem 1. Assume the scheduling matrix of a (*s*, *M*, *T*) cover-free set is denoted as *A*, a $(T-1)\times M$ matrix which is generated by removing (deleting, or squashing) redundant slot of *A* is a (*s*, *M*, *T*-1) cover-free set.

Proof: For any *s* + 1 column vectors of *A*, without loss of generality, assume them to be A_{*0}, A_{*1}, …, A_{*s}. Obviously, A_{*s} is not covered by $A_{*0} + A_{*1} + \ldots + A_{*(s-1)}$. In other word, there exist at least one integer *k* ($0 \le k \le T-1$), which satisfies $(A_{ks} = 1) \wedge (\bigcup_{i=\in[s-1]} \{A_{ki}\} = \{0\}) = true$. Therefore, *k* is not a redundant slot of *A*.

Assume a $(T-1)\times M$ matrix *B* is generated by squashing any one redundant slot of *A*, and A_{*0}, A_{*1}, …, A_{*s} are turned into B_{*0}, B_{*1}, …, B_{*s} correspondingly. Since *k* is not a redundant slot of *A*, A_{k*} is still kept in *B*. Thus, B_{*s} is not covered by $B_{*0} + B_{*1} + \ldots + B_{*(s-1)}$. Considering the generality of choosing A_{*0}, A_{*1}, …, and A_{*s}, the set of all column vectors in *B* is a (*s*, *M*, *T*-1) cover-free set.

Redundant slot results in less throughput guarantee. Assume the scheduling matrix of a (*s*, *M*, *T*) cover-free set to be *A* and *i* is one of its redundant slot. If $\bigcup_{0\le j\le M-1} \{A_{ij}\} = \{0\}$, none of nodes will transmit at slot *i*. On the other hand, if $\bigcap_{0\le j\le M-1} \{A_{ij}\} = \{1\}$, all nodes will transmit at slot *i* and no any packets can be received correctly due to the half-duplex transceiver. In a word, the throughput of redundant slots is wasted in both cases.

3.2. Redundant Slot and QoS Guarantee

Definition 4. The minimal guaranteed throughput G_{min} is defined as the ratio of the number of guaranteed successful transmissions in one frame to frame length.

Definition 5. The transmission delay under the worst traffic condition is called the maximal transmission delay, and it is defined as the ratio of frame length to the minimal number of successful transmission slots in one frame.

Theorem 2. For a (*s*, *M*, *T*) cover-free set *A* and a (*s*, *M*, *T*-1) cover-free set *A'* which is generated by squashing one redundant slot of *A*, if the minimal guaranteed throughput and the maximal transmission delay are G_{min} and $\overline{DT}_{\max}$, and $G'_{\min}$ and $\overline{DT}'_{\max}$ respectively when *A* and *A'* are adopted respectively as node scheduling codes, then $G'_{\min} / G_{\min} = T/(T-1)$, and

$\overline{DT}'_{\max} / \overline{DT}_{\max} = (T-1)/T$.

Proof: For any node, assume there are at least *k* exclusive transmission slots can be guaranteed if *A* is adopted as the MAC scheduling codes of network, based on the definition of the minimal guaranteed throughput, $G_{\min} = \frac{k}{T}$. Similarly, if *A'* is adopted, $G'_{\min} = \frac{k}{T-1}$. Therefore, $G'_{\min} / G_{\min} = T/(T-1)$.

Since the maximal transmission delay is the reciprocal of the minimal guaranteed throughput,

$\overline{DT}'_{\max} / \overline{DT}_{\max} = (T-1)/T$.

4. Algorithm and Performance Analysis

4.1. Algorithm

For selecting *N* scheduling codes from *M* candidates, there are $\binom{M}{N}$ optional subsets in total. Based on Theorem 2, QoS guarantee can be enhanced if a redundant slot is squashed. So our algorithm chooses the subset which has the maximal number of redundant slots, squashes all redundant slots of the subset, and then designates it as the node scheduling strategy.

4.2. Performance Analysis

Theorem 3. Assume the scheduling matrix of a (*s*, *M*, *T*) cover-free set to be A_{TM}, if its row vectors have equal weight *w*, then the algorithm can squash at least *i* redundant slots if the node number *N* satisfies

$M-(i+1)\times w < N \le M - i\times w$ ($0 \le i \le \frac{M}{w} - 1$, and *i* is an integer).

Proof: We prove it using mathematical induction.

1) If $M - w < N \le M$, *i.e.*, *i* = 0, Theorem 3 is obviously correct.

2) Assume when $M-(k+1)\times w < N \le M - k\times w$, at least *k* redundant slots can be squashed.

3) If $M-(k+2)\times w < N \le M-(k+1)\times w$, since $M-(k+1)\times w < N + w \le M - k\times w$, for *N* + *w* nodes, at least *k* redundant slots can be squashed from A_{TM} based on the assumption 2). If the scheduling matrix after the *k* redundant slots are squashed from A_{TM} is notated $A^{(k)}_{(T-k)(N+w)}$. Since the row weight of A_{TM} is *w*, there are at most *w* column vectors whose MSBs are 1 in $A^{(k)}_{(T-k)(N+w)}$. In other word, there are at least *N* column vectors whose

MSBs are 0 in $A^{(k)}_{(T-k)(N+w)}$.

If we select any N column vectors whose MSBs are all 0 as the node scheduling codes of network, then the *MSB* slot is obviously a redundant slot. Therefore, at least k + 1 redundant slots can be founded and squashed.

Corollary 1. For the scheduling matrix A of a (s, M, T) cover-free set, if its row vectors have equal weight M-w, then if the node number N satisfies

$M-(i+1)\times w < N \le M - i\times w$ （$0 \le i \le \frac{M}{w}-1$, and i is an integer）, the algorithm can squash at least i redundant slots.

Proof: Since the scheduling matrix is just the complementary matrix of that in Theorem 3, the proof is obvious.

5. Apply into Popular Topology-Transparent Node Scheduling Strategies

Based on Theorem 3, the performance of our algorithm can be estimated if the row vectors in scheduling matrix have equal weight. Therefore, to show its universality, we prove that the most popular topology-transparent node scheduling strategies generate scheduling matrix with equal weight.

5.1. Strategy Based on the Multinomial Theory in Galois Field

Both Chlamtac's and Ju's algorithm are based on the multinomial theory in Galois field. Based on two input parameters, the node number N and the maximal node degree D_{max}, two parameters q and k are get based on a sufficient condition of forming a cover-free set. Further, based on q and k, every node is associated with a unique vector (a_k, a_{k-1}, …, a_0), where $a_i \in [q]\,(i = 0,1,...,k)$. In other word, every node is associated with a unique multinomial $a_k x^k + a_{k-1}x^{k-1} + ... + a_1 x + a_0$.

To map from the multinomial to scheduling matrix, a frame is divided into q subframes and a subframe is further divided into q slots. Every node has transmission right only at one slot during a subframe. For example, for the subframe i, $i \in [q]$, node which is associated with the vector (a_k, a_{k-1}, …, a_0) has transmission right only at the $((a_k i^k + a_{k-1}i^{k-1} + ... + a_1 i + a_0) \mod q)\,th$ slot in the subframe i.

Theorem 4. For scheduling matrix generated by strategy based on the multinomial theory in Galois field with parameters q and k, its row vectors have equal weight q^k.

Proof: Based on the principle of the algorithm, the weight of the jth row vector in scheduling matrix is the number of node which has transmission right at the jth slot. For the slot j in subframe i, it is the number of vector (a_k, a_{k-1}, …, a_0) which satisfies

$(a_k i^k + a_{k-1}i^{k-1} + ... + a_1 i + a_0) \mod q = j$.

Assume $a_k i^k + a_{k-1}i^{k-1} + ... + a_1 i = mq + z$, $0 \le z \le q-1$ where m is an integer. For every z, there is a unique $a_0 = (j - z) \mod q$ which satisfies $(a_k i^k + a_{k-1}i^{k-1} + ... + a_1 i + a_0) \mod q = j$, *i.e.*, a_0 is determined by (a_k, a_{k-1}, …, a_1). In other word, there are k independent variables in (a_k, a_{k-1}, …, a_0). So, the number of (a_k,a_{k-1},…,a_0) which satisfies $(a_k i^k + a_{k-1}i^{k-1} + ... + a_1 i + a_0) \mod q = j$ is q^k, *i.e.*, the row vectors of scheduling matrix have equal weight q^k.

5.2. Strategy Based on the Orthogonal Array

Definition 5. A OA(k, t, v) is a $t \times v^k$ matrix with entries from [v], $0 < k \le t$, if for any $k \times v^k$ submatrix, each of its v^k column vectors is unique.

A OA(k, t, v) can be mapped into a $vt \times v^k$ scheduling matrix. A frame is composed of vt slots. Each node is assigned a unique column vector of orthogonal array as its scheduling code. For example, if a node is assigned a column vector (0, 3, 1), its scheduling code is 000110000010.

Theorem 6. For scheduling matrix generated by strategy based on OA(k, t, v), its row vectors have equal weight v^{k-1}.

Proof: For any k row vectors in OA(k, t, v), they form a $k \times v^k$ matrix. Since every column vector in the $k \times v^k$ matrix is unique, there are v^k different k-tuples. Since every entry in [v] appears equal times in any row vector, every entry appears v^{k-1} times in any row vector. Based on the mapping from orthogonal array to scheduling matrix, every row vector of scheduling matrix has equal weight v^{k-1}.

5.3. Strategy Based on Orthogonal Latin Square

Definition 7. A orthogonal latin square with order p is a $p \times p$ matrix, with every row and every column to be a full permutation of [p].

Definition 8. Two different $n \times n$ latin squares A = ($a_{i,j}$), B = ($b_{i,j}$), $i, j \in \{1, 2, ..., n\}$ is orthogonal if all 2-tuples ($a_{i,j}$, $b_{i,j}$) are different. As a generalization, r $n \times n$ latin squares $A^{(1)}$,$A^{(2)}$,…,$A^{(r)}$ form a latin squares family with order (r, n) if they are different and orthogonal between any two of them. Especially, if r = n-1, the latin squares family is a complete orthogonal latin squares family with order n.

A complete orthogonal latin squares family with order n can be mapped into a $n^2 \times n(n-1)$ scheduling matrix. For all $n(n-1)$ column vectors in the family, each of them can be associated to a node as its scheduling code.

Theorem 6. If n is a prime or prime power, for scheduling matrix generated by strategy based on a complete orthogonal latin squares family with order n, its row vectors have equal weight n-1.

Proof: Since a complete orthogonal latin squares family with order n consists of n-1 latin squares if n is a prime or prime power. For each latin square, every element appears just once in any column and row. So, the weight of the scheduling matrix is n-1 based on the mapping from latin square to scheduling matrix.

6. Experiments

We take the Chlamtac algorithm for examples to test the effect of the redundant slot squashing algorithm.

Based on the principle of Chlamtac algorithm, given the node number N and the maximum nodal degree D_{max}, the two parameters, q and k, can be get. For example, if $N = 120$, $D_{max} = 5$, then $q = 11$, $k = 2$. That is, we have $q^{k+1} = 1331$ candidate node scheduling codes for 120 nodes. In fact, there are at least 11 redundant slots can be squashed. **Figure 2** illustrates the relationship among q^{k+1}/N, N and D_{max} generated by Chlamtac algorithm. Obviously, the larger the q^{k+1}/N is, the more redundant slots can be anticipated.

Seen from **Figure 2**, q^{k+1}/N increases quickly with N or D_{max}. So it can be easily anticipated that our algorithm performs better in network with more nodes or larger node degree.

We now begin to test the effect of our algorithm under various values of N and D_{max} for Chlamtac algorithm. The range of N is 2~60, and that of the maximal node degree is 1~N-1, *i.e.*, all possibilities of maximal node degree are tested. **Figure 3** illustrates the effect of our algorithm. The three coordinates are the node number N, the maximal node degree D_{max} and the number of redundant slot. It can be found from **Figure 3** that there always exist redundant slots in any case. The larger the N or D_{max} is, the more redundant slots can be squashed, which is consistent with our anticipation.

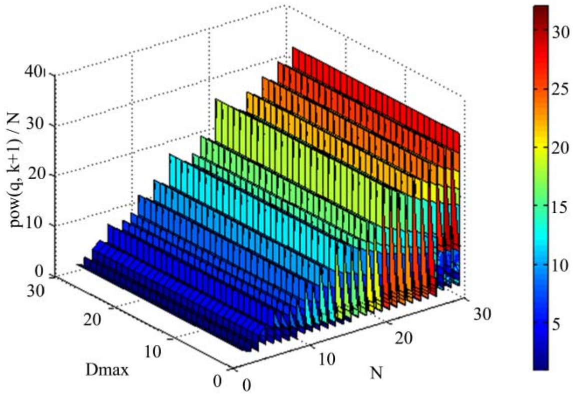

Figure 2. The relationship among q^{k+1}/N, N and D_{max} generated by the Chlmatac algorithm.

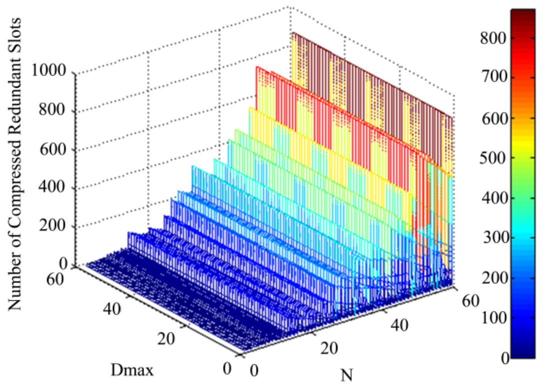

Figure 3. The effect of redundant slot squashing algorithm for the Chlamtac algorithm.

7. Conclusions

The topology-transparent node scheduling strategy is suitable for MANET because it can provide guaranteed QoS and be independent of network topology. In this paper, to improve QoS guarantee, we present a universal algorithm which can be realized as an accessory of any topology-transparent node scheduling strategy nowadays. Node scheduling codes generated by each topology-transparent node scheduling algorithm nowadays form a cover-free set. We propose the redundant slot of the cover-free set, and prove that the redundant slot has negative influence on the minimal guaranteed throughput. Further, we prove that any subset of a cover-free set is still a cover-free set after any of its redundant slots is squashed. Our algorithm chooses the subset which has the maximal number of redundant slots, squashes all redundant slots of the subset, and then designates it as the node scheduling strategy of wireless network. Both theoretical analysis and experiments prove that the increased minimal throughput and decreased maximal transmission delay are guaranteed. Simulation results reveal that the larger the network node number or the maximal node degree, the better QoS can be guaranteed by employing our algorithm.

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