Study on Productivity Model of Herringbone-Like Laterals Wells and Optimization of Morphological Parameters Considering Threshold Pressure Gradient in Heavy Oil Reservoirs

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Abstract
Compared with conventional well, herringbone-like laterals wells can increase the area of oil release, and can reduce the number of wellhead slots of platforms, and also can greatly improve the development efficiency. Based on threshold pressure gradient in heavy oil reservoir, and the applied principle of mirror reflection and superposition, the pressure distribution equation of herringbone-like laterals wells is obtained in heavy oil reservoir. Productivity model of herringbone-like laterals wells is proposed by reservoir-wellbore steady seepage. The example shows that the productivity model is great accuracy to predict the productivity of herringbone-like laterals wells. The model is used to analyze the branching length, branching angle, branching symmetry, branching position and spacing and their effects on productivity of herringbone-like laterals wells. The principle of optimizing the well shape of herringbone-like laterals wells is proposed.

Keywords
Threshold Pressure Gradient, Herringbone-Like Laterals Wells, Heavy Oil Reservoirs, Productivity Model, Optimization of Morphological Parameters

1. Introduction
Compared with the onshore oilfields, the number of wellhead slots is limited in offshore oilfields. The herringbone-like laterals wells not only increase the drainage area and single well-controlled reserves, but also increase the production of
oil wells; it has certain advantages in offshore oilfield development. Therefore, it is necessary to study the productivity prediction of herringbone-like laterals wells in reservoirs. In 1996, Salas [1] assumed each branch is divided into several segments and established analytical model of single-phase seepage in fishbone multi-branch horizontal wells. In 2004, Han Guoqin [2] and He Haifeng [3] established a steady seepage mathematical model of fishbone multi-branch wells considering the flow in the wellbore. Liu Xiangping, Yang Xiaosong, Zhao Guang, et al. [4] [5] [6] [7] [8] deduced the productivity calculation model of fishbone horizontal well by potential superposition principle. Based on the principle of equivalent seepage resistance and productivity formula of horizontal wells, a simple productivity formula for fishbone horizontal wells is derived by Li Chunlan [9]. Fan Yuping, Ye Shuangjiang, Zhang Shiming, et al. [10] [11] [12] used numerical simulation method to study the productivity of herringbone-like laterals wells. Because heavy oil is affected by threshold pressure gradient, the above research is not suitable for predicting the productivity of wells in heavy oil reservoirs. Considering the influence of threshold pressure gradient on heavy oil reservoir, herringbone-like laterals wells coupling productivity model for reservoir-wellbore steady seepage is proposed; the example shows that the productivity model is high accuracy in predicting the productivity of herringbone-like laterals wells, and the principle of well shape optimization for herringbone-like laterals wells is proposed.

2. Pressure Distribution Considering Threshold Pressure Gradient

In reference [13], porous media conditions, heavy oil reservoir seepage law is non-Darcy seepage with threshold pressure gradient. When the driving pressure gradient exceeds its initial pressure gradient, heavy oil begins to flow and its seepage characteristics are shown in Figure 1. Among them, $A$ is the minimum threshold pressure gradient, $B$ is the average threshold pressure gradient; $C$ is the maximum threshold pressure gradient.

The law of non-Darcy seepage can be described by the following formula:
\[ v = -\frac{K}{\mu} \left( \frac{dp}{dr} - \lambda \right), \quad \frac{dp}{dr} > \lambda \]  

(1)

where \( v \) is seepage velocity, \( m^3/s \); \( K \) is reservoir permeability, \( mD \); \( \frac{dp}{dr} \) is pressure gradient, \( MPa/m \); \( \lambda \) is threshold pressure gradient, \( MPa/m \).

when \( \Phi = \frac{K}{\mu} p \), have:

\[ v = -\frac{K}{\mu} \frac{d\Phi}{dr} + \lambda \frac{K}{\mu} \]  

(2)

Assuming that the formation is infinitely homogeneous and isotropic, there is a horizontal well in which the length of the horizontal well is \( L \), the coordinates of the two ends of the well are \((x_1, 0, z_w)\), \((x_1, 0, z_w)\), setting the horizontal well is homogeneous line sink, the productivity of the well is \( Q \), the productivity of per unit length \( L \) is \( q \), selecting micro-element \( dx_0 \) at \( x_0 \) of horizontal well. It can be regarded as the seepage velocity of \( M(x_0, y_0, z_0) \) at any point:

\[ v = \frac{q}{4\pi r^2} \]  

(3)

A new velocity potential function considering threshold pressure gradient at point \( M \) is obtained:

\[ d\Phi = -\frac{Q}{4\pi L} \frac{dx_0}{\sqrt{(x_0-x)^2 + (y_0-y)^2 + (z_0-z)^2}} + \lambda \frac{K}{\mu} R_D + C \]  

(4)

where \( R_D \) is the shortest distance between the micro-element and the moving boundary.

According to the superposition principle of potential, the velocity potential caused by the whole horizontal well is as follows:

\[ \Phi = \int_{x_1}^{x_0} -\frac{Q}{4\pi L} \frac{dx_0}{\sqrt{(x_0-x)^2 + (y_0-y)^2 + (z_0-z)^2}} + \lambda \frac{K}{\mu} R_D + C \]  

(5)

In this paper, a horizontal well is divided into \( N \) segments by mathematical discretization method. As the productivity of the well varies little in a small section, it can be assumed that the productivity of this section is a fixed value, and the production of each section is different:

\[ \Phi = \sum_{i=0}^{N-1} \left( -\frac{Q}{4\pi L} \frac{dx_0}{\sqrt{(x_0-x)^2 + (y_0-y)^2 + (z_0-z)^2}} \right) + \lambda \frac{K}{\mu} R_D + C' \]  

(6)

As the horizontal well is parallel to the X-axis and on the XOY plane, so \( y_0 = y, \quad z_0 = z_w \), the Formula (6) is changed to:

\[ \Phi(M) = \sum_{i=0}^{N-1} \frac{Q}{4\pi L} \ln \left( \frac{x_{i+1} - x}{x_i - x} \right) + \frac{r_i}{r_i} + \lambda \frac{K}{\mu} R_D + C' \]  

(7)

where \( r_i = \sqrt{(x_i - x)^2 + y^2 + (z_w - z)^2} \).

The herringbone-like laterals wells have \( M \) branching (including main branch-
ing). The herringbone-like laterals wells are divided into several segments. And it is setted the flow rate of section \( s \) of the branching \( t \). According to the superposition principle of mirror reflection and potential, potential produced by herringbone-like laterals wells system at any point \( M \) in reservoir:

\[
\Phi_s(M) = \sum_{k=-\infty}^{+\infty} \sum_{r=0}^{N-1} \sum_{s=1}^{M-1} \left[ \phi\left((x[r][s], y[r][s], 4kh + z_w), (x[r][s+1], y[r][s+1], 4kh + z_w), \alpha_t\right) + \phi\left((x[r][s], y[r][s], 4kh + 2h - z_w), (x[r][s+1], y[r][s+1], 4kh + 2h - z_w), \alpha_t\right) - \phi\left((x[r][s], y[r][s], 4kh - z_w), (x[r][s+1], y[r][s+1], 4kh - z_w), \alpha_t\right) - \phi\left((x[r][s], y[r][s], 4kh - 2h + z_w), (x[r][s+1], y[r][s+1], 4kh - 2h + z_w), \alpha_t\right) \right] + \lambda \frac{K}{\mu} \sqrt{\left(R_x - x\right)^2 + y^2 + z^2} \tag{8}
\]

where:

\[
\phi\left((x[r][s], y[r][s], 4kh + 2h - z_w), (x[r][s+1], y[r][s+1], 4kh + 2h - z_w), \alpha_t\right) = -\frac{q_{ts}^n \ln r_{ts} + r_{ts+1} + L_{ts}}{4\pi L_{ts}} + r_{ts} + r_{ts+1} - L_{ts} + c_y
\]

\[
r_{ts} = \sqrt{(x_{ts} - x)^2 + (y_{ts} - y)^2 + (4kh + 2h - z_w - z)^2}, \quad m; \quad L_{ts} \text{ is the length of the} \ s \text{ segment of the branching} \ t; \quad m; \quad k \text{ is infinite well row reflected by mirror image of branching in} \ Z \text{ direction.}
\]

The wellbore pressure distribution equation of herringbone-like laterals wells wells under bottom water reservoir is obtained according to Equation (8):

\[
p_{wts} = p_e + \frac{\mu}{K} \left(\Phi_{ets} - \Phi_{wts}\right) \tag{9}
\]

For bottom water reservoirs, there is:

\[
\Phi_{ets} = 0 \tag{10}
\]

The FORMULA (9) is changed to:

\[
p_{wts} = p_e - \frac{\mu}{K} \Phi_{wts} \tag{11}
\]

where \( \mu \) is oil viscosity, mPa·s; \( K \) is reservoir permeability, mD; \( p_{wts} \) is the well bottom hole flow pressure of the \( s \) segment of the branching \( t \); \( p_e \) is initial reservoir pressure, MPa.

### 3. Pressure Drop Equation of Wellbore Flow

#### 3.1. Computational Model of Flow Pressure Drop in Main and Branch Wellbore

The main wellbore and branching wellbore are divided into many units. Considering that the length of each unit is enough small, the pressure drop of the second section of the branching \( t \) of herringbone-like laterals wells is calculated in reference [14]:

\[
\Delta p_{wts} = \frac{2f_{sw} \rho}{\pi^2 D^3} \left(2Q_{ts} + q_{ts}\right)^2 \Delta x + \frac{16\rho q_{ts}}{\pi^2 D^3} \left(2Q_{ts} + q_{ts}\right) \tag{12}
\]
where \( q_{ts} \) is radial inflow of micro-element section in the \( s \) segment of the branching \( t \), \( m^3/s \); \( Q_s \) is the upstream flow of the mainstream for the \( s \) segment of branching \( t \), \( m^3/s \); \( f_{hw} \) is friction resistance coefficient of the well tube wall with radial inflow, \( \rho \) is fluid density, kg/m\(^3\); \( D \) is the wellbore diameter, m.

### 3.2. Computational Model of Convergent Flow Pressure Drop of Main and Branching Wellbore

While the fluid of the main wellbore and branching wellbore is confluencing, a mixed loss will occur, and generated local pressure drop. Based on the principle of fluid mechanics, a calculation model of local pressure drop at the confluence point of main wellbore and branching wellbore is established.

Assuming that the fluid in the wellbore flows steadily, adiabatically and isothermally, without considering the friction between the fluid and the pipe wall, and ignoring the influence of gravity, the confluence flow diagram of the branching wellbore is shown in Figure 2.

The momentum equation of the main wellbore direction of the fluid at the confluence point is as follows:

\[
p_1 \frac{\pi}{4} D^2 - p_2 \frac{\pi}{4} D^2 + F_i = \rho Q_2 V_2 - \rho Q_1 V_1
\]  

(13)

Continuity equation:

\[
V_1 \frac{\pi}{4} D^2 + q_i = V_2 \frac{\pi}{4} D^2
\]

(14)

Energy equation:

\[
\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_2
\]

(15)

where \( p_1, p_2 \) are pressure at the inflow and outflow ends along the direction of the main wellbore at the confluence point, MPA; \( D \) is main wellbore and branching wellbore diameter, m; \( q_i \) is from the branching wellbore \( t \) to main wellbore, \( m^3/s \).

The force \( F_i \) of the wall acting on the gas at the junction point can be derived from the momentum equation:

\[
F_i = \rho q_i V_i \cos \phi
\]  

(16)

Combination the Formula (13) and (16):

\[
p_1 - p_2 = \frac{4 \rho}{\pi D^2} (Q_2 V_2 - Q_1 V_1 - q_i V_i \cos \phi)
\]

(17)

Combination the Formula (14), (15) and (17), pressure drop equation at the confluence point of main and branching wellbore is:

\[
\Delta p_{ct} = p_1 - p_2 = \frac{16 \rho q_i^2}{\pi D^2} + \frac{8 V_i \rho q_i}{\pi D^2} - \frac{4 V_i \rho q_i}{\pi D^2} \cos \phi
\]

(18)

Substitute \( V_i = \frac{4Q_i}{\pi D^2} \), \( V_i = \frac{4q_i}{\pi D^2} \) into the Formula (18):
Figure 2. Confluence flow diagram of main and branch wellbore.

\[ \Delta p_{wa} = p_1 - p_2 = \frac{16 \rho q_f^2}{\pi^2 D^4} (1 - \cos \phi) + \frac{32 \rho Q q_f}{\pi^2 D^4} \]  \hspace{1cm} (19)

where \( Q \) is flow from upstream end of the point of main wellbore \( t \), m\(^3\)/s; \( q_f \) is flow from branching wellbore \( t \) to the main wellbore, m\(^3\)/s; \( \phi \) is the angle of branching, degrees; \( D \) is diameter of the main wellbore and branching wellbore, m.

4. Coupling Model of Seepage and Wellbore Flow in Herringbone-Like Laterals Wells

In addition to flowing along the length of horizontal wellbore, reservoir fluid also flows into wellbore along the horizontal wellbore direction. There is a coupling relationship between seepage flow in reservoir and in the wellbore.

4.1. Establishment of Coupling Model

The pressure distribution in wellbore can be calculated by the pressure drop calculation model as:

\[ p_{wa} = p_{wa(s-1)} + 0.5 (\Delta p_{wa(s-1)} + \Delta p_{wa}) \left( 0 \leq t \leq M - 1, 1 \leq s \leq N \right) \]  \hspace{1cm} (20)

\[ \Delta p_{wa0} = 0, \quad p_{wa0} = p_{wa} \]

where \( p_{wa} \) is the flow pressure at the heel of the branching wellbore \( t \), MPa; \( p_{wa(s)} \) is the flow pressure of \( s \) segment of the branching wellbore \( t \), MPa; \( p_{wa(s-1)} \) is the flow pressure of \( s-1 \) segment of the branching wellbore \( t \), MPa; \( \Delta p_{wa(s-1)} \) is the pressure drop of \( s-1 \) segment of the branching wellbore \( t \), MPa; \( \Delta p_{wa} \) is the pressure drop of \( s \) segment of the branching wellbore \( t \), MPa.

According to the principle of material balance, the inflow of each branching wellbore equals the sum of the inflow of each small section at the upstream:

\[ Q_w = \sum_{i=0}^{M-1} \sum_{s=1}^{N} q_{wa} \]  \hspace{1cm} (21)

As can be seen from the above, the reservoir seepage model has \( m \times n \) equation, the wellbore pressure drop model has \( m \times n \) equation, there are \( 2m \times n \) equations. The variables to be solved are \( q_{wa} \) and \( p_{wa} \)

\( 0 \leq t \leq M - 1, 1 \leq s \leq N \), which are also \( 2m \times n \), so the equations are closed.

4.2. Solution of Coupled Model

The coupling model is solved by iteration method. The specific steps are as fol-
lows: 1) Assuming that the initial value of \( p_{n0} \) is \( p_{n0}^0 \), in actual calculation it can be assumed \( p_{n0}^0 = p_{n0} \); 2) Substitute \( p_{n0} \) into the Formula (11), used gauss elimination method to find \( q_{n0} \); 3) Substitute \( q_{n0} \) into the Formula (21), find \( Q_{n0} \); 4) Substitute \( q_{n0} \) and \( Q_{n0} \) into the Formula (12) and (19), find \( \Delta p_{n0} \); 5) Substitute \( \Delta p_{n0} \) into the Formula (20), to update \( p_{n0} \). This value is taken as the initial value of the next iteration; 6) repeat (2) - (5), comparing \( p_{n0}^{n+1} \), \( q_{n0}^{n+1} \) after \( n \) iterations with \( p_{n0}^n \) and \( q_{n0}^n \) after \( n \) iterations, when both of them satisfied certain accuracy, the iteration stops, otherwise repeat (2) - (5) steps until the accuracy is satisfied; 7) Last, the Formula (21) can be used to calculate the total production of herringbone-like laterals wells.

5. Case Study and Optimization of Morphological Parameters

5.1. Example Analysis

The herringbone-like laterals wells in a heavy oil reservoir in Bohai Oilfield as an example, the well pattern is shown in Figure 3. There are four branching along the main wellbore. The distance between each branching is 50 m, the angle between the main wellbore and the branching wellbore is 45 degrees, the length of the main wellbore is 400 m, the length of each branching wellbore is 200 m, and the radius of the main wellbore and the branching wellbore is 0.119 m, reservoir thickness \( h = 6.9 \) m, distance from the well to bottom of reservoir \( Z_w = 3.45 \) m, reservoir permeability \( K = 3159 \times 10^{-3} \) \( \mu \text{m}^2 \), initial reservoir pressure \( p_e = 11.47 \) MPa, bottom hole flow pressure \( p_w = 10.47 \) MPa, production pressure drop \( \Delta p = 1 \) MPa, branching wellbore radius \( r_w = 0.119 \) m, wall roughness \( e = 0.001 \) m, oil viscosity \( \mu = 50 \) mPa-s, volume coefficient of oil \( B = 1.07 \), oil density \( \rho = 0.969 \) g/cm\(^3\), the threshold pressure gradient of the reservoir is 0.02 MPa\(^{-1}\)/m.

The productivity coupling model deduced by the author is used to predict the productivity of herringbone-like laterals wells. Compared with the actual production data, as shown in Table 1, it can be seen from Table 1 that the relative error between the calculated results and the actual production is less than that calculated by Liu Xiangping formula, which is 7.2%. The main reason is that the effect of threshold pressure gradient on productivity is considered in this paper. This formula has high practicability for predict the productivity of herringbone-like laterals wells.

<table>
<thead>
<tr>
<th>Well</th>
<th>A50H</th>
<th>The method</th>
<th>Productivity (m(^3)/d)</th>
<th>Relative error</th>
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<td>Formula of Liu Xiangping</td>
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<td></td>
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<tr>
<td>Formula in this paper</td>
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<td>Actual data</td>
<td>250</td>
<td>-</td>
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</table>
5.2. Study on Optimization of Morphological Parameters

The reservoir-wellbore steady seepage coupling model is used to optimize the shape of herringbone-like laterals wells, for give full play to the advantages of herringbone-like laterals wells.

1) Branching length optimization

The optimization of branching length mainly studies during the total wellbore length is equal, the productivity difference between equal and unequal branching length. In the study, the structure of two branching shown in Figure 4. The total length of the main wellbore and branching wellbore is 800 m, and the angle of each branching is 45 degrees.

Figure 5 shows that when the total length of the wellbore is equal, during the length of the main wellbore and branching wellbore is increased; the productivity of the well is increased. When the branching length is 0 m, the well will be transformed into a single horizontal well, that is, the productivity of a single horizontal well is greater than an equal length herringbone-like laterals wells.

2) Branching angle optimization

Using the three branching structures as an example, the effect of branching angle on productivity is studied. As can be seen from Figure 6, the total angle of the three branching structures is 135 degrees. The productivity variation law is studied by changing the angle of each branching. The main wellbore and branching wellbore length are 400 m and 200 m respectively.

From Figure 7, it can be seen that the productivity of the well is the smallest when the branching angle is equal and the productivity is increased with the increase of the angle difference. Generally, the change of branching angle has little effect on the total productivity of the well, less than the influence of branching length.

3) Branching symmetry optimization

In order to study the effect of branching symmetry on productivity of the well, four branching structures are designed as shown in Figure 8. This paper mainly analyses whether there are common convergence points between the branching and the influence of the branching on the productivity of the well. The main wellbore length is 400 m, the branching wellbore length is 200 m, and the angle of each branching is 45 degrees.

As can be seen from Figure 9, that the ipsilateral branching structure will
contribution more productivity than opposite side branching structure. During the number of branching on one side of the main wellbore is increasing, the interference on the one side of the main wellbore is increasing, and the driving area of each branching is increasing. The result of comprehensive action increases the total productivity of the well.

4) Branching location and spacing optimization

In order to study the influence of branching location and spacing on the productivity of the well, four branching structures are designed as shown in Figure 10. The main wellbore length is 400 m, the branching wellbore length is 200 m, the angle of branching is 45 degrees, and the distance between the main wellbore and the branching heel is shown in Figure 10.

As can be seen from Figure 11, that during the branching is closer to the heel of the main wellbore, the productivity of the well is larger. When branching spacing is increased the interference of branching is decreased, but the interference range of main wellbore is increased, the result of comprehensive action decreases the total productivity of the well.

![Figure 4](image-url)
Figure 4. Schematic diagrams of herringbone-like laterals wells with different branching length.

![Figure 5](image-url)
Figure 5. The productivity of herringbone-like laterals wells with different branching length.
Figure 6. Schematic diagrams of herringbone-like laterals wells with different branching angle.

Figure 7. The productivity of herringbone-like laterals wells with different branching angle.

Figure 8. Schematic diagrams of herringbone-like laterals wells with different branching symmetry.
Figure 9. The productivity of herringbone-like laterals wells with different branching symmetry.

Figure 10. Schematic diagrams of herringbone-like laterals wells with different branching location and spacing.

Figure 11. The productivity of herringbone-like laterals wells with different branching location and spacing.
6. Conclusion

Based on the threshold pressure gradient, the productivity coupling model of herringbone-like laterals wells is established in heavy oil reservoir-wellbore steady seepage. The productivity coupling model is suitable for predicting the productivity of herringbone-like laterals wells in heavy oil reservoir. Using the productivity coupling model in this paper, the well shape parameters of the well are optimized, and the principle of optimizing the well shape of herringbone-like laterals wells is proposed.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References


