

Differential Cross Section of Electron Scattering from ^3He and ^3H Nuclei with Considering Pionic Contribution

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ABSTRACT

The excess pion inside the nucleus could change not only nucleons structure function, but also electron scattering cross section from nuclei. In this paper, we calculate pionic contribution in nuclear structure functions of ^3He and ^3H nuclei and the differential cross sections of electron scattering from these nuclei. At first, we calculate the Fermi motion and binding energy contribution as an important nuclear medium effect in scattering cross sections and then we add pionic contribution to structure functions and differential cross sections of electron scattering from these nuclei that the Fermi motion and binding energy are considered.

Keywords: Pionic Contribution; Structure Function; Differential Cross Section

1. Introduction

One of the important questions in nuclear physics is difference between a free nucleon structure function with a bound nucleon structure function. In this paper, we have investigated one of the main reasons of this difference in nucleus structure function in $0.1 \leq x \leq 0.3$ range by considering pion sea around nucleons inside nucleus. A nucleus consists of nucleons, mesons, and objects like Δ particle [1]. Nucleons and mesons made of quarks and sea quarks. Quarks inside both of nuclei and mesons could not be marked and distinguished, so the electron scattering from bound nucleons are different from free nucleons [2-4]. Therefore, the free nucleon structure function is different from bound nucleon structure function. Pions are the lightest mesons, and presences of them in a nucleus are more than other mesons. So, we investigate presence of pions in nucleus. Since, a free nucleon is a particle that virtual pions embedded around nucleons and creating a pion cloud around nucleons. When a nucleon is inside nuclei, pion cloud is increasing around the nucleon. When electrons scatter from nuclei, the probability of the interaction between a virtual photon of incident electrons with a bound nucleon is increasing, because interaction between virtual photon with pions is increasing. The virtual photon scatters from not only a quark insides nucleons, but also a quark or sea quark from pion cloud [5]. This effect is obvious in range $x \leq 0.2$. Therefore, pion cloud plays a role in nuclei structure functions. In this paper, first, we calculate pionic contri-

butions in ^3He and ^3H nuclei, and then we calculate cross sections by considering these contributions.

2. Pion Structure Function

The nucleus structure function, in nuclear conventional theory, with respecting the pion cloud effect is [5]:

$$F_2^A(x, Q^2) = \int_x^A f_\pi(z) F_2^\pi\left(\frac{x}{z}, Q^2\right) dz + \int_x^A f_N(z) F_2^N\left(\frac{x}{z}, Q^2\right) dz, z \geq x \quad (1)$$

First term indicates the pionic contribution and the next term indicates the nucleon contribution in the nucleus. First term contribution is noticeable around $x \cong 0.2$. x is Bjorken variable. z is the light-cone momentum fraction of the nucleus carried by nucleon, and its distribution is inside nucleus $f_N(z)$ taken from [6].

$F_2^N\left(\frac{x}{z}, Q^2\right)$ is the nucleon structure function [7]. The momentum distribution $f_\pi(z)$ in a free nucleon case is given as follow [8]:

$$f_\pi^A(z) = \frac{3g^2}{16\pi^2} \Delta\lambda z \left(\frac{1}{\lambda} \exp\left(-2\lambda \frac{t_0 + m_\pi^2}{m_\pi^2}\right) + \frac{1}{2} Ei\left(-2\lambda \frac{t_0 + m_\pi^2}{m_\pi^2}\right) \right) \quad (2)$$

where

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$$Ei(-z) = -\int_z^\infty \frac{e^{-t}}{t} dt \quad (3)$$

which $g = 13.5$ is the coupling constant. The cut off parameter λ plays the most substantial role. When the nucleon is embedded in a nucleus, several modifications such as the polarization of nuclear medium occur. They may be expressed by an effective change of λ . We have taken $\lambda = 0.026$ [8] and pion mass $m_\pi = 139.570$ MeV.

We take $\Delta\lambda > 0$ to have

$$\eta_\pi = \int_0^{M_\pi/m} dz z f_\pi^A(z) \quad (4)$$

where η_π is the momentum fraction of pion. We have taken $\eta_\pi = 0.130$ for ${}^2\text{H}$ and $\eta_\pi = 0.155$ for ${}^3\text{He}$ and ${}^3\text{H}$ nuclei from [8], also $\Delta\lambda = 0.0031$ for ${}^2\text{H}$ and $\Delta\lambda = 0.00367$ for ${}^3\text{He}$ and ${}^3\text{H}$ nuclei.

For pion structure function, we used follow parameterization [9,10]:

$$F_2^\pi(x) = \frac{5}{9} x V_\pi(x) + \frac{4}{3} x S_\pi(x) \quad (5)$$

$$x V_\pi(x) = \frac{\Gamma(\alpha + \beta + 1)}{\Gamma(\alpha)\Gamma(\beta + 1)} x^\alpha (1-x)^\beta \quad (6)$$

$$x S_\pi(x) = \frac{1}{6} A(p+1)(1-x)^p \quad (7)$$

$$\alpha = 0.36 - 0.074\bar{S}, \quad \beta = 0.99 + 0.60\bar{S} \quad (8)$$

$$\bar{S} = \ln \left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right) \quad (9)$$

$$Q_0^2 = 25(\text{GeV}c^{-1})^2, \quad \Lambda = 0.2\text{GeV}c^{-1}, \quad p = 8.7, \quad (10)$$

$$A = 0.51 - 2\alpha/(\alpha + \beta + 1)$$

In **Figure 1**, we plotted the momentum distribution of pions inside ${}^2\text{H}$, ${}^3\text{He}$ and ${}^3\text{H}$ nuclei.

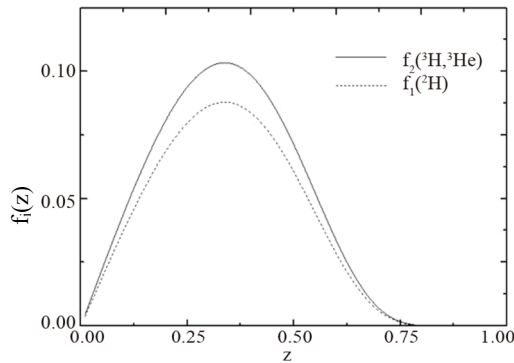


Figure 1. The momentum distribution of pions for ${}^2\text{H}$, i.e. $i = 1$ in figure, shown with dotted curve, ${}^3\text{He}$ and ${}^3\text{H}$, i.e. $i = 2$ in figure, shown with full curve.

3. Calculating Differential Cross Section of Electron Scattering from Nucleus

The structure functions for charged lepton scattering from a nucleon are related to cross section by [11]:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 (E'')^2 \cos^2\left(\frac{\theta}{2}\right)}{Q^4} \times \left[\frac{F_2(x, Q^2)}{\nu} + \frac{2F_1(x, Q^2) \tan^2\left(\frac{\theta}{2}\right)}{M} \right] \quad (11)$$

$$Q^2 = -q^2 = -4EE' \sin^2\left(\frac{\theta}{2}\right), \quad \nu = E - E'$$

where

$$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

is the fine structure constant, four-momentum transfer squared is Q^2 . Initial and scattered lepton energies are E and E' , respectively. Energy of the virtual photon is $\nu = E - E'$, and

$$x = \frac{Q^2}{2M\nu}$$

is Bjorken scaling variable. M is the nucleon rest mass. θ is the detected lepton scattering angle. F_1 and F_2 are the deep inelastic structure functions.

4. Results and Discussion

In **Figures 2** and **3**, nucleus structure functions for ${}^3\text{He}$

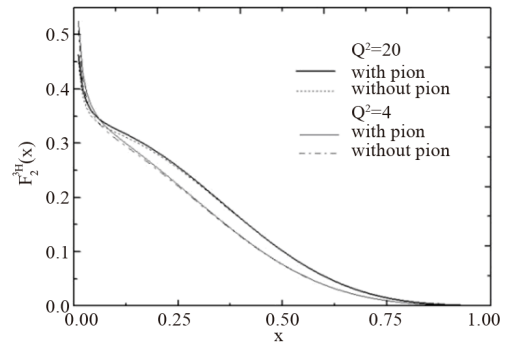


Figure 2. This figure shows ${}^3\text{H}$ structure functions. The full curve is plotted for $Q^2 = 4 \text{ GeV}^2$ with considering the Fermi motion, the binding energy, and the pionic contribution effect, the dotted curve is plotted with considering the Fermi motion, and binding energy effect. The full curve is plotted for $Q^2 = 20 \text{ GeV}^2$ with considering the Fermi motion, the binding energy, and the pionic contribution effect, the line-dash is plotted with considering the Fermi motion, and the binding energy effect.

and ^3H nuclei have been plotted by considering the Fermi motion, the binding energy and the pionic contribution. In this calculation we do not consider any difference between pionic cloud around the protons and neutrons and the contribution is equal ^3H and ^3He nuclear structure functions. This figure shows that the pionic contribution has maximum around $x \approx 0.2$ that one expected.

In addition, the full curve in **Figures 4** and **5** show differential cross sections of electron scattering from ^3He and ^3H nuclei with considering the Fermi motion, the binding energy and the pionic contribution. The dotted curves in **Figures 4** and **5** show differential cross sections of electron scattering from ^3He and ^3H nuclei with considering only the Fermi motion and the binding energy effects. These figures show the pionic contribution

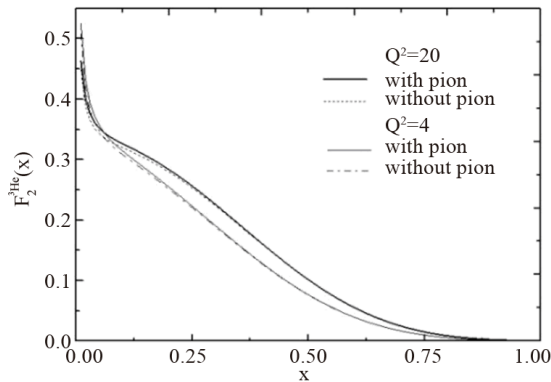


Figure 3. This figure shows ^3He structure functions. The full curve is plotted for $Q^2 = 4 \text{ GeV}^2$ with considering the Fermi motion, the binding energy, and the pionic contribution effect, the dotted curve is plotted with considering the Fermi motion, and binding energy effect. The full curve is plotted for $Q^2 = 20 \text{ GeV}^2$ with considering the Fermi motion, the binding energy, and the pionic contribution effect, the line-dash is plotted with considering the Fermi motion, and the binding energy effect.

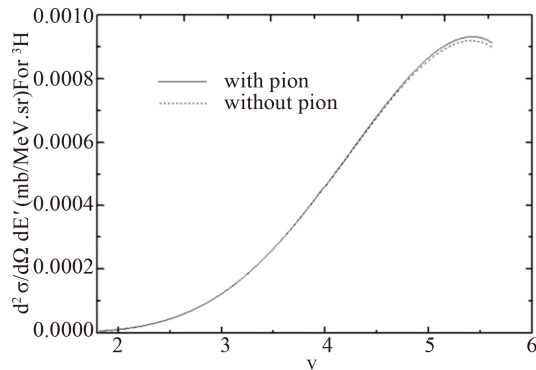


Figure 4. Differential cross section of electron scattering from ^3H at $\theta = 8^\circ$, $Q^2 = 4 \text{ GeV}^2$, and $E = 7.26 \text{ GeV}$. The full curve is plotted with considering the Fermi motion, the binding energy, and the pionic contribution. The dotted curve is plotted with considering only the Fermi motion and the binding energy.

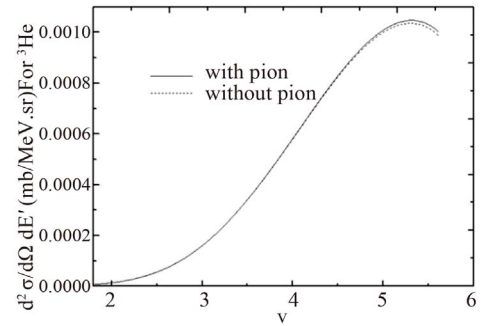


Figure 5. Differential cross section of electron scattering from ^3He at $\theta = 8^\circ$, $Q^2 = 4 \text{ GeV}^2$, and $E = 7.26 \text{ GeV}$. The full curve is plotted with considering the Fermi motion, the binding energy, and the pionic contribution. The dotted curve is plotted with considering only the Fermi motion and the binding energy.

cloud improve cross section up to %1 or %2, which is perceptible in small x . In these figures pionic contribution increases when ν goes up because pion cloud inside nuclei plays a role like sea quark inside nucleons.

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