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Lepton Scattering Cross Section and Nuclear Structure Function of ^{4}He and ^{12}C Nuclei

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ABSTRACT

This paper is an effort to extract the structure function, the EMC ratio, and the lepton scattering cross section in the convolution nuclear theory framework for ^{4}He and ^{12}C nuclei. We suppose that, in conventional approach, based on harmonic oscillator model, one could consider for a nucleus shell different $\hbar\omega$ parameters which are associated with the square root of the mean radius of the nucleus shells. We use GRV free nucleon structure function, which has good agreement with the proton and neutron structure function, extracted from experimental results. In addition, the lepton scattering cross sections of ^{4}He and ^{12}C nuclei are calculated in energy higher than 1 GeV. The extracted results show good agreement with experimental data.

Keywords: Structure Function; Differential Cross Section; The EMC Effect

1. Introduction

One of the best methods of acquiring information about the structure of the bound nucleons inside the nuclei is the studying of the leptons scattering from the nuclei. This kind of scattering is due to weak electromagnetic interactions of quarks inside nuclei which gain a great deal of momentum from transferred momentum of virtual photon. In this process, the change of the whole structure of the nuclei is small and ignorable. The weak interacttions do not change the structure of the nuclei, so lepton scattering can be utilized to determine the structure of the nuclei [1]. The experimental data, obtained from the deep inelastic leptons scattering from free and bound nucleons, have been get considerable information about the structure function of the free and bound nucleons. This accomplishment was published in 1983 by EMC group under Aubert supervision [2]. The EMC group found out that the bound nucleons inside the nuclei have different momentum distribution in comparison with the free nucleons [2] there is a lot of explained theory about this difference such as Fermi motion [3,4] and the binding energy [5,6], quark exchange effect [7], pionic contribution [8,9], shadowing effect [10] and presence of the Δ particles [11]. In this paper, with considering Fermi motion and binding energy effects, which have major roles in difference between the structure function of the free and bound nucleons, the structure function and EMC ratio in the conventional nuclear theory for ^{4}He and ^{12}C nuclei are calculated by using harmonic oscillator model with considering different $\hbar\omega$ parameters for different shells inside the nucleus have been investigated. For calculating the structure function and the EMC effect, we imply the GRV structure function of free nucleon [12]. Finally, we calculate the differential cross section leptons scattering from ^{4}He and ^{12}C nuclei. It is shown that the obtained results have good agreement with experimental data.

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2. Convoluted Nuclear Structure Function

The structure function of nucleus in convolution model can be obtained by using the following equation [5]:

$$F_2^A(x) = \sum_{N=n,p} \sum_{nl} \int_x^A dz \cdot g_{nl}^N f^N(z)_{nl} F_2^N\left(\frac{x}{z}\right) \qquad (1)$$

where the first sum is over proton and neutron cases. The second sum is over the quantum number of states. The g_{nl}^N is the occupation number of energy level ε_{nl} for proton $(N=P)$ and neutron $(N=n)$. $F_2^N(x)$ is the structure function of nucleons which we have used the GRV structure function of neutron and proton.

The momentum distribution function of the nucleons inside the nucleus is given by [5]:

$$f^N(z)_{nl} = \int_{|m_N(z-1)-\varepsilon_{nl}|}^{\infty} dp \cdot p \cdot m_N |\varphi_{nl}(p)|^2 / (2\pi)^2 \quad (2)$$

where $z = \frac{p_{nl}q}{m_N q_0}$ and $x = \frac{Q^2}{2m_N q_0}$ is the bjorken variable for the free nucleon. The effects of the momentum and energy distribution of the nucleon in the nucleus are included in Equation (2) through $\varphi_{nl}(p)$ and ε_{nl}, respectively. The magnitude of the nuclear binding energy ε_{nl} mainly affects the structure function in the intermediate *x* region. The momentum and energy distribution of a nucleon inside a nucleus is expressed by $f^N(z)_{nl}$ that satisfies the normalization rule as below [3]:

$$\sum_{N=n,p} \sum_{\text{nl}} \int_0^{\infty} \text{dz} \cdot g_{nl}^N f^N(z)_{\text{nl}} = A . \quad (3)$$

When the gluons and sea quark are considered, sum rule becomes as:

$$\int_0^1 dx \cdot F_2^N(x) = 1 . \quad (4)$$

Equation (2) describes the momentum and energy distribution of nucleons inside nuclei. In order to calculate $f^N(z)_{nl}$ the harmonic-oscillator nuclear wave function $\varphi_{nl}(p)$ is used. Since in the heavy nucleus the deeper closed shells have different root mean square radius, each shell of a nucleus has different root mean square radius from others as [13]:

$$\langle r^2 \rangle_{\text{nl}} = \frac{1}{\alpha^2}(2n + l + 3/2) , \quad (5)$$

Where $\alpha^2 = \frac{m_n \omega}{\hbar}$ and in the natural unite, we have:

$$\hbar\omega = \frac{42.2}{\langle r^2 \rangle_{nl}}(2n + l + 3/2) \quad (6)$$

where m_n = 938.905 MeV is a nucleon mass, $\langle r^2 \rangle^{1/2}$ and $\hbar\omega$ are expressed in Fermi and MeV unit, respectively.

The calculated data for $\langle r^2 \rangle^{1/2}$ and $\hbar\omega$ are expressed in **Table 1** for ^{2}H, ^{4}He and ^{12}C nuclei.

Table 1. Brackets contain $\left(\langle r^2 \rangle^{1/2}, \hbar\omega, g_{nl}^n, g_{nl}^p, \varepsilon_{nl}(\text{MeV})\right)$ for studied nuclei that oscillator-model parameter $\hbar\omega$ calculated from Equation (6). $\langle r^2 \rangle^{1/2}$ is taken from [14] for each shell.

Shell	^{2}H	^{4}He	^{12}C
0*s*	(2.09, 7.21, 1, 1, −1)	(1.67, 11.3, 2, 2, −15)	(1.67, 11.3, 2, 2, −20)
0*p*			(2.44, 8.8, 4, 4, −20)

$f^N(z)_{nl}$ describes the momentum distribution and energy of nucleons inside a nucleus. Instead of using Equation (2) we use the below expression to describe the momentum distribution and energy of the nucleons in a nucleus as [5]:

$$f^N(z)_{nl} = \frac{1}{2}\left(\frac{m_N}{\hbar\omega}\right)^{1/2} \times \frac{n!}{\Gamma(n+l+3/2)}$$
$$\sum_{t_1=0}^{n} \sum_{t_2=0}^{n} \frac{(-1)^{t_1+t_2}}{t_1!t_2!} \binom{n+l+1/2}{n-t_1} \binom{n+l+1/2}{n-t_2}$$
$$\times\Gamma\left[l + t_1 + t_2 + l, \frac{m_N}{\hbar\omega}\left(z - 1 - \frac{\varepsilon_{nl}}{m_N}\right)^2\right] \quad (7)$$

For $F_2^N(x/z)$ we use the GRV free proton and neutron structure functions [12], and we ignore the contribution of strange quark, the structure function of ^{2}H, ^{4}He and ^{12}C nuclei are plotted in **Figure 1** based on Ref. [12].

3. The EMC Ratio

The experimental results showed that the structure function of a nucleus could not be obtained by summing all of the structure function of the nucleons inside the nucleus [2]. Since there are some factors like Fermi motion [3,4], binding energy [5,6], quark exchange [7], pionic contribution [8-9], and the shadowing effect [10], presence of Δ particle [11], that cause to this difference. One of the main reasons to this difference is the Fermi motion effect that impresses the structure function of nucleus in *x*→1. The nuclear structure function is incoherent sum of the structure function of constituted nucleons, therefore the Fermi motion has effect on the nuclear structure function. In addition, the nuclear wave function is the convolution of the free nucleon wave function by momentum distribution of nucleon, therefore the binding energy also has effect on nuclear structure function.

To show the difference between the free nucleon with the bound nucleon inside the nucleus, the EMC ratio could be expressed as follows [2,5]:

$$R_{EMC}^A(x) = F_2^A(x)_{\text{per nucleon}} / F_2^{^2\text{H}}(x)_{\text{per nucleon}} \quad (8)$$

Figure 2 shows the EMC ratio, *R*(*x*), when Fermi motion and binding energy are considered. When *x* arise to 1, it is seen that $R_{EMC}^A(x) > 1$. In this region, one can see that the structure function of ^{4}He and ^{12}C are very small, however the structure function of free nucleon vanishes, so when *x* arise to 1 the $R_{EMC}^A(x)$ goes to above 1. Because of the structure function of nucleus vanishes when *x* approach to *A* (*A* > 1), which is atomic number of nucleus. We should remark that *x* is the fraction of momentum that is carried by nucleons inside the nucleus. *x* > 0.8 is the Fermi motion region, and in this region nucleons have Fermi-like motions [17,18].

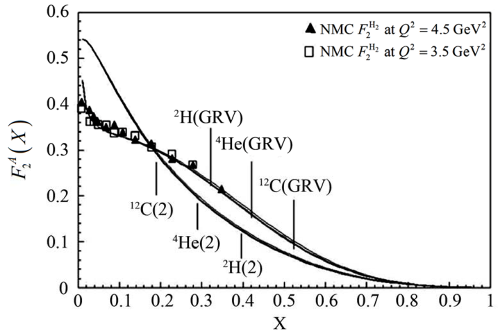


Figure 1. The structure function of ^{2}H, ^{4}He and ^{12}C nuclei. In order to obtain the nucleus structure function we have used the GRV structure function of the free proton. The experimental data are taken from [15,16].

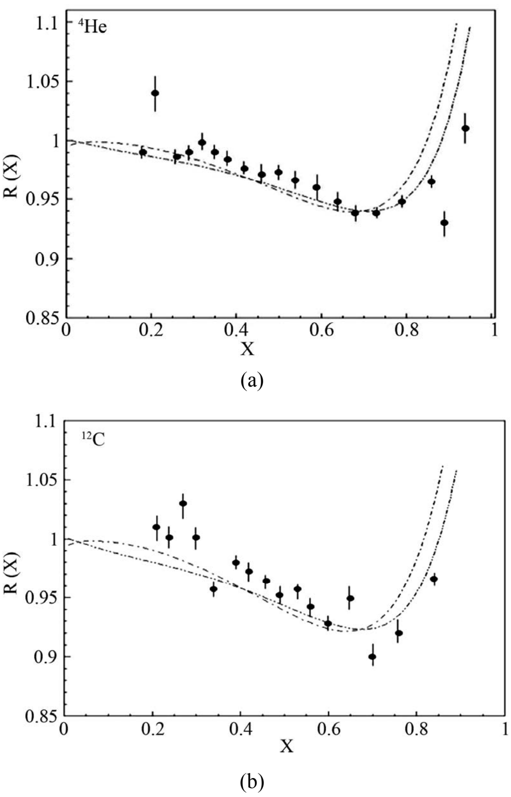


Figure 2. The $R_{EMC}^A(x)$ for ^{4}He and ^{12}C nuclei are plotted. We use two different nucleon structure functions. One of them is nucleon structure function based on GRV nucleon structure function of free proton and neutron [12] *i.e.* dashed and the other have been taken from Ref. [5] *i.e.* dash-double dotted.The experimental data have been taken from Ref. [19].

4. Deep Inelastic Scattering

Particle collision in low energy is elastic scattering; in the other word this kind of collision doesn't produce new particles. Otherwise, if particles have enough energy when they collide to each other they create new particles. This kind of collision is called inelastic scattering. In deep inelastic scattering, the number of scattered particles with large angles demonstrates that the protons have inner structures. When leptons incident on nuclei and produce new particles, this deep inelastic scattering could be seen as the elastic scattering of leptons from nuclei constituted particles *i.e.* partons [20].

The differential cross section could be expressed by [21]:

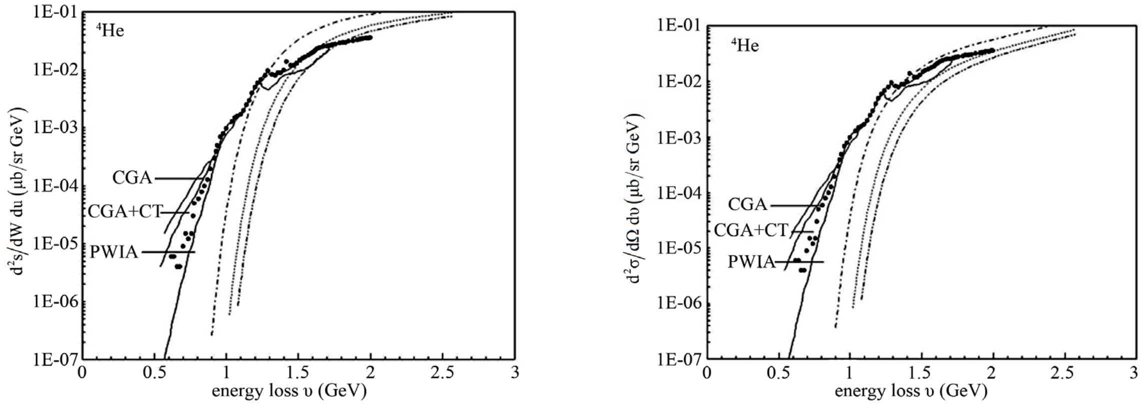
$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 (E')^2}{Q^4}\cos^2\left(\frac{\theta}{2}\right) \times \left[\frac{F_2(x,Q^2)}{\upsilon} + \frac{2F_1(x,Q^2)}{M}\tan^2\left(\frac{\theta}{2}\right)\right]. \quad (9)$$

where $\alpha = e^2/4\pi \sim 1/137$ is the fine structure constant, four-momentum transfer squared is Q^2. Initial and scattered lepton energies are E and E', respectively. Energy of the virtual photon is $\nu = E - E'$, and $x = \frac{Q^2}{2M\nu}$ is Bjorken scaling variable. M is the nucleon rest mass. θ is the detected lepton scattering angle. F_1 and F_2 are the deep inelastic structure functions.

Figure 3 shows the scattering differential cross section of leptons from the ^{4}He nucleus, for E = 3.595 GeV, θ = 30˚, and $0.97 \lesssim |q|^2 \lesssim 2.42$ GeV^2 is plotted with dash-double-dotted, also, for θ = 29˚, $0.91 \lesssim |q|^2 \lesssim 2.32$ GeV^2 is plotted with dot and for θ = 27˚, and $0.79 \lesssim |q|^2 \lesssim 2.11$ GeV^2 show with dash-dotted as well. These curves show our main results. Of course we should mention that, in order to obtain the results of **Figure 3(a)** we have used the GRV free nucleon structure function [12], and to obtain the results of **Figure 3(b)** we have used the nucleon structure function used in Ref. [5]. Also **Figure 4** demonstrates the scattering differential cross section of leptons from the ^{12}C nucleus, for E = 5.86 GeV, θ = 30˚, and $3.07 \lesssim |q|^2 \lesssim 5.52$ GeV^2 depicted with dash-double-dotted, and for θ = 29˚, and $2.88 \lesssim |q|^2 \lesssim 5.26$ GeV^2 depicted with dot and θ = 27˚, also for $2.50 \lesssim |q|^2 \lesssim 4.8$ GeV^2 show with dash-dotted as well. These figures show our main results. Of course we should mention that, in order to obtain the results of **Figure 4(a)** we have used the GRV free nucleon structure function [12], and to obtain the results of **Figure 4(b)** we have used the nucleon structure function used in Ref. [5].

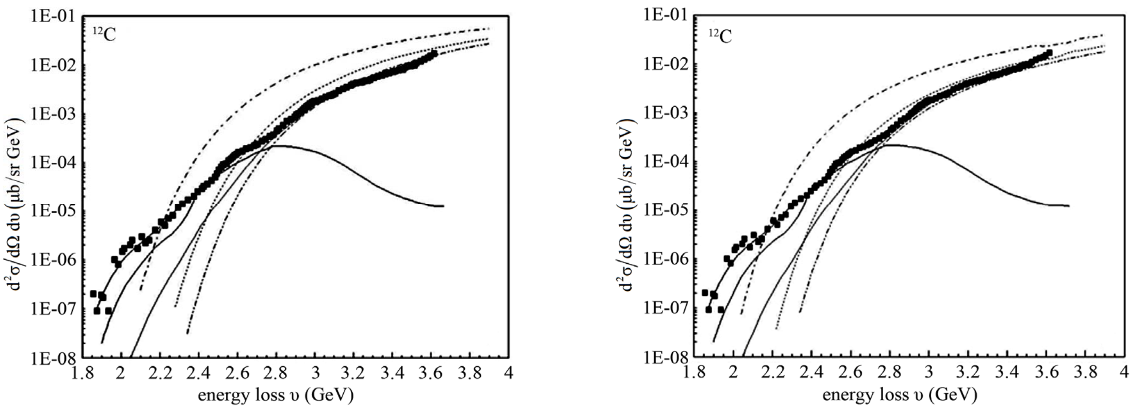
5. Results and Discussion

Based on the calculations and comparing our results with the previous works, we conclude that nuclear interactions specially binding energy effects and the new assumptions considered in this paper can clarify both qualitatively and quantitatively the deep inelastic lepton-nuclear scattering without considering the nucleon's quark structure changes.



(a) (b)

Figure 3. The (a) and (b) diagrams show the deep inelastic cross section of electron beam from ^{4}He nuclei. We have used the GRV nucleon structure function [12] for calculation of nucleus structure function that plotted in (a) and nucleon structure function from [5] for calculating of nucleus structure function that plotted in (b) diagram. The dash-double-dotted and dot and dash-dotted cures shows our calculation. The CGA, CGA + CT, PWIA curves, and experimental data have been taken from [22].



(a) (b)

Figure 4. The (a) and (b) diagrams show the deep inelastic cross section of electron beam from ^{12}C nuclei. We have used the GRV nucleon structure function [12] for calculation of nucleus structure function that plotted in (a) and nucleon structure function from [5] for calculating nucleus structure function that plotted in (b) diagram The dash-double-dotted and dot and dash-dotted curves shows our calculation. The three full curves and experimental data have been taken from [23].

The obtained results in **Figures 2-4** are slightly different from experience or are located in experimental error domain, which are due to cloud meson, shadow effect, delta particle and other nuclear effects, which are neglected in this paper.

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