

Mixed Convection MHD Flow of Viscoelastic Fluid in a Porous Medium past a Hot Vertical Plate

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ABSTRACT

The boundary layer flow of a steady incompressible and visco-elastic fluid with short memory (obeying Walters' B fluid model) passing over a hot vertical porous plate has been investigated in the presence of transverse magnetic field. The momentum and energy equations are reduced to couple non-linear partial differential equations along with the boundary conditions by using a suitable similarity transformation. These partial differential equations are transformed to a system of coupled non-linear ordinary differential equations by employing a perturbation technique. The system is solved by developing a suitable numerical procedure such as implicit finite difference scheme along with Newton's linearization method. The computational results for the flow quantities have presented graphically for the effects of thermal radiation, viscous dissipation, heat generation/absorption, visco-elasticity, Hartmann number and the permeability parameter. Results demonstrated that Prandtl number has more pronouncing effect on the temperature distribution rather than the viscosity parameter as well as the thermal radiation parameter. Further the velocity gradient changes significantly due to the presence of temperature dependent variable viscosity.

Keywords: MHD Flow; Variable Viscosity; Visco-Elastic Fluid; Viscous Dissipation; Thermal Radiation

1. Introduction

The mixed convection boundary layer flow of non-Newtonian fluid in the presence of strong magnetic field has wide range of application in nuclear engineering and industries. In astrophysical and geophysical studies, the MHD boundary layer flows of an electrically conducting fluid through porous media have also enormous applications. These studies are also used for modeling and simulation. Many researchers have studied the transient laminar natural convection flow past a vertical porous plate for the application in the branch of science and technology such as in the field of agriculture engineering and chemical engineering. In petroleum refineries, movement of oil, water and gas through porous media for purification and filtration are bright applied areas of research. With the advancement of science and technology, MHD study on any fluid flow phenomenon exhibits some results which have constructive application for the design of devices. MHD heat transfer has great importance in the liquid metal flows, ionized gas flow in a nuclear reactor and electrolytes. Research works on radiation of heat in natural convection flow are very limited, though these have many modern applications viz. missile

technology used in army, nuclear power plant, parts of aircraft and ceramic tiles. Thus more investigation is required by considering the terms like magnetic field, heat source/sink, thermal radiation and variable viscosity.

Considering the application in underground water resource and seepage of water under dam, Raptis *et al.* [1] examined the free convection flow through a vertical porous media. In the other context Raptis *et al.* [2,3] studied the similar problem by introducing mass transfer, constant suction and constant heat flux. Lai and Kulacki [4] shown an interesting problem regarding free convective flow of Newtonian fluid through a vertical porous medium with varying permeability. Taking into account the temperature dependent viscosity and thermal conductivity Rani and Kim [5] studied the flow of Newtonian fluid over a semi-infinite isothermal vertical cylinder. In the recent past several authors have studied the radiation effect on the flow past a parallel plate [6,7]. Rahman *et al.* [8] presented an interesting result on a natural convection flow past a vertical plate considering the temperature dependent thermal conductivity and heat conduction. Using implicit finite difference scheme Laganathan *et al.* [9] solved a coupled non-linear momentum and energy equations for the investigation of the effect of thermal conductivity on the free convective flow over a semi-

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infinite vertical plate under the influence of transverse magnetic field. Shit and Haldar [10] investigated the effects of thermal radiation on the magnetohydrodynamic flow and heat transfer over nonlinear shrinking porous sheet. Jang and Leu [11] analyzed vortex instability in the Newtonian fluid flow and estimated the effect of variable viscosity. Unsteady boundary layer flow over a stretching sheet has rigorously studied by Misra *et al.* [12] in the presence of variable thermal conductivity. Sharma *et al.* [13] in two subsequent study showed that the variable permeability increase the flow velocity and heat source has revised effect on Nusselt number for air and water. They elaborately discussed the effect of Grashoff number on Newtonian fluid due to the presence of magnetic field. In polymer processing industries, researchers deals with non-Newtonian fluids in which stress strain relations are non-linear. The differential types of fluid flow exhibits a little effect of deformation gradient on the stress. In the constitutive relations stress is just a function of velocity gradient and its higher power and the stress relaxation time is very small. Rivlin-Eriction fluid shows small relaxation time factor in addition to a convective time derivative in its constitutive relation. The boundary layer flow of non-Newtonian Rivlin-Eriction fluid past a wedge was investigated by Massoudi and Ramezan [14]. They presented results for velocity and shear stress at the wall for different suction/injection rates and different wedge angle.

It is well known that most of the fluid property changes in the presence of heat generation/absorption. Therefore, it is highly meaningful to study the fluid flow and heat transfer taking into account the temperature dependent viscosity. Singh and Gorla [15] carried out the boundary layer analysis of free convective flow of conducting Newtonian fluid over an infinite vertical porous plate. In this investigation they studied the effect of viscous dissipation, Joule heating, thermal diffusion and Hall current. To study the effect of viscosity which varies with temperature and viscous dissipation, Pal and Mondal [16] adopted a problem of mixed convective heat transfer over a stretching surface under the influence of non-uniform heat source/sink. In a similar study Shateyi and Motsa [17] used successive linearization method to solve a set of non-linear ordinary differential equation arises in formulation of unsteady heat transfer over a stretching sheet with Hall Effect. Taking into account the variable viscosity and thermal conductivity Mahanti and Gaur [18] put forwarded some interesting results on free convective flow and heat transfer due to heat sink along an isothermal vertical plate. Numerical solutions for non-Newtonian fluid flow differential equation have been presented by Cortell [19,20]. He compared the results with Rajagopal *et al.* [21] and found close similari-

ties. Successively the same author presented similarity solutions when the flow takes place over a stretching sheet. Adopting a second order fluid he studied two cases viz., constant surface temperature and prescribed surface temperature along with the examination of effect of viscoelasticity on heat transfer. Very recently, he studied the viscoelastic fluid flow and heat transfer over a stretching sheet [22]. Incorporating viscous dissipation, work due to deformation, heat radiation and heat generation/absorption terms in energy equation he presented several results and compared among those. Takhar *et al.* [23] investigated the mixed convection flow over a hot vertical plate in the presence of non-Darcian porous media.

In order to study the flow of viscoelastic fluids of longer relaxation time we consider Walters liquid B' model and investigated the flow and heat transfer through a vertical porous plate in the presence of transverse magnetic field. In the present problem heat and fluid flow has been investigated for viscoelastic fluid when the radiation of heat occurs from the flow domain and flow field has some heat source/sink. Introducing some similarity variables in the governing equations the reducing equations are found to be non-linear partial differential equations. Taking into account the boundary layer approximation length of the plate is adopted as the perturbation parameter. A set of non-linear ordinary differential equations are solved by finite difference techniques with the use of boundary conditions prescribed in the problem. Interesting effects of viscoelasticity, radiation parameter, variable viscosity, magnetic field and heat source are to be observed on account of fluid flow and heat transfer.

2. Mathematical Formulation and Analysis

Considering the steady two dimensional flow of viscoelastic fluid past a porous hot vertical plate in which the x -axis is taken in the vertical direction and y -axis perpendicular to it as horizontal. The fluid is assumed to be falling down vertically over a vertical plane. The constitutive equation of Walter's B' liquid can be written as (cf. Mahapatra and Gupta [24])

$$\tau_{ik} = -p\delta_{ik} + \tau'_{ik} \quad (1)$$

$$\tau'^{ik}(x, t) = 2 \int_{-\infty}^t \Psi(t-t') \frac{\partial x^i}{\partial x'^m} \frac{\partial x^k}{\partial x'^r} e^{(1)mr}(x', t') dt', \quad (2)$$

where

$$\Psi(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{-(t-t')/\tau} d\tau \quad (3)$$

$N(\tau)$ is denoted as the distribution function of relaxation time, τ_{ij} the isotropic stress tensor, p the hydrostatic pressure, δ_{ij} the metric tensor, x'^i the position at time t' at the point x^i , $e_{ik}^{(1)}$ the rate of strain

tensor. Taking into account the short relaxation time the Equation (2) becomes

$$e^{(1)}_{ik} \tau^{ik} = 2\mu e^{(1)ik} - 2k_0 \frac{D}{Dt} e^{(1)ik} \tag{4}$$

with

$$\mu = \int_0^\infty N(\tau) d\tau, \quad k_0 = \int_0^\infty \tau N(\tau) d\tau$$

and
$$\frac{D\Gamma^{ik}}{Dt} = \frac{\partial \Gamma^{ik}}{\partial t} + v^m \frac{\partial \Gamma^{ik}}{\partial x^m} - \frac{\partial v^k}{\partial x^m} \Gamma^{im} - \frac{\partial v^i}{\partial x^m} \Gamma^{mk}$$

Considering boundary layer approximation in which the pressure term becomes order of boundary layer thickness $\delta(x)$ and the governing equations such as continuity, momentum and energy equations are reducing to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\mu}{\rho} \frac{\partial u}{\partial y} \frac{\partial \mu}{\partial y} + g\beta(T - T_\infty) - \frac{gu}{K_1^*} - \frac{\sigma B_0^2}{\rho} u - \frac{k_0}{\rho} \left(\frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right) \tag{6}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \lambda \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial \mu}{\partial y} \right)^2 - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{q'''}{\rho C_p} - \frac{k_0}{\rho C_p} \frac{\partial u}{\partial y} \frac{\partial}{\partial y} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \tag{7}$$

The boundary conditions are

$$u = 0, \quad v = 0, \quad T = T_w \quad \text{at } y = 0 \tag{8}$$

$$u \rightarrow U_0, \quad v \rightarrow 0, \quad T \rightarrow 0 \quad \text{when } y \rightarrow \infty$$

where u and v are the velocity components along and perpendicular to the plate, μ is the viscosity of the fluid, ρ the density of the fluid, β the volumetric coefficient of thermal expansion, K the permeability of the porous medium (being a constant), g the field of gravity, k_0 the visco-elastic constant, λ the thermal diffusivity, C_p the specific heat at constant pressure, T the temperature variable, T_w the constant temperature at the plate, T_∞ the ambient temperature, q_r is the radiative heat flux and q''' is the rate of internal heat generation (>0). The second and fifth terms on the right hand side of the Equation (8) represent the effects of viscous dissipation and elastic deformation respectively.

The non-linear radiative heat flux may be expressed as

$$q_r = -\frac{4\sigma}{3k} \frac{\partial T}{\partial y} (T - T_\infty)$$

where σ and k are the Stefan Boltzmann constant and mean absorption coefficient respectively.

The term q''' is considered to be

$$q''' = \frac{ku}{\nu} \left[A^* (T - T_\infty) f'(\eta) + B^* (T - T_\infty) \right]$$

with k , the thermal conductivity, A^* and B^* are the parameters of space-dependent and temperature dependent internal heat generation ($A^* > 0, B^* > 0$) and absorption ($A^* < 0, B^* < 0$) respectively.

The temperature dependent fluid viscosity is given by $\mu = \mu^* e^{-A(T - T_\infty)}$, where μ^* is the constant viscosity away from the ambient of temperature field and A is the controlling coefficient.

Let us introduce the similarity variable

$$\eta = \frac{y}{\delta(x)}$$

and dimensionless variables f and θ as follows:

$$\psi = U_0 \delta(x) f(\xi, \eta); \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\xi(x) = g\beta \Delta T_0 (x - x_0) / U_0^2$$

where,

$$\Delta T_0 = T_w - T_\infty \quad \text{and} \quad \delta(x) = \left[2(x - x_0) \nu / U_0^2 \right]^{1/2}$$

With the use of above variables into the Equation (6) we obtain

$$u = \frac{\partial \psi}{\partial y} = U_0 \frac{\partial f}{\partial \eta}$$

and

$$v = - \left[U_0 f \frac{d\delta}{dx} + U_0 \delta \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} - \eta U_0 \frac{\partial f}{\partial \eta} \frac{d\delta}{dx} \right] \tag{9}$$

Introducing the above mentioned non-dimensional variables and u, v from (9) into the Equations (6) and (7) it is found that

$$\begin{aligned} & ff'' + f'''(1 - A_1\theta) - A_1 f''\theta' + \\ & \left[2f' \frac{\partial f}{\partial \xi} - K_0^* G_r \left(f'^2 - 2f'' \frac{\partial^3 f}{\partial \eta^2 \partial \xi} - ff'''' - 2f'''' \frac{\partial f}{\partial \xi} \right) \right. \\ & \left. + 2\theta - K_1^* f'(1 - A_1\theta) - \frac{M}{G_r} f' = 0 \right] \xi \end{aligned} \tag{10}$$

$$\begin{aligned} & 2\xi P_r f' \frac{\partial \theta}{\partial \xi} + P_r f \theta' - 2\xi P_r \frac{\partial f}{\partial \xi} + P_r \eta f' \theta' \\ & = \theta'' + P_r E_c \xi (1 - A_1\theta) f''^2 \\ & + K_0^* f'' \left[ff'' + ff'' + f'' \frac{\partial^2 f}{\partial \eta \partial \xi} + f'''' \frac{\partial f}{\partial \xi} \right] \\ & + S_1 (\theta'^2 + \theta\theta'') - \xi (A^* f' + B^* \theta) \end{aligned} \tag{11}$$

where

$$P_r = \frac{\mu C_p}{k} \quad (\text{Prandtl number});$$

$$E_c = \frac{U_0^2}{C_p \Delta T_0} \quad (\text{Eckert number});$$

$$G_r = \frac{g \beta \Delta T_0 \nu}{U_0} \quad (\text{Grashof number});$$

$$M = \frac{\sigma B_0^2 U_0}{\rho} \quad (\text{Hartmann number});$$

$$K_0 = k_0 g \beta U_0 / k \quad (\text{Viscoelastic parameter});$$

$$R = \frac{4\sigma \Delta T_0}{3k^2} \quad (\text{Thermal radiation parameter});$$

$$K_1^* = \frac{K}{1 - \gamma(T - T_\infty)}$$

$$\varepsilon = \gamma(T - T_\infty), \quad A_1 = A(T_w - T_\infty)$$

Since ξ is a small variable in magnitude so that we can expand the unknown functions in powers of ξ and thus

$$f(\xi, \eta) = f_0(\eta) + \xi f_1(\eta) + \xi^2 f_2(\eta) + \dots \quad (12)$$

$$\theta(\xi, \eta) = \theta_0(\eta) + \xi^2 \theta_2(\eta) + \dots \quad (13)$$

Using (12) and (13) into (10) and equating the like powers of ξ we get,

$$f_0 f_0'' + (1 - A_1 \theta_0) f_0''' - A_1 f_0'' \theta_0' = 0 \quad (14)$$

$$f_1'' f_0 + f_0'' f_1 + f_1''' (1 - A_1 \theta_0) - A_1 f_1'' \theta_0' + 2 f_1 f_0' + K_0^* G_r (2 f_0 f_1'' + 2 f_0^{iv} f_1 - f_0^{n2} + f_0 f_0^{iv}) \quad (15)$$

$$+ 2 \theta_0 - K_1^* (1 - \varepsilon \theta_0) f_0' - \frac{M}{G_r} f_0' = 0$$

$$(f_2'' f_0 + f_0'' f_2 + 2 f_1 f_1'') + f_2''' (1 - A_1 \theta_0) - A_1 f_2'' \theta_0' - A_1 (f_2'' \theta_0' + f_0'' \theta_2') + 2 (f_1 f_1' + 2 f_2 f_0') - K_0^* G_r (2 f_0 f_1'' + 2 f_0 f_2'' - f_1^{n2}) \quad (16)$$

$$- f_0 f_2^{iv} + f_2 f_0^{iv} + 2 f_1 f_1^{iv} - 2 f_2^{iv} f_1 - 4 f_2 f_0^{iv})$$

$$+ 2 \theta_2 - K_1^* \{ \varepsilon f_0' \theta_2' - (1 - \varepsilon \theta_2) f_1' \} - \frac{M}{G_r} f_1' = 0$$

Using (12) and (13) into (11) and equating the coefficients of ξ^0 and ξ^2 we get

$$\theta_0'' + P_r (f_1 \theta_0' + f_0 \theta_2') - P_r \eta f_0' \theta_0' + K_0^* [f_0' f_0'' + f_0 f_0^{n2} + 2 f_0^{n2} f_1' + 2 f_0^{n2} f_1] + R (\theta_0^{l2} + \theta_0 \theta_0'') = 0 \quad (17)$$

$$\begin{aligned} & \theta_2'' + P_r (f_1 \theta_0' + f_0 \theta_2') - 2 P_r \theta_2 f_0' + 2 P_r f_1 \\ & - P_r \eta (f_0' \theta_2' + \theta_0' f_1') + P_r E_c (1 - A_1 \theta_0) f_0^{n2} \\ & = + K_0^* [f_1' f_0^{n2} + 2 f_0' f_0'' f_1'' + 2 f_0^{n2} f_1 + f_0^{n2} f_0 f'' \\ & + 4 f_1' f_1'' f_0 + 4 f_2' f_0'' + f_1 f_1'' f_0'' + f_1 f_0'' f_1'' + 2 f_2 f_0'' f_0''] \\ & + R (\theta_0' \theta_2' + \theta_2 \theta_0'' + \theta_2'' \theta_0) - (A^* f_0' + A^* \theta_0') \end{aligned} \quad (18)$$

and the perturbed boundary conditions reduces to

$$f_0 = f_1 = f_2 = 0; f_0' = f_1' = 0; \theta_0 = 1, \theta_2 = 0 \text{ at } \eta = 0 \quad (19)$$

$$f_1' \rightarrow 1; f_1'' \rightarrow 0, f_2' \rightarrow 0; \theta_0 \rightarrow 0, \theta_2 \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (20)$$

Equations (14)-(18) are coupled and highly non-linear ordinary differential equations, finding exact solutions are not possible. So to get into the insight of the problem, above equations with the boundary conditions (19) and (20) are solved numerically with the use of finite difference scheme developed by Misra and Shit [25].

Another important studies of this investigation are two physical quantities such as local skin friction (C_f) and Nusselt number (Nu_x) defined by

$$C_f = \frac{2\tau_w}{\rho U_0^2} \quad \text{and} \quad Nu_x = \frac{x q_x}{k(T_w - T_\infty)}$$

where

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

is the wall shear stress and

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

is the surface heat flux. The non-dimensional form of these two quantities may be expressed as

$$C_f R_e^{1/2} = 2 f''(0) \quad \text{and} \quad Nu_x R_e^{1/2} = -\theta'(0)$$

3. Results and Discussions

The system of coupled non-linear ordinary differential Equations (14)-(18) subject to the boundary conditions (19) and (20) are solved numerically by using the finite difference scheme along with the Newton's linearization technique [25-27] to linearize the discretized equations. The essential features of this technique is that it is based on a finite difference scheme, which has better stability, simple, accurate and more efficient. Finite difference technique leads to a system which is tri-diagonal and therefore speedy convergence as well as economical memory space to store the coefficients.

Figure 1 gives the variation of f along with the distance from the plate. The variation of A_1 , the parameter which is associated with the viscosity function of

the temperature produce a substantial change in a single valued function f of η . It is observed that the values of f increases sharply in magnitude as the values of A_1 increases. In our problem the temperature dependent viscosity vary exponentially with temperature multiplied by a negative constant. The presented graphs are in the sense that as A_1 increases, the viscosity of the fluid decreases and hence resistance in the flow becomes weaker. It enhances the flow velocity in a substantial manner. Besides, velocity increases at a distance away from the point of commencing the horizontal velocity. This characteristic of velocity appears because the effect of temperature diminishes along with the distance from the plate.

To study the velocity of fluid along the plate for the effect of Grashof number is presented in **Figure 2**. From this figure we observed that the change is insignificant but the distinction can be made easily. Basic structure of the curves is similar and rectilinear. It is interesting to note that the velocity changes sharply with respect to the distance from the plate. Also the Grashof number reduces the friction then it attributed to increase the flow velocity in moderate magnitude.

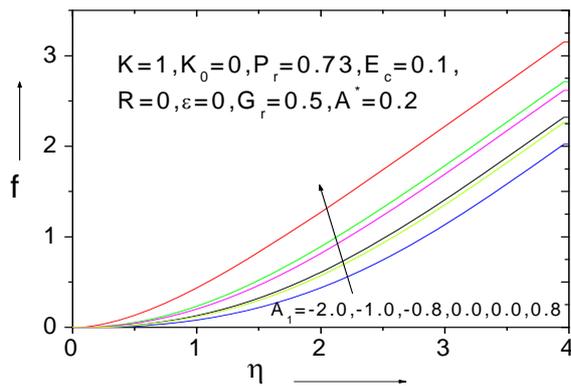


Figure 1. Variation of f with η for different values of A_1 .

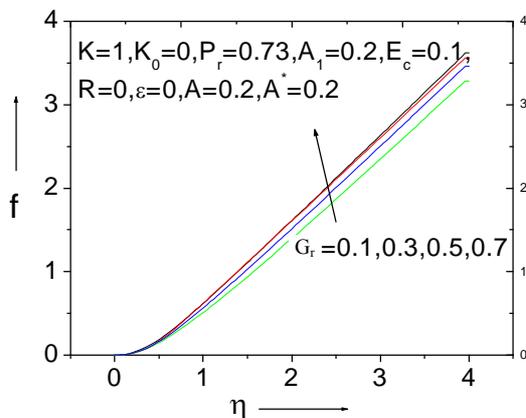


Figure 2. Variation of f with η for different values of G_r .

The temperature profile at different point of the plate along y -direction is shown in **Figure 3**. The effect of increasing the constant A_1 is prominent on the temperature variation. The temperature at some point on the plate increases with the decreasing values of A_1 . This reveals that the temperature distribution has adverse effect with the viscosity parameter. Thus for any values of A_1 the temperature distribution vanishes asymptotically. The appreciable change in temperature (θ) with Prandtl number is found to be presented in **Figure 4**. It is to be noted that the Prandtl numbers have profound impression in a certain neighborhood of the point of commencement of flow along the horizontal y -direction. This is also expected because the Prandtl number is associated with the convective part of the heat transfer equation and the effect of this part on temperature distribution becomes weaker as the fluid moves on the plate for a low Reynolds number flow. **Figure 5** represents the temperature variation along the horizontal plate for different radiation parameter. It is observed that the temperature decreases sharply with the increasing distance away from the plate. The sharpness increases with the increasing values of R . It is interesting to note from this figure that as heat radiates into the fluid, the temperature increases immediately with a short distance which is quite natural in heat trans-

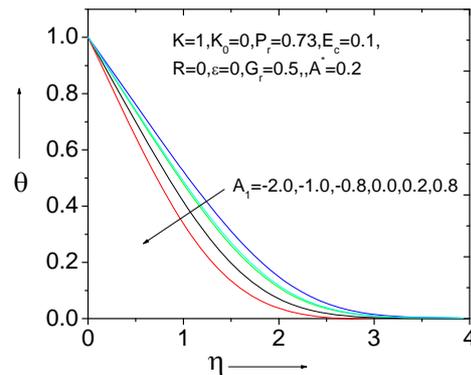


Figure 3. Variation of θ with η for different values of A_1 .

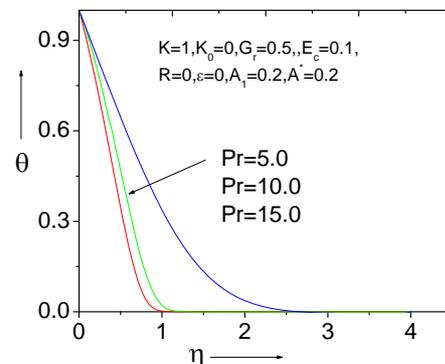


Figure 4. Variation of θ with η for different values of Pr .

fer can be obtained in Chen [28]. The velocity gradient increases with the increase of the viscosity parameter shown in **Figure 6**. At some fixed point on the horizontal plate temperature increases with the increasing values of the parameter A_1 . This figure also depicts that the variation of viscosity has significant effect on the velocity gradient. Thus, if viscosity decreases then velocity gradient as well as motion of fluid accelerated. A comparison has also been made with Takhar *et al.* [23]. **Figure 7** represents the variation of f_1' with the viscosity parameter A_1 . Comparing two **Figures 6** and **7** it is observed that both the figures have same attitude *i.e.* f_1' and f_1' both decrease with the increasing values of A_1 . For $A_1 = 0$, graphs of **Figures 6** and **7** are totally different. In **Figure 7**, it may predict that for $A_1 = -7.0$ or -8.0 , f_1' becomes zero which is absent in **Figure 6**. This characteristic appears due to the boundary conditions used to solve for f_0' and f_1' .

Figure 8 is an interesting figure as this predicts a critical point on the plate after which back flow occur. This figure shows that the velocity gradient f_1' has positive

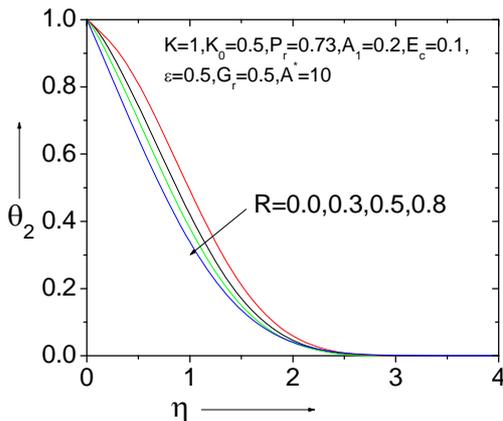


Figure 5. Temperature profile for various values of R .

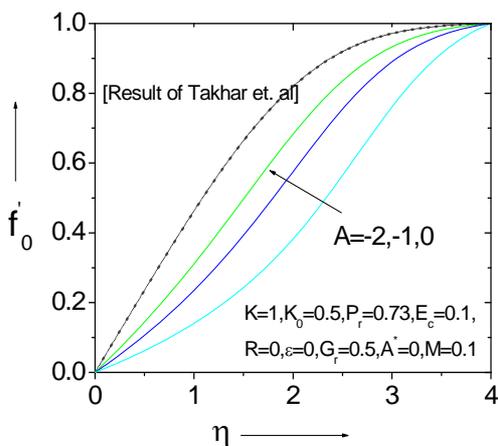


Figure 6. First order velocity distribution for various A_1 .

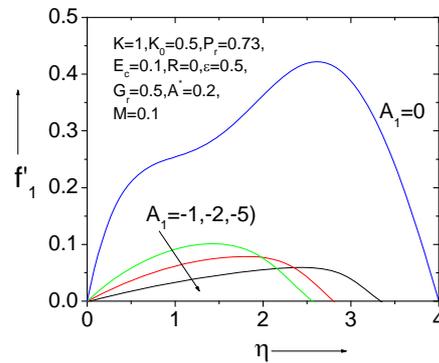


Figure 7. Second order velocity distribution for various A_1 .

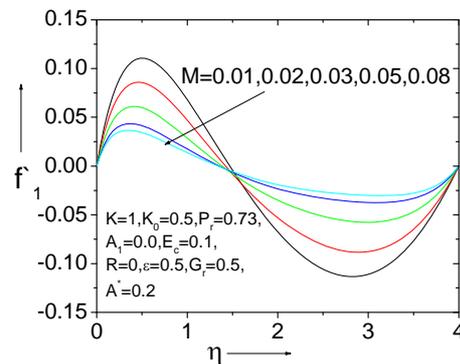


Figure 8. Second order velocity distribution for different values of M .

and negative values before and after $\eta = 1.5$.

It is worthwhile to mention here that f_1' decreases with the increasing values of M . Thus it may conclude that the influence of magnetic field on the velocity gradient is predominantly vanishes after certain distance from the plate.

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In the present study we have considered a flow past a porous vertical plate where the permeability is a function of θ . **Figure 9** gives the distribution of second order temperature for various permeability constants. The structure of this figure is similar for positive and negative values of ε but the trend is reversed in the case of its values in magnitude. It is found that the effect of ε is more prominent in the vicinity of the vertical wall and it is nearly same for $\eta = 1.0$ onwards. Therefore heating and cooling both are possible by controlling the values of

ε . The Grashof number plays an important role on the flow and heat transfer of a viscoelastic fluid. To interpret the effect of Gr on second order velocity gradient the **Figure 10** is being presented. This observation illustrates that f_1' decreases slowly with the decreasing values of Gr. Also the values of f_1' cannot become zero at any point on the horizontal plate as well as for any value of Gr. The second order temperature distribution with η for different values of G_r has been presented in **Figure 11**. For the same values of G_r , there is always cooling and heating in the fluid before and after the plate positioned at $\eta = 0.5$.

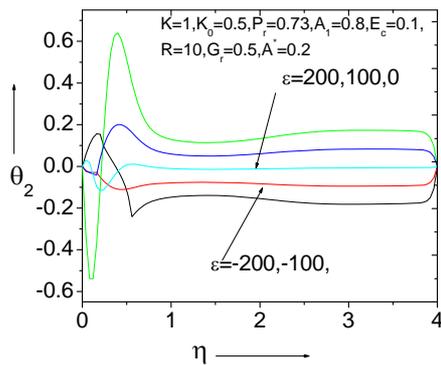


Figure 9. Second order velocity distribution for various M .

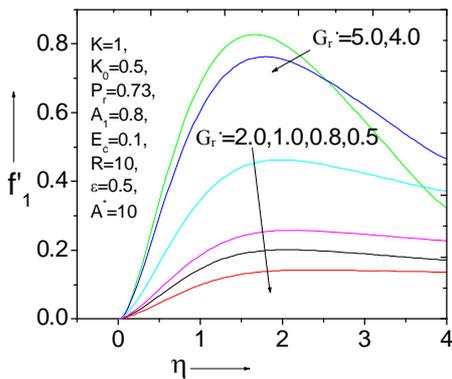


Figure 10. Second order velocity for various G_r .

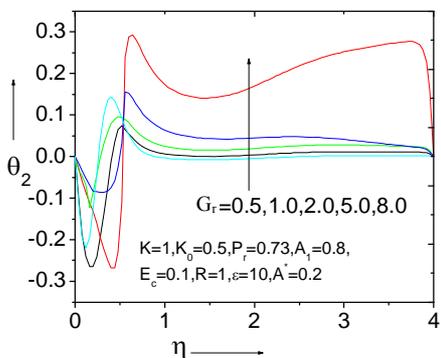


Figure 11. Second order temperature profile for various G_r .

Figure 12 gives the variation of second order velocity component for different permeability parameter K . It is seen that f_1 decrease with the increasing values of K . Here the permeability of the fluid effectively decreases the flow velocity. It is interesting to mention here that the fluid velocity can never be zero as we advance along the plate. **Figure 13** depicts the change in f_1 for various values of the viscoelastic parameter K_0 . With the increasing values of K_0 back flow becomes more prominent. It is observed that at the beginning of motion flow appears to be positive; however, with the advancement of the flow over the horizontal plate back flow appears. Therefore the viscoelastic parameter retards the motion significantly. To furnish the influence of source and sink parameter A^* on θ_2 , we presented it in **Figure 14**. This parameter has mild effect on the second order temperature distribution, however in the neighborhood of the vertical plate, the magnitude of θ_2 has substantial distinction between different curves. Also, for the critical value $\eta = 1.0$, θ_2 becomes negative *i.e.* there is cooling in the fluid which is an important characteristic. Thus, for positive source constant, A^* fluid may lose its heat after crossing over the critical distance.

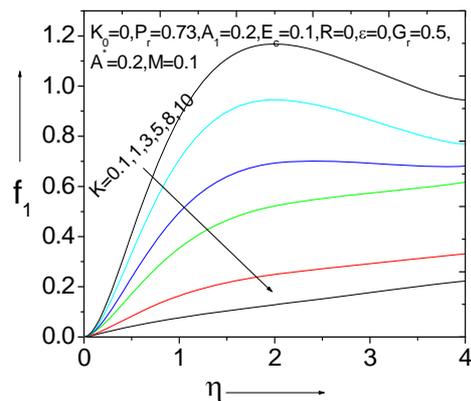


Figure 12. Variation of f_1 for various values of K .

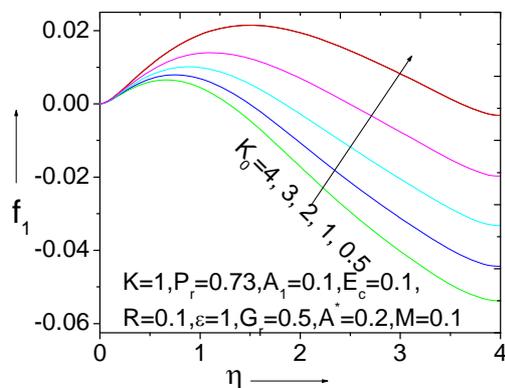


Figure 13. Variation of f_1 for various K_0 .

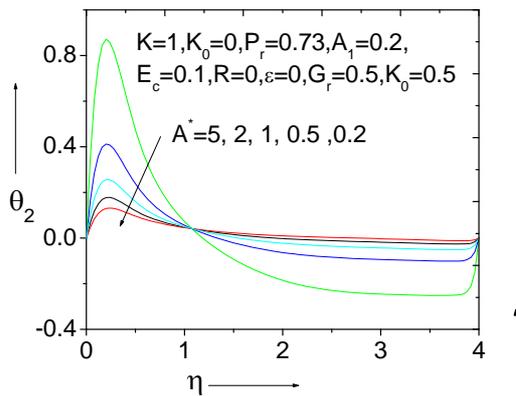


Figure 14. Second order temperature profile for different A^* .

From **Table 1** it is found that the zeroth order and first order skin-friction varies significantly with various viscosity parameter. As the exponential coefficient of viscosity increase when other parameters are fixed, the magnitude of frictional resistance increase on the plate. Also it has decreasing effect for the increasing values of magnetic parameter because this parameter creates more resistance on the flow. **Table 2** illustrates the variation of

Nusselt number for different values of the viscosity parameter A_1 . We observed that the rate temperature distribution in the fluid increases with the increasing values of viscosity parameter. This is due to the fact that the viscosity parameter depends on temperature. Moreover, the magnitude of local Nusselt number decreases with the increasing of radiation parameter.

4. Conclusion

In the present paper a numerical investigation has been conducted for the flow of visco-elastic fluid past a porous vertical plate in the presence of transverse magnetic field. Results are presented graphically for different non-dimensional parameters such as M the magnetic parameter (Hartmann number), G_r the Grashof number, E_c the Eckert number, A_1 the constant appeared in the exponentially variable viscosity, R the radiation parameter, A^* the source parameter, K_0 the visco-elastic parameter and K the permeability parameter. Using perturbation technique the momentum and energy equations are reduced to a coupled non-linear ordinary differential equation and those are solved by finite difference method by already developed scheme. Results extracted from this

Table 1. Values of skin-friction coefficient $f_0''(0)$, $f_1''(0)$ for different values of viscosity parameter A and magnetic parameter M with $K = 1.0$, $K_0 = 0.5$, $E_c = 0.1$, $R = 0.0$, $\epsilon = 0.0$, $G_r = 0.5$, $A^* = 0.2$, $M = 0.2$.

A_1	-5.0		-3.0		0.0		0.2	
	$f_0''(0)$	$f_1''(0)$	$f_0''(0)$	$f_1''(0)$	$f_0''(0)$	$f_1''(0)$	$f_0''(0)$	$f_1''(0)$
0.1	0.8341	1.4520	1.0034	3.2981	6.8743	9.2897	11.4532	13.0943
0.5	0.7234	1.0756	1.3420	2.4861	4.2091	7.9824	11.0126	13.0018
0.7	0.6213	1.0002	1.0217	1.8749	3.6849	6.4320	9.9682	11.9828
0.9	0.5103	0.8321	0.9092	1.0502	2.4703	5.6027	7.2581	10.3904
1.2	0.4579	0.7480	0.7769	0.8062	1.6729	4.6802	6.0361	8.3096
1.5	0.2495	0.6103	0.5302	0.5481	0.7302	2.6819	5.0318	6.9906

Table 2. Numerical values of the local Nusselt number $\theta_0'(0)$, $\theta_2'(0)$ for different viscosity parameter A_1 and magnetic parameter M . $K = 1.0$, $K_0 = 0.5$, $E_c = 0.1$, $R = 0.0$, $\epsilon = 0.0$, $G_r = 0.5$, $A^* = 0.2$, $M = 0.2$.

A_1	-5.0		-3.0		0.0		0.2	
	$\theta_0'(0)$	$\theta_2'(0)$	$\theta_0'(0)$	$\theta_2'(0)$	$\theta_0'(0)$	$\theta_2'(0)$	$\theta_0'(0)$	$\theta_2'(0)$
0.1	0.3591	2.5020	1.7310	3.2082	4.0835	6.0807	8.0391	10.021
0.3	0.3109	2.0031	1.1200	2.9080	3.6117	5.8024	7.0816	9.0826
0.9	0.2713	1.5514	0.8279	1.7999	3.0010	5.1320	6.0321	7.9397
1.5	0.1030	0.9831	0.2036	0.8593	1.7903	4.2731	5.5081	6.4501
2.3	0.0709	0.3804	0.1005	0.3362	0.2419	2.0255	3.6091	4.0906
2.8	0.0125	0.1038	0.0502	0.0998	0.07677	0.9512	1.0085	1.9862

study concluded that the temperature distribution as well as fluid velocity substantially depends on variable viscosity parameter which is the function of temperature. Another important feature is the paramount effect of Grashof number on the temperature and velocity. Heat radiation into the fluid enhances the temperature significantly in this study.

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