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A Novel Approach for Finding a Shortest Path in a Mixed Fuzzy Network

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Abstract

We present a novel approach for computing a shortest path in a mixed fuzzy network, network having various fuzzy arc lengths. First, we develop a new technique for the addition of various fuzzy numbers in a path using a -cuts. Then, we present a dynamic programming method for finding a shortest path in the network. For this, we apply a recently proposed distance function for comparison of fuzzy numbers. Four examples are worked out to illustrate the applicability of the proposed approach as compared to two other methods in the literature as well as demonstrate the novel feature offered by our algorithm to find a fuzzy shortest path in mixed fuzzy networks with various settings for the fuzzy arc lengths.

Keywords: Fuzzy Numbers, a -Cut; Shortest Path, Dynamic Programming

1. Introduction

Determination of shortest distance and shortest path between two vertices is one of the most fundamental problems in graph theory. Let *G* = *(V, E)* be a graph with *V* as the set of vertices and *E* as the set of edges. A *path* between two vertices is an alternating sequence of vertices and edges starting and ending with vertices, and no vertices or no edges appear more than once in the sequence. The *length* of a path is the sum of the weights of the edges on the path. There may exist more than one path between a pair of vertices. The *shortest path problem* is to find the path with minimum length between a specified pair of vertices. In classical graph theory, the weight of each edge is taken as a crisp real number. But, practically weight of each edge of the network may not be a fixed real number and it may well be imprecise.

The shortest path problem involves addition and comparison of the edge weights. Since, the addition and comparison between two interval numbers or between two triangular fuzzy numbers are not alike those between two precise real numbers, it is necessary to discuss them at first. Interval arithmetic was developed in Moore [1]. The case of optimization with interval valued and fuzzy constraints were discussed in Delgado *et al*., Ishibuchi and Tanaka, Sengupta, and Shaocheng [2–5]. Various ranking methods for interval numbers were introduced by several researchers. An extensive survey of the order relations along with a new proposal are given by Sengupta and Pal [6]. There are also ranking methods for fuzzy numbers available in the literature. Dubois and Prade [7] introduced a ranking of fuzzy numbers in the setting of possibility theory, and Chen [8] ranked fuzzy numbers using maximizing and minimizing sets. Ranking of fuzzy numbers was also studied by Bortolan and Degani, Cheng, and Delgado *et al*. [2,9,10].

Fuzzy graph problems were addressed in Blue *et al*. and Koczy, Klein, Li *et al*., Lin and Chen, Okada and Gen [11–17] paid special attention to fuzzy shortest paths. In a recent development, Okada and Soper [18] proposed an algorithm to find the shortest path in a network with fuzzy edge weights. The algorithm gives a family of non-dominated shortest paths for a specified satisfaction level, but it does provide any guideline to the decision-maker to choose the best amongst the paths according to his/her view; *i.e.*, optimistic, pessimistic, etc.

The shortest path (SP) problem has received lots of attention from researchers in the past decades, because it is important to many applications such as communication, transportation, scheduling and routing. In a network, the arc length may represent time or cost. Conventionally, it is assumed to be crisp. However, it is difficult for decision makers to specify the arc lengths. For example, using the same modem to transmit the data from node *a* to *b* in a network, the data transmission time may not be the same every time. Therefore, in real world, the arc length could be uncertain. Fuzzy set theory, as proposed by Zadeh [19], is frequently utilized to deal with uncertainty. Zadeh presented the possibility theory using membership functions to describe uncertainties.

Considering a directed network that is composed of a finite set of nodes and a set of directed arcs, we denote each arc by an ordered pair (i, j), where i and j are two different nodes. The arc length represents the distance needed to traverse (i, j) from node i to j. We denote it by $l(i, j)$ or $L(i, j)$, when it is crisp or fuzzy, respectively. We call $L(i, j)$ as "fuzzy arc length".

The shortest path problem is the following: given a weighted, directed graph and two special vertices s and t, compute the weight of the shortest path between s and t. The shortest path problem is one of the most fundamental network optimization problems. This problem comes up in practice and arises as a subproblem in many network optimization algorithms. Algorithms for this problem have been studied for a long time [20–22]. However, advances in the method and theory of shortest path algorithms are still being made [23–25].

In the network we consider here, the lengths of the arcs are assumed to represent transportation times or costs rather than geographical distances. As time or cost fluctuate with traffic conditions, payload and so on, it is not practical to represent the arcs as crisp values. Thus, it is appropriate to utilize fuzzy numbers based on fuzzy set theory. In proposing an algorithm for solving the problem, we are first faced with the comparison or ordering of fuzzy numbers, a task not considered to be routine. For this reason, fuzzy shortest path problems have rarely been studied despite their potential application to many problems [18,26]. The problem turns out to be even more complicated in our more general case of allowing various fuzzy arc lengths.

Here, we propose a new approach and an algorithm to find a shortest path in a mixed network having various fuzzy arc lengths. The remainder of the paper is organized as follows. In Section 2, basic concepts and definitions are given. A dynamic programming algorithm for finding a fuzzy shortest path in a network is presented in Section 3. There, we make use of a -cuts for computing approximations for the addition of two different types of fuzzy numbers and apply a distance function for the comparison of fuzzy numbers. Comparative examples are given in Section 4. Section 5 works out an example to show the novel feature of our algorithm to find fuzzy shortest paths in mixed fuzzy networks with various settings for the fuzzy arc lengths. We conclude in Section 6.

2. Concepts and Definitions

We start with basic definitions of some well-known fuzzy numbers.

Definition 1. An *LR* fuzzy number is represented by $\tilde{A} = (m, a, b)_{LR}$, with the membership function, $m_{\tilde{a}}(x)$, defined by the expression,

$$m_{\tilde{a}}(x) = \begin{cases} L\left[\dfrac{m-x}{a}\right] & x \le m \\ R\left[\dfrac{x-m}{b}\right] & x \ge m, \end{cases}$$

where L and R are non-increasing functions from R^+ to [0,1], $L(0)=R(0)=1$, m is the center, a is the left spread and b is the right spread.

Note that if $L(x)=R(x)=1-x$ with $0<x<1$, then x is a triangular fuzzy number and is represented by the triplet $\tilde{a} = (a_1, a_2, a_3)$, with the membership function, $m_{\tilde{a}}(x)$, defined by

$$m_{\tilde{a}}(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 < x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 < x \le a_3 \\ 0 & x > a_3. \end{cases}$$

A triangular fuzzy number is shown in Figure 1.

Definition 2. A trapezoidal fuzzy number $\tilde{a}$ is shown by $\tilde{a} = (a_1, a_2, a_3, a_4)$, with the membership function as follows:

$$m_{\tilde{a}}(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ 0 & a_4 \le x. \end{cases}$$

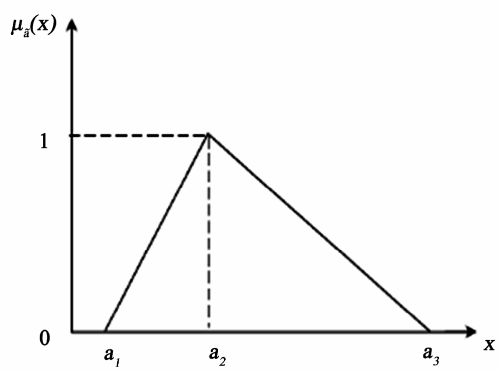

Figure 1. A triangular fuzzy number.

A general trapezoidal fuzzy number is shown in Figure 2. It is apparent that a triangular fuzzy number is a special trapezoidal fuzzy number with $a_2 = a_3$.

Definition 3. If $L(x)=R(x)= e^{-x^2}$, with $x \in \Re$, then x is a normal fuzzy number that is shown by (m, s) and the corresponding membership function is defined to be:

$$m_{\tilde{a}}(x) = e^{-(\frac{x-m}{s})^2}, \quad x \in \Re,$$

where m is the mean and s is the standard deviation. A normal fuzzy number is shown in Figure 3.

Definition 4. The a-cut and strong a-cut for a fuzzy set $\tilde{A}$ are shown by $\tilde{A}_a$ and $\tilde{A}_a^+$, respectively, and for $a \in [0,1]$ are defined to be:

$$\tilde{A}_a = \left\{ x \mid m_{\tilde{A}}(x) \ge a, x \in X \right\},$$

$$\tilde{A}_a^+ = \left\{ x \mid m_{\tilde{A}}(x) > a, x \in X \right\},$$

where X is the universal set.

Upper and lower bounds for any a-cut $(\tilde{A}_a)$ are shown by $\sup \tilde{A}_a$ and $\inf \tilde{A}_a$, respectively. Here, we assume that the upper and lower bounds of a-cuts are finite values and for simplicity we show $\sup \tilde{A}_a$ by A_a^+ and $\inf \tilde{A}_a$ by A_a^-.

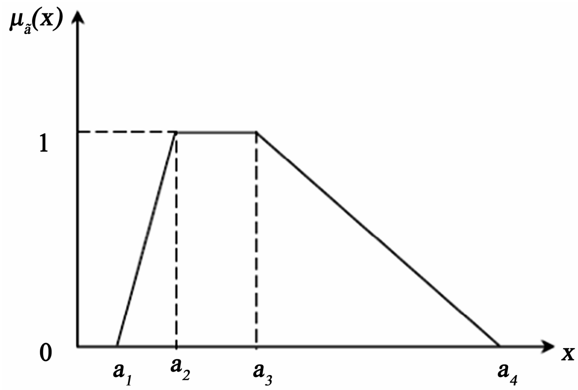

Figure 2. A trapezoidal fuzzy number.

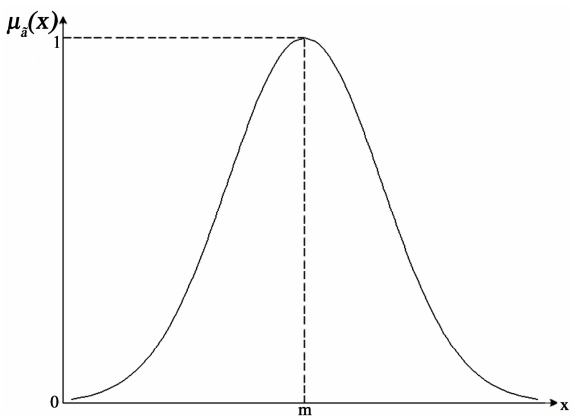

Figure 3. A normal fuzzy number.

2.1. Computing a-Cuts for Fuzzy Numbers

For an *LR* fuzzy number with *L* and *R* as invertible functions, the a-cuts are:

$$a = L[\frac{m-x}{b}] \;\Rightarrow\; \frac{m-x}{b} = L^{-1}(a) \;\Rightarrow x = m - bL^{-1}(a),$$

$$a = R[\frac{x-m}{g}] \;\Rightarrow\; \frac{x-m}{g} = R^{-1}(a) \;\Rightarrow x = m + gR^{-1}(a).$$

For specific *L* and *R* functions, the following cases are discussed. Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number. The a-cut for $\tilde{a}$, $(\tilde{a}_a)$, is computed as:

$$m_{\tilde{a}}(x) = \begin{cases} 0 & x \le a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ 0 & a_4 \le x \end{cases}$$

$$\Rightarrow \begin{array}{l} a = \frac{x-a_1}{a_2-a_1} \Rightarrow \quad x = (a_2-a_1)a + a_1 \\ a = \frac{a_4-x}{a_4-a_3} \Rightarrow \quad x = a_4 - (a_4-a_3)a \end{array}$$

$$\Rightarrow \tilde{a}_a = \begin{cases} \tilde{a}_a^+ = a_4 - (a_4-a_3)a \\ \\ \tilde{a}_a^- = (a_2-a_1)a + a_1 \end{cases}, \quad 0 \le a \le 1. \qquad (1)$$

Note that the a-cut for triangular fuzzy numbers is simply obtained by using the above equations considering $a_2 = a_3$:

$$\tilde{a}_a = \begin{cases} \tilde{a}_a^+ = a_3 - (a_3-a_2)a \\ \tilde{a}_a^- = (a_2-a_1)a_a + a_1 \end{cases}, \quad 0 \le a \le 1. \qquad (2)$$

For $\tilde{a} = (m, s)$, a normal fuzzy number, $\tilde{a}_a$, is computed as:

$$a = e^{-(\frac{m-x}{s})^2} \Rightarrow \sqrt{-\ln(a)} = \frac{m-x}{s}$$

$$a = e^{-(\frac{x-m}{s})^2} \Rightarrow \sqrt{-\ln(a)} = \frac{x-m}{s}$$

$$\Rightarrow \tilde{a}_a = \begin{cases} \tilde{a}_a^L = m - s\sqrt{-\ln a} \\ \\ \tilde{a}_a^R = m + s\sqrt{-\ln a} \end{cases}, \quad 0 < a \le 1. \qquad (3)$$

2.2. Fuzzy Sum Operators

Here, we propose an approach for summing various fuzzy numbers approximately using α -cuts. The approximation is based on fitting an appropriate model for the sum using α -cuts of the addition by a set of α_i values. Let us divide the α -interval [0,1] into *n* equal subintervals by letting $\alpha_0 = 0$, $\alpha_i = \alpha_{i-1} + \Delta\alpha_i$, *i=1,…,n* with $\Delta\alpha_i = \frac{1}{n}$, *i=1,…,n*. This way, we have a set of *n+1* equidistant α_i points. For addition of two different fuzzy numbers, we add the set of corresponding α_i-cut points of the two numbers to yield the α_i-cuts of the sum as an approximation for the fuzzy addition.

3. An Algorithm for Fuzzy Shortest Path in a Network

3.1. Distance between Fuzzy Numbers

Knowing that we can obtain a good approximation of the addition of various fuzzy numbers by use of α -cuts, we compute the distance between two fuzzy numbers using the resulting points from the α -cuts. For $\tilde{a}$ and $\tilde{b}$ as two different fuzzy numbers, we use a new fuzzy ranking method for the fuzzy numbers. Let us consider the fuzzy *min* operation to be defined as follows

$$
\begin{aligned}
\widetilde{MV} &= \text{Min}\quad \text{value}(\tilde{a},\tilde{b}) \\
&= (\min(a_1,b_1),\ \min(a_2,b_2),\ \min(a_3,b_3), \min(a_4,b_4)).
\end{aligned} \tag{4}
$$

It is evident that, for non-comparable fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the fuzzy *min* operation results in a fuzzy number different from both of them. For example, for $\tilde{a} = (5,10,13,19)$ and $\tilde{b} = (6,9,15,20)$, we get from (4) a fuzzy $\widetilde{MV} = (5,9,13,19)$ which is different from both $\tilde{a}$ and $\tilde{b}$. To alleviate this drawback, we use a method based on the distance between fuzzy numbers. We use the distance function introduced by Sadegh-pour-Gildeh and Gien [27]. The main advantages of this distance function are the generality of its usage on various fuzzy numbers, and its reliability in distinguishing unequal fuzzy numbers. Indeed, the use of this distance function worked out to be quite appropriate for our approach here as well as in a different context before where we considered the arc lengths in the network to be all of the same type (see Mahdavi *et al.* [28]).

The proposed $D_{p,q}$ -distance, indexed by parameters $1 < p < \infty$ and $0 < q < 1$, between two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ is a nonnegative function given by:

The analytical properties of $D_{p,q}$ depend on the first

$$
D_{p,q}(\tilde{a},\tilde{b}) = \begin{cases} \left[(1-q)\int_0^1 \left| a_\alpha^- - b_\alpha^- \right|^p d\alpha + q\int_0^1 \left| a_\alpha^+ - b_\alpha^+ \right|^p d\alpha \right]^{\frac{1}{p}}, & p < \infty, \\ (1-q) \sup\limits_{0<\alpha\le 1} \left(\left| a_\alpha^- - b_\alpha^- \right| \right) + q \inf\limits_{0<\alpha\le 1} \left(\left| a_\alpha^+ - b_\alpha^+ \right| \right), & p = \infty. \end{cases} \tag{5}
$$

parameter *p*, while the second parameter *q* of $D_{p,q}$ characterizes the subjective weight attributed to the end points of the support; *i.e.*, (a_α^+, a_α^-) of the fuzzy numbers. If there is no reason for distinguishing any side of the fuzzy numbers, then $D_{p,\frac{1}{2}}$ is recommended. Having *q* close to 1 results in considering the right side of the support of the fuzzy numbers more favorably. Since the significance of the end points of the support of the fuzzy numbers is assumed to be the same, then we consider $q = \frac{1}{2}$.

For two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ with corresponding α_i-cuts, the $D_{p,q}$ distance is approximately proportional to:

$$
D_{p,q}(\tilde{a},\tilde{b}) = \left[(1-q)\sum_{i=1}^{n} \left| a_{\alpha_i}^- - b_{\alpha_i}^- \right|^p + q\sum_{i=1}^{n} \left| a_{\alpha_i}^+ - b_{\alpha_i}^+ \right|^p \right]^{\frac{1}{p}}. \tag{6}
$$

If $q = \frac{1}{2}, p = 2$, then the above equation turns into:

$$
D_{2,\frac{1}{2}}(\tilde{a},\tilde{b}) = \sqrt{\frac{1}{2}\sum_{i=1}^{n} (a_{\alpha_i}^- - b_{\alpha_i}^-)^2 + \frac{1}{2}\sum_{i=1}^{n} (a_{\alpha_i}^+ - b_{\alpha_i}^+)^2}, \tag{7}
$$

To compare two fuzzy arc lengths $\tilde{a}$ and $\tilde{b}$ with α_i-cuts as their approximations, since they are supposed to represent positive values, we compare them with $\widetilde{MV} = (0,0,....,0)$. In fact, we use (7) to compute $D_{2,\frac{1}{2}}(\tilde{a},0)$ and $D_{2,\frac{1}{2}}(\tilde{b},0)$ and use these values for comparison of the two numbers.

3.2. An Algorithm for Computing s Shortest Fuzzy Path

The following dynamic programming algorithm is for computing the shortest path in a network. The algorithm is based on Floyd's dynamic programming method to find a shortest path, if it exists, between every pair of nodes *i* and *j* in the network (see Floyd [29]).

We make use of the following optimal value function $f_k(i,j)$ and the corresponding labeling function $P_k(i,j)$:

$f_k(i,j)$ = length of the shortest path from node *i* to node *j* when the path is considered to use only the nodes from the set of nodes $\{1,....,k\}$.

$P_k(i,j)$ = the last intermediate node on the shortest path from node *i* to node *j* using $\{1,....,k\}$ as intermediate node.

The dynamic updating for the optimal path length and its corresponding labeling are:

$$f_k(i,j) = \min\{f_{k-1}(i,j),\ f_{k-1}(i,k) + f_{k-1}(k,j)\},$$

$$P_k(i,j) = \begin{cases} P_{k-1}(i,j) & \text{if k is not on shortest path from i to j using \{1,...,k\}} \\ P_{k-1}(k,j) & \text{otherwise.} \end{cases}$$

We are now ready to give the steps of the algorithm.

Algorithm: A dynamic programming method for computing a shortest path in a fuzzy network $G=(V,A)$, where V is the set of nodes with $|V| = N$, and A is the set of arcs.

Step 1: Let *k=0* and $\tilde{f}_k(i,j) = \tilde{d}_{ij}$, for all $(i,j) \in A$, $\tilde{f}_k = (i,j) = \infty$, for all $(i,j) \notin A$. **If** an arc exists from node *i* to node *j* **then** let $P_k(i,j) = i$.

Step 2: Let $k = k+1$.

Do the following steps for $i = 1,2,3,....,N,\ j = 1,2,3,....,N,\ i \neq j$.

2.1 Compute the value of $f_k(i,j) = \min$ $[f_{k-1}(i,j), f_{k-1}(i,k) + f_{k-1}(k,j)]$ (for the addition, our proposed method discussed in Subection 2.2 and for comparison of fuzzy numbers the $D_{p,q}$ method of Subection 3.1 are applied).

2.2 If node *k* is not on the shortest path using nodes $\{1,2,...,k\}$ as intermediate nodes, **then** let $P_k(i,j) =$ $P_{k-1}(i,j)$ **else** let $P_k(i,j) = P_{k-1}(k,j)$

Step 3: **If** $k < N$ **then go to Step 2.**

Step 4: Obtain the shortest path using the $P_k(i,j)$. If $f_N(i,j) = \infty$, then there is no path between *i* and *j*. The shortest path from node *i* to *j*, if it exists, is identified backwards and read by the nodes: *j*, $P_N(i,j) = k$ followed by $P_N(i,k),....,P_N(i,l) = i$, where l is the node immediately after *i* in the path.

3.3. Termination and Complexity of the Algorithm

The proposed algorithm terminates after *N* outer iterations corresponding to *k*. A total of $N(N-1)^2$ additions and comparisons are needed for every *k*. For each addition, *n* fuzzy additions for the a_i-cuts should be performed resulting in $2n(N)(N-1)^2$ additions. For comparisons, we have $(2n+1)N(N-1)^2$ additions and $(2n+1)\ N(N-1)^2$ multiplications using (7). Therefore, the total needed operations are $(6n+2)\ N(N-1)^2$ additions and multiplications, with $N(N-1)^2$ comparison

4. Comparative Examples

Here, we illustrate examples for a comparison of our proposed method and two other approaches.

Example 1:

Consider the following network Figure 4 considered by Chuang and Kung [30]. The triangular arc lengths are presented in Table 1. The results obtained by the approach ($\tilde{f}_6$ and P_6) of Chuang and Kung [30] are shown in Tables 2 and 3.

The shortest path and the corresponding length using the proposed approach in Chuang and Kung [30] are reported below:

$$\text{Shortest path from 1 to 6}: 1 \to 2 \to 4 \to 6.$$

$$\text{Shortest path length from 1 to 6}: (177,195,256)$$

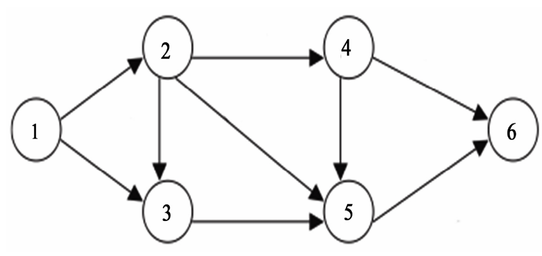

Figure 4. The network for Example 1.

Table 1. The arc lengths for example 1.

Arc	lengths	Arc	lengths	Arc	lengths
(1,2)	(33,45,50)	(1,3)	(42,57,61)	(2,3)	(50,52,61)
(2,4)	(56,58,72)	(2,5)	(51,79,85)	(3,5)	(43,55,60)
(4,5)	(32,40,46)	(4,6)	(88,92,134)	(5,6)	(75,110,114)

Table 2. The $\tilde{f}_6$ values obtained by the approach of Chuang and Kung.

i/j	1	2	3	4	5	6
1	-	(33,45,50)	(42,57,61)	(89,103,122)	(85,112,121)	(177,195,256)
2	-	-	(50,52,61)	(56,58,72)	(51,79,85)	(144,150,206)
3	-	-	-	-	(43,55,60)	(118,165,174)
4	-	-	-	-	(32,40,46)	(88,92,134)
5	-	-	-	-	-	(75,110,114)
6	-	-	-	-	-	-

Table 3. The P_6 values obtained by the approach of Chuang and Kung.

i/j	1	2	3	4	5	6
1	-	1	1	2	3	4
2	-	-	2	2	2	4
3	-	-	-	-	3	5
4	-	-	-	-	4	4
5	-	-	-	-	-	5
6	-	-	-	-	-	-

Here, we solve the same problem using our proposed algorithm givenin Subection 3.2 using the ranking method of Sadeghpour-Gildeh and Gien [27]. The results of the proposed approach for $\tilde{f}_6$ and P_6 are given in Tables 4 and 5.

Here, the shortest path obtained and the corresponding length are exactly the same as the ones we obtained by the approach of Chuang and Kung [30].

Example 2:

Consider the following network Figure 5 considered by Hernandes *et al.* [32]. The fuzzy triangular arc lengths are given in Table 6. The results ($\tilde{f}_6$ and P_6) for the approach of Hernandes *et al.* [32] are given in Tables 7 and 8.

Table 4. The $\tilde{f}_6$ values obtained by our proposed algorithm

i/j	1	2	3	4	5	6
1	-	(33,45,50)	(42,57,61)	(89,103,122)	(85,112,121)	(177,195,256)
2	-	-	(50,52,61)	(56,58,72)	(51,79,85)	(144,150,206)
3	-	-	-	-	(43,55,60)	(118,165,174)
4	-	-	-	-	(32,40,46)	(88,92,134)
5	-	-	-	-	-	(75,110,114)
6	-	-	-	-	-	-

The shortest path and the corresponding length using the proposed approach of Hernandes *et al.* [32] are reported below:

Shortest path from 1 to 11: $1 \rightarrow 9 \rightarrow 7 \rightarrow 11$.

Shortest path length from 1 to 11: $(860, 902, 990)$

We solved the same problem using our proposed algorithm of Subsection 3.2 using the ranking method of Sadeghpour-Gildeh and Gien [27]. The results of our proposed approach ($\tilde{f}_{11}$ and P_{11}) are given in Tables 9 and 10.

The shortest path and the corresponding length are reported below:

Shortest path from 1 to 11: $1 \rightarrow 9 \rightarrow 7 \rightarrow 11$.

Shortest path length from 1 to 11: $(840, 882, 990)$.

Clearly, the proposed algorithm computes almost the same solution as obtained by Hernandes *et al.* [32].

Table 5. The P_6 values obtained by our proposed algorithm.

i/j	1	2	3	4	5	6
1	-	1	1	2	3	4
2	-	-	2	2	2	4
3	-	-	-	-	3	5
4	-	-	-	-	4	4
5	-	-	-	-	-	5
6	-	-	-	-	-	-

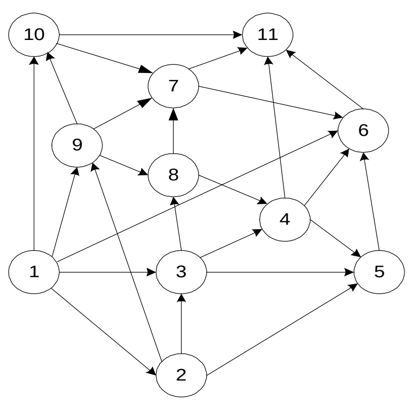

Figure 5. The network for Example 2.

Table 6. The arc lengths for Example 2.

Arc	lengths	Arc	lengths	Arc	lengths
(1,2)	(800,820,840)	(3,5)	(730,748,770)	(8,4)	(710,730,735)
(1,3)	(350,361,370)	(3,8)	(425,443,465)	(8,7)	(230,242,255)
(1,6)	(650,677,683)	(4,5)	(190,199,210)	(9,7)	(120,130,150)
(1,9)	(290,300,350)	(4,6)	(310,340,360)	(9,8)	(130,137,145)
(1,10)	(420,450,470)	(4,11)	(710,740,770)	(9,10)	(230,242,260)
(2,3)	(180,186,193)	(5,6)	(610,660,690)	(10,7)	(330,342,350)
(2,5)	(495,510,525)	(6,11)	(230,242,260)	(10,11)	(1250,1310,1440)
(2,9)	(900,930,960)	(7,6)	(390,410,440)	(3,4)	(650,667,883)
(7,11)	(450,472,490)				

Table 7. The $\tilde{f}_{11}(i,j)$ values obtained by Hernandes *et al*.

i/j	1	2	3	4	5	6	7
1	-	(800,820,840)	(350,361,370)	(1000,1028,1253)	(1080,1109,1140)	(650,677,683)	(410,430,500)
2	-	-	(180,186,193)	(830,853,1076)	(495,510,525)	(1105,1170,1215)	(835,871,913)
3	-	-	-	(650,667,883)	(730,748,770)	(960,1007,1243)	(655,685,720)
4	-	-	-	-	(190,199,210)	(310,340,360)	-
5	-	-	-	-	-	(610,660,690)	-
6	-	-	-	-	-	-	-
7	-	-	-	-	-	(390,410,440)	-
8	-	-	-	(710,730,735)	(900,929,945)	(620,652,695)	(230,242,255)
9	-	-	-	(840,867,880)	(1030,1066,1090)	(510,540,590)	(120,130,150)
10	-	-	-	-	-	(720,752,790)	(330,342,350)
11	-	-	-	-	-	-	-

Table 7. continued.

i/j	8	9	10	11
1	(420,437,495)	(290,300,350)	(420,450,470)	(860,902,990)
2	(605,629,658)	(900,930,960)	(1130,1172,1220)	(1285,1343,1403)
3	(425,443,465)	-	-	(1105,1157,1210)
4	-	-	-	(540,582,620)
5	-	-	-	(840,902,950)
6	-	-	-	(230,242,260)
7	-	-	-	(450,472,490)
8	-	-	-	(680,714,745)
9	(130,137,145)	-	(230,242,260)	(570,602,640)
10	-	-	-	(780,814,840)
11	-	-	-	-

Example 3: A wireless sensor network

Consider a mobile service company which handles 23 geographical centers. A configuration of a telecommunication network is presented in Figure 6. Assume that the distance between any two centers is a trapezoidal fuzzy number (the arc lengths are given in Table 11). The company wants to find a shortest path for an effective message flow amongst the centers.

The results obtained by our approach ($\tilde{f}_{23}$ and P_{23}) are given in Tables 12 and 13.

The shortest path and the corresponding length are reported below:

Shortest path from 1 to 23 : $1 \rightarrow 5 \rightarrow 11 \rightarrow 17 \rightarrow 21 \rightarrow 23$.

$$\text{Shortest path length from 1 to 23}: (38, 49, 58, 65) \cdot$$

5. Discussion

We can also apply the proposed algorithm to networks having possibly a mixture of different fuzzy numbers as arc lengths. To see how the steps of the proposed algorithm are carried out on such networks, a small sized mixed fuzzy network with 4 nodes as shown in Figure 7 is considered, where the arc lengths are considered to be a mixture of trapezoidal and normal fuzzy numbers.

Example 4: Consider the mixed fuzzy network in Figure 4 with four nodes and five arcs having two trapezoidal and three normal arc lengths as specified in Table 14.

Step 1: We gain the $\tilde{f}_k(i,j) = \tilde{d}_{ij}$ for *k=0* as specified in Table 14.

Table 8. The $P_{11}(i,j)$ values obtained by Hernandes *et al*.

i/j	1	2	3	4	5	6	7	8	9	10	11
1	-	1	1	3	3	1	9	9	1	1	9
2	-	-	2	3	2	5	8	3	2	9	8
3	-	-	-	3	3	4	8	3	-	-	8
4	-	-	-	-	4	4	-	-	-	-	6
5	-	-	-	-	-	5	-	-	-	-	6
6	-	-	-	-	-	-	-	-	-	-	6
7	-	-	-	-	-	7	-	-	-	-	7
8	-	-	-	8	4	7	8	-	-	-	7
9	-	-	-	8	8	7	9	9	-	9	7
10	-	-	-	-	-	7	10	-	-	-	7
11	-	-	-	-	-	-	-	-	-	-	-

Table 9. The $\tilde{f}_{11}(i,j)$ values obtained by our proposed algorithm.

i/j	1	2	3	4	5	6	7
1	-	(800,820,840)	(350,361,370)	(1000,1028,1253)	(1080,1109,1140)	(650,677,683)	(410,430,500)
2	-	-	(180,186,193)	(830,853,1076)	(495,510,525)	(1105,1170,1215)	(835,871,913)
3	-	-	-	(650,667,883)	(730,748,770)	(960,1007,1243)	(655,685,720)
4	-	-	-	-	(190,199,210)	(310,340,360)	-
5	-	-	-	-	-	(610,660,690)	-
6	-	-	-	-	-	-	-
7	-	-	-	-	-	(390,410,440)	-
8	-	-	-	(710,730,735)	(900,929,945)	(620,652,695)	(230,242,255)
9	-	-	-	(840,867,880)	(1030,1066,1090)	(510,540,590)	(120,130,150)
10	-	-	-	-	-	(720,752,790)	(330,342,350)
11	-	-	-	-	-	-	-

Table 9. continued.

i/j	8	9	10	11
1	(420,437,495)	(290,300,350)	(420,450,470)	(840,882,990)
2	(605,629,658)	(900,930,960)	(1130,1172,1220)	(1265,1323,1403)
3	(425,443,465)	-	-	(1085,1137,1210)
4	-	-	-	(540,582,620)
5	-	-	-	(840,902,950)
6	-	-	-	(230,242,260)
7	-	-	-	(430,452,490)
8	-	-	-	(660,694,745)
9	(130,137,145)	-	(230,242,260)	(550,582,640)
10	-	-	-	(760,794,840)
11	-	-	-	-

Table 10. The $P_{11}(i,j)$ values obtained by our proposed algorithm.

i/j	1	2	3	4	5	6	7	8	9	10	11
1	-	1	1	3	3	1	9	9	1	1	9
2	-	-	2	3	2	5	8	3	2	9	8
3	-	-	-	3	3	4	8	3	-	-	8
4	-	-	-	-	4	4	-	-	-	-	6
5	-	-	-	-	-	5	-	-	-	-	6
6	-	-	-	-	-	-	-	-	-	-	6
7	-	-	-	-	-	7	-	-	-	-	7
8	-	-	-	8	4	7	8	-	-	-	7
9	-	-	-	8	8	7	9	9	-	9	7
10	-	-	-	-	-	7	10	-	-	-	7
11	-	-	-	-	-	-	-	-	-	-	-

Table 11. The arc lengths for Example 3.

Arc	lengths	Arc	lengths	Arc	lengths
(1,2)	(12,13,15,17)	(1,3)	(9,11,13,15)	(1,4)	(8,10,12,13)
(1,5)	(7,8,9,10)	(2,6)	(5,10,15,16)	(2,7)	(6,11,11,13)
(3,8)	(10,11,16,17)	(4,7)	(17,20,22,24)	(4,11)	(6,10,13,14)
(5,8)	(6,9,11,13)	(5,11)	(7,10,13,14)	(5,12)	(10,13,15,17)
(6,9)	(6,8,10,11)	(6,10)	(10,11,14,15)	(7,10)	(9,10,12,13)
(7,11)	(6,7,8,9)	(8,12)	(5,8,9,10)	(8,13)	(3,5,8,10)
(9,16)	(6,7,9,10)	(10,16)	(12,13,16,17)	(10,17)	(15,19,20,21)
(11,14)	(8,9,11,13)	(11,17)	(6,9,11,13)	(12,14)	(13,14,16,18)
(12,15)	(12,14,15,16)	(13,15)	(10,12,14,15)	(13,19)	(17,18,19,20)
(14,21)	(11,12,13,14)	(15,18)	(8,9,11,13)	(15,19)	(5,7,10,12)
(16,20)	(9,12,14,16)	(17,20)	(7,10,11,12)	(17,21)	(6,7,8,10)
(18,21)	(15,17,18,19)	(18,22)	(3,5,7,9)	(18,23)	(5,7,9,11)
(19,22)	(15,16,17,19)	(20,23)	(13,14,16,17)	(21,23)	(12,15,17,18)
(22,23)	(4,5,6,8)				

Table 12. The $\tilde{f}_{23}(i, j)$ values obtained by our algorithm.

i/j	1	2	3	4	5	6	7	8
1	-	(12,13,15,17)	(9,11,13,15)	(8,10,12,13)	(7,8,9,10)	(17,23,30,33)	(18,24,26,30)	(13,17,20,23)
2	-	-	-	-	-	(5,10,15,16)	(6,11,11,13)	-
3	-	-	-	-	-	-	-	(10,11,16,17)
4	-	-	-	-	-	-	(17,20,22,24)	-
5	-	-	-	-	-	-	-	(6,9,11,13)
6	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-
19	-	-	-	-	-	-	-	-
20	-	-	-	-	-	-	-	-
21	-	-	-	-	-	-	-	-
22	-	-	-	-	-	-	-	-
23	-	-	-	-	-	-	-	-

Table 12. continued.

i/j	9	10	11	12	13	14	15
1	(23,31,40,44)	(27,34,38,43)	(14,18,22,24)	(17,21,24,27)	(16,22,28,33)	(22,27,33,37)	(29,35,39,43)
2	(11,18,25,27)	(15,21,23,26)	(12,18,19,22)	-	-	(20,27,30,35)	-
3	-	-	-	(15,19,25,27)	(13,16,24,27)	(28,33,41,45)	(23,28,38,42)
4	-	(26,30,34,37)	(6,10,13,14)	-	-	(14,19,24,27)	-
5	-	-	(7,10,13,14)	(10,13,15,17)	(9,14,19,23)	(15,19,24,27)	(22,27,30,33)
6	(6,8,10,11)	(10,11,14,15)	-	-	-	-	-
7	-	(9,10,12,13)	(6,7,8,9)	-	-	(14,16,19,22)	-
8	-	-	-	(5,8,9,10)	(3,5,8,10)	(18,22,25,28)	(13,17,22,25)
9	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-
11	-	-	-	-	-	(8,9,11,13)	-
12	-	-	-	-	-	(13,14,16,18)	(12,14,15,16)
13	-	-	-	-	-	-	(10,12,14,15)
14	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-
19	-	-	-	-	-	-	-
20	-	-	-	-	-	-	-
21	-	-	-	-	-	-	-
22	-	-	-	-	-	-	-
23	-	-	-	-	-	-	-

Table 12. continued.

i/j	16	17	18	19	20	21	22	23
1	(29,38,49,54)	(20,27,33,37)	(37,44,50,56)	(33,40,47,53)	(27,37,44,49)	(26,34,41,47)	(40,49,57,65)	(38,49,58,65)
2	(17,25,34,37)	(18,27,30,35)	-	-	(25,37,41,47)	(24,34,38,45)	-	(36,49,55,63)
3	-	-	(31,37,49,55)	(30,34,43,47)	-	(39,45,54,59)	(34,42,56,64)	(36,44,58,66)
4	(38,43,50,54)	(12,19,24,27)	-	-	(19,29,35,39)	(18,26,32,37)	-	(30,41,49,55)
5	-	(13,19,24,27)	(30,36,41,46)	(26,32,38,43)	(20,29,35,39)	(19,26,32,37)	(33,41,48,55)	(31,41,49,55)
6	(12,15,19,21)	(25,30,34,36)	-	-	(21,27,33,37)	(31,37,42,46)	-	(34,41,49,54)
7	(21,23,28,30)	(12,16,19,22)	-	-	(19,26,30,34)	(18,23,27,32)	-	(30,38,44,50)
8	-	-	(21,26,33,38)	(20,23,27,30)	-	(29,34,38,42)	(24,31,40,47)	(26,33,42,49)
9	(6,7,9,10)	-	-	-	(15,19,23,26)	-	-	(28,33,39,43)
10	(12,13,16,17)	(15,19,20,21)	-	-	(21,25,30,33)	(21,26,28,31)	-	(34,39,46,50)
11	-	(6,9,11,13)	-	-	(13,19,22,25)	(12,16,19,23)	-	(24,31,36,41)
12	-	-	(20,23,26,29)	(17,21,25,28)	-	(24,26,29,32)	(23,28,33,38)	(25,30,35,40)
13	-	-	(18,21,25,28)	(17,18,19,20)	-	(33,38,43,47)	(21,26,32,37)	(23,28,34,39)
14	-	-	-	-	-	(11,12,13,14)	-	(23,27,30,32)
15	-	-	(8,9,11,13)	(5,7,10,12)	-	(23,26,29,32)	(11,14,18,22)	(13,16,20,24)
16	-	-	-	-	(9,12,14,16)	-	-	(22,26,30,33)
17	-	-	-	-	(7,10,11,12)	(6,7,8,10)	-	(18,22,25,28)
18	-	-	-	-	-	(15,17,18,19)	(3,5,7,9)	(5,7,9,11)
19	-	-	-	-	-	-	(15,16,17,19)	(19,21,23,27)
20	-	-	-	-	-	-	-	(13,14,16,17)
21	-	-	-	-	-	-	-	(12,15,17,18)
22	-	-	-	-	-	-	-	(4,5,6,8)
23	-	-	-	-	-	-	-	-

Table 13. The $P_{23}(i, j)$ values obtained by our algorithm.

i/j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	-	1	1	1	1	2	2	5	6	7	5	5	8	11	12	9	11	15	13	17	17	18	21
2	-	-	-	-	-	2	2	-	6	7	7	-	-	11	-	9	11	-	-	17	17	-	21
3	-	-	-	-	-	-	-	3	-	-	-	8	8	12	13	-	-	15	13	-	14	18	18
4	-	-	-	-	-	-	4	-	-	7	4	-	-	11	-	10	11	-	-	17	17	-	21
5	-	-	-	-	-	-	-	5	-	-	5	5	8	11	12	-	11	15	13	17	17	18	21
6	-	-	-	-	-	-	-	-	6	6	-	-	-	-	-	9	10	-	-	16	17	-	20
7	-	-	-	-	-	-	-	-	-	7	7	-	-	11	-	10	11	-	-	17	17	-	21
8	-	-	-	-	-	-	-	-	-	-	-	8	8	12	13	-	-	15	13	-	14	18	18
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	-	-	-	16	-	-	20
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10	10	-	-	16	17	-	20
11	-	-	-	-	-	-	-	-	-	-	-	-	-	11	-	-	11	-	-	17	17	-	21
12	-	-	-	-	-	-	-	-	-	-	-	-	-	12	12	-	-	15	15	-	14	18	18
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13	-	-	15	13	-	18	18	18
14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	14	-	21
15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	15	15	-	18	18	18
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	16	-	-	20
17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	17	17	-	21
18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	18	18	18
19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	19	22
20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	20
21	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	21
22	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	22
23	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

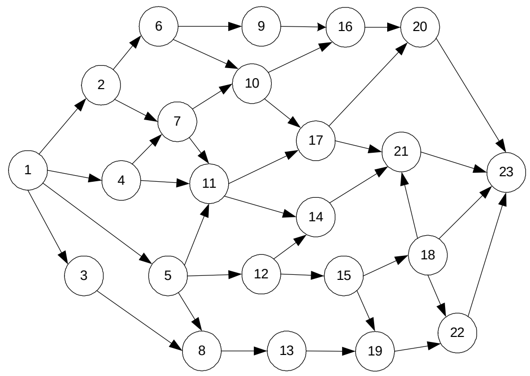

Figure 6. The telecommunication network proposed for Example 3.

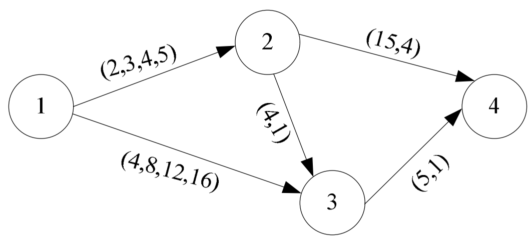

Figure 7. A small sized network having various fuzzy arc lengths.

Table 14. The $\tilde{f}_0(i, j)$ matrix for *k*=0.

i/j	1	2	3	4
1	-	(2,3,4,5)	(4,8,12,16)	2
2	-	-	(4,1)	(15,4)
3	-	-	-	(5,1)
4	-	-	-	-

Therefore, with $P_k(i, j) = i$, we have Table 15.

Step 2: Here, we consider *k*=1 and compute the value of $f_k(i, j) = \min\left[f_{k-1}(i, j), f_{k-1}(i,k) + f_{k-1}(k, j)\right]$. The result is shown in Table 16.

Therefore, for $P_k(i, j) = i$, we have Table 17.

Table 15. The $P_0(i, j)$ matrix for *k*=0.

i/j	1	2	3	4
1	-	1	1	-
2	-	-	2	2
3	-	-	-	3
4	-	-	-	-

Table 16. The $\tilde{f}_1(i, j)$ matrix for *k*=1.

i/j	1	2	3	4
1	-	(2,3,4,5)	(4,8,12,16)	-
2	-	-	(4,1)	(15,4)
3	-	-	-	(5,1)
4	-	-	-	-

Table 17. The $P_1(i, j)$ matrix for *k*=1.

i/j	1	2	3	4
1	-	1	1	-
2	-	-	2	2
3	-	-	-	3
4	-	-	-	-

If node *k* is not on the shortest path using $\{1, 2, ..., k\}$ as intermediate nodes, then we consider $P_k(i, j) = P_{k-1}(i, j)$, otherwise we let $P_k(i, j) = P_{k-1}(i, k)$. We now report the results obtained for other values of *k* in Tables 18-23. Note that, the sets V_i and W_i are the points obtained by a -cut additions, where the *V* and *W* values are obtained by the a_i-cuts considering *n=10*. It includes 10 points for the $a^-_{a_i}$ and 10 points for the $a^+_{a_i}$:

V_1={(4.58257,10.4174), (4.93136,10.0686), (5.20274,9.79726), (5.44277,9.55723), (5.66745,9.33255), (5.88528,9.11472), (6.10278,8.89722), (6.32762,8.67238), (6.57541,8.42459), (7,8)}

W_1={(11.0303,25.9697), (12.1255,24.8745), (12.911,24.089), (13.5711,23.4289), (14.1698,22.8302), (14.7411,22.2589), (15.3111,21.6889), (15.9105,21.0895), (16.6016,20.3984), (18,19)}

V_2={(4.58257,10.4174),(4.93136,10.0686), (5.20274,9.79726), (5.44277,9.55723), (5.66745,9.33255), (5.88528,9.11472), (6.10278,8.89722), (6.32762,8.67238), (6.57541,8.42459), (7,8)}

W_2={(8.06515,16.9349), (8.66273,16.3373), (9.10549,15.8945), (9.48554,15.5145), (9.83489,15.1651), (10.1706,14.8294), (10.5056,14.4944), (10.8552,14.1448), (11.2508,13.7492), (12,13)}

Table 18. The $\tilde{f}_2(i, j)$ matrix for *k*= 2.

i/j	1	2	3	4
1	-	(2,3,4,5)	V_1	W_1
2	-	-	(4,1)	(15,4)
3	-	-	-	(5,1)
4	-	-	-	-

Table 19. The $P_2(i, j)$ matrix for *k*= 2.

i/j	1	2	3	4
1	-	1	2	2
2	-	-	2	2
3	-	-	-	3
4	-	-	-	-

Table 20. The $\tilde{f}_3(i, j)$ matrix for *k*= 3

i/j	1	2	3	4
1	-	(2,3,4,5)	V_2	W_2
2	-	-	(4,1)	(9,2)
3	-	-	-	(5,1)
4	-	-	-	-

Table 21. The $P_3(i, j)$ matrix for *k*= 3.

i/j	1	2	3	4
1	-	1	2	3
2	-	-	2	3
3	-	-	-	3
4	-	-	-	-

Table 22. The $\tilde{f}_4(i,j)$ matrix for *k*= 4.

i/j	1	2	3	4
1	-	(2,3,4,5)	V_3	W_3
2	-	-	(4,1)	(9,2)
3	-	-	-	(5,1)
4	-	-	-	-

Table 23. The $P_4(i,j)$ matrix for *k*= 4.

i/j	1	2	3	4
1	-	1	2	3
2	-	-	2	3
3	-	-	-	3
4	-	-	-	-

V_3={(4.58257,10.4174), (4.93136,10.0686), (5.20274,9.79726), (5.44277,9.55723), (5.66745,9.33255), (5.88528,9.11472), (6.10278,8.89722), (6.32762,8.67238), (6.57541,8.42459), (7,8)}

W_3={(8.06515,16.9349), (8.66273,16.3373), (9.10549,15.8945), (9.48554,15.5145), (9.83489,15.1651), (10.1706,14.8294), (10.5056,14.4944), (10.8552,14.1448), (11.2508,13.7492), (12,13)}

Finally, when $k = N$, we identify the shortest path as follows:

Shortest path from 1 to 4: 1-2-3-4.

Shortest path length from 1 to 4:

(8.06515,16.9349),(8.66273,16.3373),(9.10549,15.8945),(9.48554,15.5145),(9.83489,15.1651),(10.1706,14.8294),(10.5056,14.4944),(10.8552,14.1448),(11.2508,13.7492),(12,13)

6. Conclusions

We presented a novel approach for computing a shortest path in a mixed network having various fuzzy arc lengths. First, we developed a new technique for the addition of various fuzzy numbers in a path using α-cuts. Then, we applied a dynamic programming method for finding a shortest path in the network, using a recently proposed distance function to compare fuzzy numbers in the proposed algorithm. Four comparative examples were worked out to illustrate the applicability of our proposed approach as compared to two other methods in the literature as well as demonstrate the additional novel feature offered by our algorithm to find a fuzzy shortest path in mixed fuzzy networks having various settings for the fuzzy arc lengths.

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