

# Visco-Elastic Boundary Layer Flow past a Stretching Plate and Heat Transfer with Variable Thermal Conductivity

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## Abstract

In the present paper, the boundary layer flow of Walters Liquid B Model over a stretching plate has been considered to solve heat flow problem with variable conductivity. First, using similarity transformation, the velocity components have been obtained. Then, the heat flow problem has been considered in two ways: 1) prescribed surface temperature (PST), and 2) prescribed stretching plate heat flux (PHF) in case of variable conductivity. Due to variable conductivity, temperature profile has its two part- one mean temperature and other temperature profile induced due to variable conductivity. The related results have been discussed with the help of graphs.

**Keywords:** Boundary Layer Flow, Similarity Solution, Variable Thermal Conductivity, Prandtl Number

## 1. Introduction

Due to number of applications in industrial manufacturing process, the problem of boundary layer flow past a stretching plate has attracted considerable attention of researchers during the past few decades. Examples of such technological process are hot rolling, wire drawing, glass-fiber and paper production. In the process of drawing artificial fibers the polymer solution emerges from orifice with a speed which increases from almost zero at the orifice up to a plateau value at which it remains constant. The moving fiber, which is of great technical importance, is governed by the rate at which the fiber is cooled and this, in turn affects the final properties of the yarn. A number of works are presently available that follow the pioneering classical work of Sakiadis [1], F. K. Tsou, E. M. Sparrow, R. J. Goldstein [2] and Crane[3]. The following Table lists some relevant works that pertain to cooling liquids, *i.e.*, heat transfer for stretching surface:

Author/s	Type of vis-co-elastic fluid	Remarks
K. R. Rajagopal, T. Y. Na, A. S. Gupta [4]	Second order fluid	Not heat transfer
K. R. Rajagopal, T. Y. Na, A. S. Gupta [5]	Second order fluid	Not heat transfer
N. M. Bujurke, S. N. Biradar, P. S. Hiremath [6]	Second order fluid	Heat transfer

B. S. Dhanpat, A. S. Gupta [7]	Second order fluid	Heat transfer
N. Ahmad, G. S. Patel and B. Siddappa [8]	Walter's liquid B	Heat transfer
D. Rollins, K. Vajravelu [9]	Second order fluid	Heat transfer
N. Ahmad, G. S. Patel, B. Siddappa [10]	Walter's liquid B	Heat transfer
S. P. Lawrence, N. B. Roa [11]	Second order fluid	Heat transfer
M. I. Char [12]	Second order fluid	Heat transfer
A. Naseem [13]	Walter's liquid B	Heat transfer
N. Ahmad [14]	Walter's liquid B	Heat transfer
D. Kelly, K. Vjavelu, L. Andrews [15]	Walters' liquid B	Heat transfer
N. Ahmad, K. Marwah [16]	Walter's liquid B	Heat transfer
N. Ahmad, K. Marwah [17]	Walter's liquid B	Heat transfer
A. M. subhas, A. Joshi, R. M. Sonth [18]	Walter's liquid B	Heat transfer
R. M. Sonth, S. K. Khan, A. M. Subhas [19]	Walter's liquid B	Heat transfer
Siddheshwar, Mahabaleswar [20]	Walter's liquid B	Heat transfer
M. Subhas Abel, P. G. Siddheshwar, Mahantesh M. Nandeppanavar [21]	Walter's liquid B	Heat transfer

There are liquid metals whose thermal conductivity varies with temperature in an approximately linear manner in the range from 0° F to 400° F. In 1996, T. C. Chiam [22] considered heat transfer problem with variable thermal conductivity in stagnation-point flow towards stretching sheet. N. Ahmad and K. Marwah [16,17] also studied boundary layer flow of Walters Liquid B Model with heat transfer for linear stretching plate with variable thermal conductivity numerically.

In almost all the flow problems over stretching sheet with heat transfer referred so far where closed form solution is obtained, the thermal conductivity of liquid has been taken constant. In this paper, we try to solve the boundary layer flow of Walters Liquid B Model over a stretching plate and heat transfer with variable thermal conductivity in the following two cases:

1) prescribed surface temperature (PST), and 2) prescribed stretching plate heat flux (PHF). The temperature

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial x^2} - k_0 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\} \tag{2}$$

where  $u$ , the horizontal velocity component;  $v$ , the vertical velocity component;  $\nu$ , the kinematic viscosity;  $k_0$ , the coefficient of visco-elasticity The relevant boundary conditions are:

$$\begin{aligned} y = 0, u = u_w = mx, v = 0, m > 0 \\ y \rightarrow \infty, u = 0 \end{aligned}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - k_1 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\} \tag{4}$$

where  $k_1 = \frac{k_0}{h^2}$ ,

with boundary conditions

$$\begin{aligned} y = 0, u = mx, v = 0, m > 0 \\ y \rightarrow \infty, u = 0 \end{aligned} \tag{5}$$

$$f'^2(y) - f''(y)f'(y) = \frac{1}{m} f'''(y) - k_1 \{ 2f'(y)f'''(y) - f(y)f^{iv}(y) - f''^2(y) \} \tag{7}$$

which is non-linear differential equation of order three.

The boundary conditions (5) reduce to

$$y = 0, f' = 1, f = 0 \tag{8}$$

$$y \rightarrow \infty, f' = 0 \tag{9}$$

Boundary conditions suggest that the velocity function may be of the form  $f'(y) = e^{-ry}$  where  $r$  is complex number with positive real part. Thus,

$$v = -\frac{m}{r} (1 - e^{-ry})$$

field has been obtained in each case and the effect on it, of Visco-elasticity and Prandtl number has been discussed graphically.

## 2. Mathematical Formation and Solution

The problem considered here is the steady boundary layer flow due to a moving flat plate in a quiescent Walters Liquid B Model. The flow is two dimensional where x-axis is along the plane of moving plate and y-axis is normal to it, respectively. We assume that the surface is moving continuously with the velocity  $u_w = U_w(x)$  in the positive x-direction. Under these assumptions, the boundary layer along moving plate is governed by the equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Introducing the dimensionless variables

$$\bar{y} = \frac{y}{h}, \bar{u} = \frac{uh}{\nu}, \bar{x} = \frac{x}{h}, \bar{v} = \frac{vh}{\nu}$$

the Equations (1) and (2) reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

where bar has been dropped for convenience.

Setting the similarity solution of the form  $u = mx f'(y)$ , we have

$$v = -mf'(y) \tag{6}$$

Putting  $u$  and  $v$  in the Equation (2), we have

Now, from the Equation (7), we get  $r = \frac{1}{\sqrt{\frac{1}{m} - k_1}}$ .

Therefore, the velocity components become as follows:

$$u = mx e^{-ry} \text{ and } v = -\frac{m}{r} (1 - e^{-ry}) \tag{8}$$

## 3. Heat Transfer Problem

In absence of viscous dissipation and heat generation, the

energy equation for two dimensional heat flow is given by

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) \quad (9)$$

subject to boundary conditions

$$\begin{aligned} y = 0, T &= T_p \\ y \rightarrow \infty, T &= T_\infty \end{aligned} \quad (10)$$

where  $T_p$  is plate temperature,  $T_\infty$  is temperature of surrounding fluid,  $C_p$  is specific heat at constant pressure and  $k$  is thermal conductivity.

### 3.1. Case A: Prescribed Power Law Surface Temperature (PST)

Let the surface temperature be of the form

$$y = 0, T = T_p = T_\infty + A \left( \frac{x}{l} \right)^2$$

while the temperature out side the dynamic region be  $y \rightarrow \infty, T = T_\infty$ . Now, we define the dimensionless temperature by

$$\theta(\eta) = \frac{T - T_\infty}{T_p - T_\infty}$$

where  $\eta = ry$ .

For liquid metals, it has been found that the thermal conductivity varies with temperature in an approximately linear manner in the range from  $0^\circ$  F to  $400^\circ$  F. Therefore, we assume  $k$  as  $k = k_\infty (1 + \varepsilon\theta)$  where  $\varepsilon = \frac{k_p - k_\infty}{k_\infty}$ .

Now, substituting  $u$  and  $v$  and changing the independent variable  $y$  to  $\eta = ry$ , we have

$$\theta'' + \frac{mPr}{r^2} (1 - e^{-\eta}) \theta' + \varepsilon (\theta\theta'' + \theta'^2) = 0 \quad (11)$$

with boundary conditions

$$\eta = 0, \theta = 1 \quad (12)$$

$$\eta \rightarrow \infty, \theta = 0 \quad (13)$$

From Equation (11), we note that the heat transfer takes place in two parts, that is, one part of heat transfer is due to temperature difference and the other part is due to variable thermal conductivity. We denote the first part by  $\theta_m$  and second by  $\theta_v$ . Thus, equating the terms independent of  $\varepsilon$  and the terms involving  $\varepsilon$ , we have

$$\theta_m'' + \frac{mPr}{r^2} (1 - e^{-\eta}) \theta_m' = 0 \quad (14)$$

$$\theta_m(0) = 1, \theta_m \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (15)$$

and

$$\theta_v \theta_v'' + \theta_v'^2 = 0 \quad (16)$$

$$\theta_v(0) = 1, \theta_v \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (17)$$

The solution of the Equation (14) is

$$\theta_m = \frac{\int_0^\eta \exp \left\{ -\frac{mPr}{r^2} (\eta + e^{-\eta}) \right\} d\eta}{\int_0^\infty \exp \left\{ -\frac{mPr}{r^2} (\eta + e^{-\eta}) \right\} d\eta} \quad (18)$$

Equation (17) is a non-linear differential equation of order two. Let the solution of this equation be of the form

$$\theta_v(\eta) = (1 - \eta)^\alpha$$

Putting this solution in the Equation (18), we have

$$2\alpha^2 - \alpha = 0 \quad (19)$$

The roots of this equation are 0 and 1/2. Therefore,

$$\theta_v(\eta) = A + B\sqrt{1 - \eta} \quad (20)$$

The general solution (20) of the Equation (16) is real only when  $0 \leq \eta \leq 1$ . Therefore, the heat transfer due to variable thermal conductivity takes place within the dynamic region  $0 \leq \eta \leq 1$ . Hence, the boundary conditions (17) may be presented as

$$\begin{aligned} \eta = 0, \theta_v &= 1 \\ \eta \rightarrow 1, \theta_v &\rightarrow 0 \end{aligned} \quad (21)$$

The solution (20) finally reduces to

$$\theta_v = (1 - \eta)^{\frac{1}{2}} \quad \forall \eta \in [0, 1[ \quad (22)$$

### 3.2. Case B: Prescribed Power Law Surface Heat Flux (PHF Case)

The power law heat flux on the surface of stretching plate is considered to be a quadratic power of  $x$  in the form

$$-k \frac{\partial T}{\partial y} = D \left( \frac{x}{l} \right)^2 \text{ at } y = 0 \quad (23)$$

$$T \rightarrow T_\infty, \text{ as } y \rightarrow \infty \quad (24)$$

where  $D$  is a constant,  $k$  is the thermal conductivity. Now we define dimensionless temperature  $g(\eta)$  by

$$g(\eta) = \frac{T - T_\infty}{T_p - T_\infty}, \quad (25)$$

where

$$T_p - T_\infty = \frac{D}{kr} \left( \frac{x}{l} \right)^2 \quad \text{when } g'(0) = -1$$

and  $\eta = ry$ .

Writing the Equation (11) in terms of  $g(\eta)$ , we get the following differential equation

$$g''(\eta) + \frac{m}{r^2} \text{Pr}(1 - e^{-\eta})g' + \epsilon (gg'' + g'^2) = 0 \quad (26)$$

together with boundary conditions:

$$\begin{aligned} g' &= -1 \text{ at } \eta = 0 \\ g &\rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \quad (27)$$

Equating the terms independent of  $\epsilon$  and the terms involving  $\epsilon$  from Equation (26), we get the following two boundary value problems:

$$g_m''(\eta) + \frac{m}{r^2} \text{Pr}(1 - e^{-\eta})g_m'(\eta) = 0 \quad (28a)$$

$$g_m'(0) = -1, \quad g_m \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (28b)$$

and,

$$g_v(\eta)g_v''(\eta) + g_v'^2(\eta) = 0 \quad (29a)$$

$$g_v'(0) = -1, \quad g_v \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (29b)$$

The solution of the Equation (28a) together with boundary conditions (28b) is given by

$$g_m(\eta) = -e^{r^2 \text{Pr} \int_0^\eta \exp\left(-\frac{m}{r^2} \text{Pr}(\eta + e^{-\eta})\right) d\eta} \quad (30)$$

The general solution of the Equation (29a) is

$$g_v(\eta) = C + D\sqrt{1-\eta} \quad (31)$$

where  $C$  and  $D$  are arbitrary constants to be determined. Here, we observe that the dynamic region for this temperature field is  $0 \leq \eta \leq 1$  as it was in PST case. Therefore, the boundary conditions may be taken as

$$\begin{aligned} g_v' &= -1 \text{ at } \eta = 0 \\ g_v &\rightarrow 0 \text{ as } \eta \rightarrow 1 \end{aligned} \quad (32)$$

Hence finally the solution becomes

$$g_v(\eta) = 2\sqrt{1-\eta} \quad (32)$$

### 4. Discussion and Results

A boundary layer flow of visco-elastic fluid (Walters Liquid B Model) over a stretching plate and heat transfer has been analyzed through out this paper. The boundary layer equations of momentum and heat transfer have been solved analytically. Two different analytical expressions have been obtained for dimensionless tem-

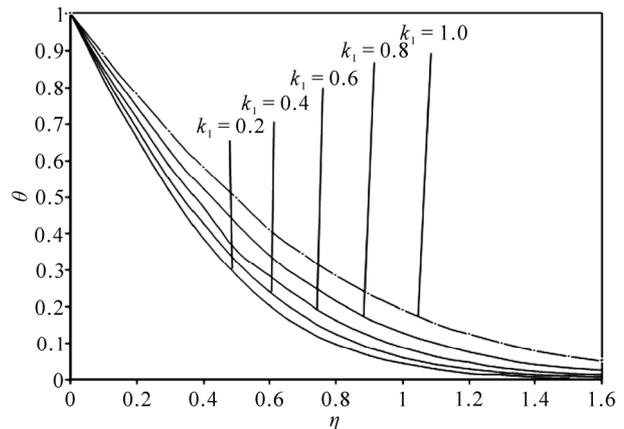
perature fields for two general cases of boundary conditions namely 1) PST case, and 2) PHF respectively. The equation governing the induced temperature field due to variable thermal conductivity has been solved with modified boundary conditions in both the cases PST and PHF. The related results have been discussed in the subsequent paragraphs in detail.

**Figure 1** is a graph drawn for mean temperature field  $\theta_m(\eta)$  versus  $\eta$  in PST case taking different values of visco-elastic parameter  $k_0$ . It is observed that the temperature  $\theta_m(\eta)$  at the surface of stretching plate is invariant with respect to physical parameters. Further, we observe that as we move away the stretching plate within dynamic region, the temperature field increases as visco-elastic parameter  $k_0$  increases. Physically, when  $k_0$  increases, the fluid absorbs more heat which causes the increase of temperature.

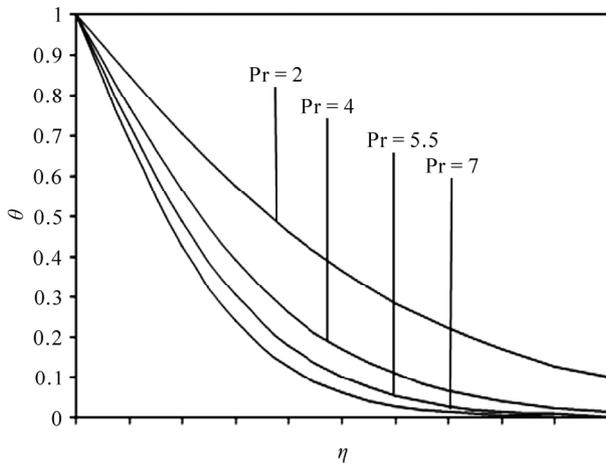
**Figure 2** exhibits variation pattern of mean temperature field  $q_m(h)$  with regard to Prandtl number Pr in PST case. We notice that as Prandtl number Pr increases, the mean temperature decreases.

**Figure 3** exhibits the variation pattern of induced temperature field  $q_v$  in PST case. This temperature field is independent of  $k_0$  and Pr. We see that the induced temperature decreases as we move away the plate. The variation pattern is represented by a parabolic curve having focus at  $h = 1$ .  $q_v$  decreases as we move away the stretching plate and it becomes almost zero at  $h = 1$  which end upper edge of boundary layer thickness.

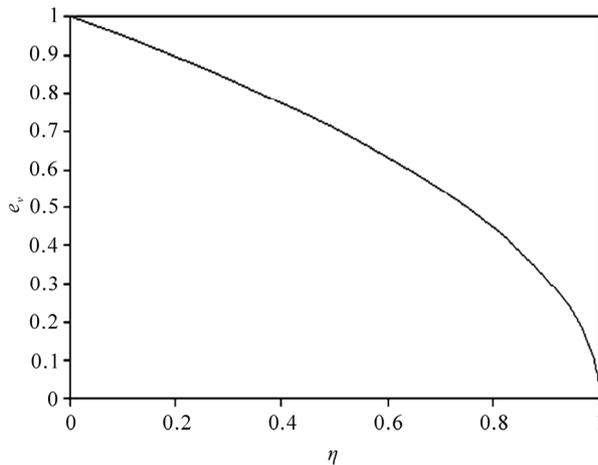
**Figure 4** is the graph of mean temperature field  $g_m(\eta)$  versus  $\eta$  in PHF case. Here we observe that this temperature field attains the maximum value at the surface of stretching plate which is more than PST case. The reason of this maximum value is that the direction of heat flow is from fluid to stretching plate. Temperature field is approaching to zero asymptotically. The variation pattern is almost same as in PST case.



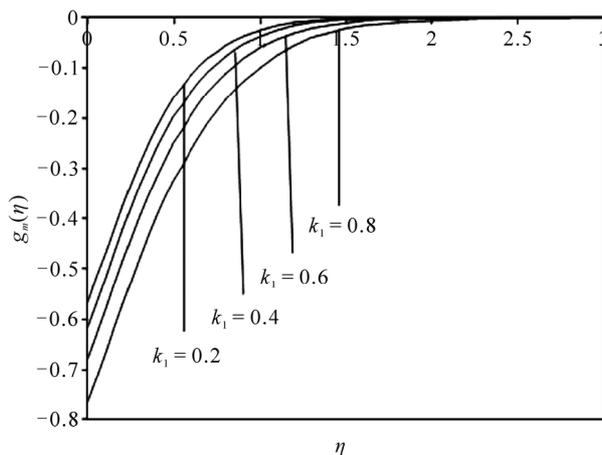
**Figure 1.** Mean temperature  $\theta_m(\eta)$  for different values of visco-elastic parameter  $k_1$  in PST case.



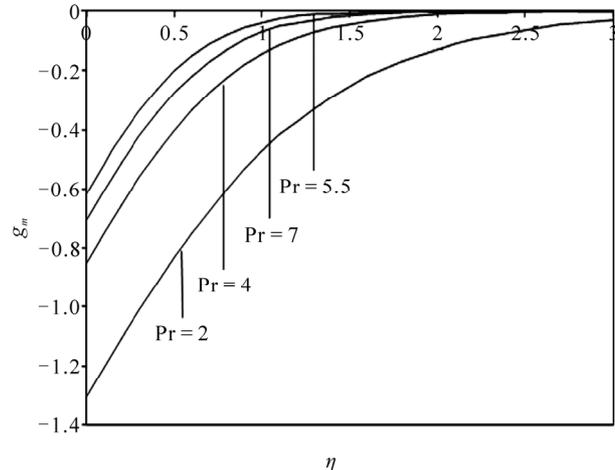
**Figure 2.** Effect of Prandtl number  $Pr$  mean temperature field  $\theta_m(\eta)$  keeping visco-elasticity  $k_1 = 0.4$  fixed in PST case.



**Figure 3.** Variation pattern of  $\theta$ , within dynamic region  $[0,1]$  PST case.



**Figure 4.** Mean temperature  $g_m(\eta)$  for different values of visco-elastic parameter  $k_1$  in PHF case.



**Figure 5.** variation pattern of mean temperature  $g_m(\eta)$  for different values of Prandtl number  $Pr$  for  $k_1 = 0.4$  in PHF case.

**Figure 5** shows the graph of  $g_m(\eta)$  versus  $\eta$  for the different values of Prandtl number  $Pr$  in PHF case. It has been observed that as  $Pr$  increases, temperature field decreases absolutely. Temperature field is tending to zero asymptotically.

Equation (32) represents the temperature field induced by variable thermal conductivity in PHF case. The geometry of this equation is parabola having focus at  $h = 1$ . This temperature field is independent of  $k_1$  and  $Pr$ . We see that the induced temperature decreases as we move away the plate. It varies from maximum at the plate to zero at the upper end of boundary layer.

Putting  $k_0 = 0$ , we get the results due to Naseem Ahmad, Z. U. Siddiqui and M. K. Mishra [23].

### 5. Conclusion

The boundary layer flow of Walters Liquid B over linear stretching plate has been studied together with heat transfer with variable thermal conductivity. The heat transfer has been studied by considering two cases: PST and PHS. The effect of visco-elasticity  $k_0$  has been looked upon in the **Figures 1** and **2**. This paper is an extension of the work due to Naseem Ahmad, Z. U. Siddiqui and M. K. Mishra [23].

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