World Journal of Engineering and Technology, 2017, 5, 117-125
http://www.scirp.org/journal/wjet
ISSN Online: 2331-4249
ISSN Print: 2331-4222

Application of a Fuzzy Analytical Hierarchy Process for Predicting the Grindability of Granite

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How to cite this paper: Zhang, Z.M., Wang, J., Cao, H.Y., Lu, Q.X. and Ding, M.W. (2017) Application of a Fuzzy Analytical Hierarchy Process for Predicting the Grindability of Granite. *World Journal of Engineering and Technology*, **5**, 117-125.
https://doi.org/10.4236/wjet.2017.54B013

Received: September 24, 2017
Accepted: October 9, 2017
Published: October 12, 2017

Abstract

The ranking system of grindability is the key technology for high-efficiency grinding granite. A new classification system is presented to evaluate and ranking the grindability of granite. On account of the complicated relation between the mineral composition and mechanical properties with the grindability of granite, a new method by the combination of Fuzzy Analytic Hierarchy Process (FAHP) method with TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) methods is developed to establish the dependence function and fuzzy relationship between SiO_2 content, quartz content, Shore hardness, density, compressive strength, flexural strength and abrasion resistance of granite with grinding force. The grindability of ten types of granite was evaluated and classified by this method. With the fuzzy ranking system established and the grindability classification, it is very convenient to evaluate the grindability and select a suitable diamond tools and proper grinding parameters for a new granite type by only the petrographic analysis and mechanical properties testing.

Keywords

Grindability, Ranking, Granite, Fuzzy Analysis, Hierarchy Process

1. Introduction

The granite products have been widely used for its exquisite appearance, luxurious and elegant tones and wear resistance, corrosion resistance and other stable physical and chemical properties in the fields of architectural decoration, craft and art, life appliances, precision machine and development toward high-level, art and precision. But the granite is a natural rock material that consists of several minerals, so the grindability of different types of granite is different. The ranking system of granite grindability is very important for high-efficiency grinding key technology. Many experts have been studied the sawability of rocks and tried to conceive the ranking method for rocks. Reza Mikaeil proposed changes of motor power while cutting stone to stone sawing performance evaluation [1] [2] [3] [4]. Saffet Yagiz proposed to evaluate the brittleness of stone sawing of stone, and the use of fuzzy inference system and nonlinear regression to establish a mathematical model of sawing force [5]. Bulent Tiryaki used the specific cutting energy index to evaluate the machinability of the stone, and using artificial neural network prediction model [6] [7].

The aim of this paper is developing a new method by the combination of Fuzzy Analytic Hierarchy Process (FAHP) method with TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) methods is developed to establish the dependence function and fuzzy relationship between SiO_2 content, quartz content, Shore hardness, density, compressive strength, flexural strength and abrasion resistance of granite with grinding force.

2. Applied Theoretical Concept

2.1. Theory of Triangular Fuzzy

In this study, the Fuzzy Analytic Hierarchy Process (FAHP) has been used. Let $X=\{x_1,x_2,x_3\cdots,x_n\}$ be an object set, and $G=\{g_1,g_2,g_3\cdots,g_n\}$ be a goal set, each object is taken, and extent analysis for each goal performed respectively. Therefore, m extent analysis values for each object can be obtained with M_{gi}^1, M_{gi}^2, ···, M_{gi}^m (i = 1, 2, ···, n), where $M_{gi}^j\left(j=1,2,\cdots,m\right)$ all are triangular fuzzy numbers(TFN). The steps of extent analysis can be given as in the following:

Step 1. The value of fuzzy synthetic extent with respect to the i object is defined as

$$S_i=\sum_{j}^{m}M_{gi}^j\otimes\left[\sum_{i=1}^{n}\sum_{j=1}^{m}M_{gi}^j\right]^{-1} \tag{1}$$

To obtain, the $\sum_{j=1}^{m}M_{gi}^j$ fuzzy addition operation of m extent analysis values for a particular matrix is performed as follows

$$\sum_{j}^{m}M_{gi}^j=\left(\sum_{j=1}^{m}l_j,\sum_{j=1}^{m}m_j,\sum_{j=1}^{m}u_j\right) \tag{2}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}M_{gi}^j=\left(\sum_{i=1}^{n}l_i,\sum_{i=1}^{n}m_i,\sum_{i=1}^{n}u_i\right) \tag{3}$$

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m}M_{gi}^j\right]^{-1}=\left(\frac{1}{\sum_{i=1}^{n}u_i},\frac{1}{\sum_{i=1}^{n}m_i},\frac{1}{\sum_{i=1}^{n}l_i}\right) \tag{4}$$

Step 2. As $M_1\left(l_1,m_1,u_1\right)$ and $M_2\left(l_2,m_2,u_2\right)$ are two triangular fuzzy numbers, the degree of possibility of $M_1 \ge M_2$ is defined as

$$V\left(M_1 \ge M_2\right) = \sup_{x \ge y}\left[\min\left(\mu_{M_1}\left(x\right), \mu_{M2}\left(y\right)\right)\right] \tag{5}$$

and can be expressed as follows

$$V\left(M_1 \ge M_2\right) = \mu\left(d\right) = \begin{cases} 1, & m_1 \ge m_2 \\ \dfrac{l_2 - u_1}{\left(m_1 - u_1\right) - \left(m_2 - l_2\right)}, & m_1 \le m_2, \quad u_1 \ge l_2 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Step 3. The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy M_i ($i = 1, 2, \cdots, k$) numbers can be defined by

$$V\left(M \ge M_1, M_2, \cdots M_k\right) = \min V\left(M \ge M_i\right), \quad i = 1, 2, \cdots k \tag{7}$$

Assume that $d'\left(A_i\right) = \min V\left(S_i \ge S_k\right) \quad \left(k = 1, 2, \cdots m,\ k \ne i\right)$, then the weight vector is given by

$$W' = \left(d'\left(A_1\right), d'\left(A_2\right), \cdots, d'\left(A_m\right)\right)^T \tag{8}$$

where A_i ($i = 1, 2, \cdots, n$) are i elements.

Step 4. Via normalization, the normalized weight vectors are

$$W = \left(d\left(A_1\right), d\left(A_2\right), \cdots, d\left(A_m\right)\right)^T \tag{9}$$

where W is a non-fuzzy number.

2.2. TOPSIS Method

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is one of the useful multi-attribute decision making (MADM) techniques to manage real-world problems. According to this technique, the best alternative would be the one that is nearest to the positive ideal solution and farthest from the negative ideal solution. The positive ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria. In short, the positive ideal solution is composed of all best attainable values of criteria, whereas the negative ideal solution consists of all worst attainable values of criteria. In this paper TOPSIS method is used for determining the final ranking of the sawability of carbonate rocks. TOPSIS method is performed in the following steps:

Step 1. Decision matrix is normalized via Equation (10)

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{i=1}^{n} f_{ij}^2}} \qquad i = 1, 2, \cdots n; \ j = 1, 2, \cdots, m \tag{10}$$

Step 2. Weighted normalized decision matrix is formed

$$v_{ij} = W_j \times r_{ij} \quad i = 1, 2, \cdots n; \ j = 1, 2, \cdots, m \tag{11}$$

Step 3. Positive Ideal Solution and Negative Ideal Solution are determined

$$A^{+}=\left\{v_1^{+},v_2^{+},v_3^{+},\cdots,v_m^{+}\right\}^T,v_j^{+}=\max_i\left\{v_{ij}\right\},j=1,2,\cdots,m \tag{12}$$

$$A^{-}=\left\{v_1^{-},v_2^{-},v_3^{-},\cdots,v_m^{-}\right\}^T,v_j^{-}=\min_i\left\{v_{ij}\right\},j=1,2,\cdots,m \tag{13}$$

Step 4. The distance of each alternative from A^{+} and A^{-} are calculated

$$D_i^{+}=\sqrt{\sum_{i=1}^{n}\left(v_j^{+}-v_{ij}\right)^2}\quad D_i^{-}=\sqrt{\sum_{i=1}^{n}\left(v_j^{-}-v_{ij}\right)^2} \tag{14}$$

Step 5. The closeness coefficient of each alternative is calculated

$$C_i=\frac{D_i^{-}}{D_i^{+}+D_i^{-}}\qquad 0\le C_i\le 1 \tag{15}$$

Step 6. By comparing C_i values, the ranking of alternatives are determined.

3. Application of FAHP-TOPSIS Method to Multi-Criteria Comparison of Grindability

3.1. Granite Materials and Parameters

Test workpieces are selected typical granite materials. The SiO_2 content, quartz content, Shore hardness, density, compressive strength, flexural strength and abrasion resistance are as the most granite important characteristics that affect grindability. In order to get the universal research conclusion, ten kinds of granite that widely used are chosen to experiment. These parameters are shown in **Table 1**.

3.2. Determination of Criteria Weights

The fuzzy judgment matrix is established about SiO_2 content (C_1), quartz content (C_2), Shore hardness (C_3), density (C_4), compressive strength (C_5), flexural

Table 1. Material parameters of granites.

Granite type	SiO_2 (%)	Quartz content (%)	Shore hardness (HSD)	Density ($g\cdot cm^{-3}$)	Compressive strength (MPa)	Flexural strength (MPa)	Abrasion resistance ($g\cdot cm^{-2}$)
Wulian Red	67.55	24.3	85	2.68	92.89	8.542	0.6
Cherry Blossom Red	75.25	45.06	104	2.58	149.9	14.1	2.572
Shidao Red	64.29	44.19	110	2.7	234.37	13.63	2.68
Qilu Red	69.01	29.7	87	2.661	162	13.97	3.822
Liubu Red	75.64	43.36	60.4	2.61	203.97	20.89	1.607
Laoshan Red	71.88	29.76	99.7	2.59	208.29	18.06	3.828
Lu Grey	67.25	47.11	104	2.58	149.9	14.1	2.735
Marshal Red	70.22	25.2	90	2.65	147.88	12.87	3.76
China Grey	70.19	38.41	115	2.65	214.40	15.00	3.036
Wulian Flower	62.28	22	85	2.65	90.77	8.35	5.473

strength (C_6) and abrasion resistance (C_7) using pair-wise comparison. In the fuzzy AHP, fuzzy ratio scales are used to indicate the relative strength of the factors in the corresponding criteria. Therefore, a fuzzy judgment matrix can be constructed. The final scores of alternatives are also represented by fuzzy triangular numbers depicted over Saaty's nine-point fundamental scale. A summary of the fuzzy linguistic variable set with triangular fuzzy numbers as well as with the definitions for aiding comparisons is provided in **Table 2**.

According to the grinding process goal of granite, the weights for the parameters of granites are analyzed. A comprehensive triangular fuzzy pair-wise comparison matrix is built as in **Table 3**.

The triangular fuzzy synthesis values S_i are calculated by using Equation (2), as in **Table 4**.

The fuzzy values are compared by using Equation (6), and the values of V are obtained. Then, priority weights are calculated by using Equation (7). After normalizing the priority weights the standardized weights are extracted, the results of priority weights and standardized weights are shown in **Table 5**.

Table 2. Fuzzy linguistic variable set and underlying fuzzy numbers.

Linguistic variable	Fuzzy number	Membership function	Definition
Equally important	1	(1, 1, 2)	Practical knowledge and experience assert that criterion *i* is equally important when compared to criterion *j*
Moderately important	3	(2, 3, 4)	Practical knowledge and experience assert that criterion *i* seems moderately more important when compared to criterion *j*
More important	5	(4, 5, 6)	Practical knowledge and experience assert that criterion *i* is more important when compared to criterion *j*
Strongly important	7	(6, 7, 8)	Practical knowledge and experience assert that criterion *i* is strongly important when compared to criterion *j*
Extremely important	9	(8, 9, 9)	Practical knowledge and experience assert that criterion *i* is extremely important when compared to criterion *j*, and totally out weights it

Table 3. Triangular fuzzy pair-wise comparison matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	(1, 1, 1)	(1/5, 1/4, 1/3)	(1, 2, 3)	(3, 4, 5)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(2, 3, 4)
C_2	(3, 4, 5)	(1, 1, 1)	(4, 5, 6)	(6, 7, 8)	(1, 2, 3)	(2, 3, 4)	(5, 6, 7)
C_3	(1/3, 1/2, 1)	(1/6, 1/5, 1/4)	(1, 1, 1)	(2, 3, 4)	(1/5, 1/4, 1/3)	(1/4, 1/3, 1/2)	(1, 2, 3)
C_4	(1/5, 1/4, 1/3)	(1/8, 1/7, 1/6)	(1/4, 1/3, 1/2)	(1, 1, 1)	(1/7, 1/6, 1/5)	(1/6, 1/5, 1/4)	(1/3, 1/2, 1)
C_5	(2, 3, 4)	(1/3, 1/2, 1)	(3, 4, 5)	(5, 6, 7)	(1, 1, 1)	(1, 2, 3)	(4, 5, 6)
C_6	(1, 2, 3)	(1/4, 1/3, 1/2)	(2, 3, 4)	(4, 5, 6)	(1/3, 1/2, 1)	(1, 1, 1)	(3, 4, 5)
C_7	(1/4, 1/3, 1/2)	(1/7, 1/6, 1/5)	(1/3, 1/2, 1)	(1, 2, 3)	(1/6, 1/5, 1/4)	(1/5, 1/4, 1/3)	(1, 1, 1)

Table 4. Triangular fuzzy synthesis values.

	l	*m*	*u*
C_1	0.0673	0.1221	0.2181
C_2	0.1902	0.3086	0.4999
C_3	0.0428	0.0803	0.1483
C_4	0.0192	0.0286	0.0507
C_5	0.1412	0.2369	0.3970
C_6	0.1002	0.1745	0.3014
C_7	0.0272	0.0490	0.0850

Table 5. Results of priority weights and standardized weights.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Priority weights	0.1301	1	0	0	0.7427	0.4533	0
Standardized weights	0.0559	0.4299	0	0	0.3193	0.1949	0

3.3. Ranking the Grindability of Granite

The weights of C_3, C_4 and C_7 are zero from **Table 5**, it means that these parameters are nonobviously to affect the grindability of granite. The greatest significance parameters of C_1, C_2, C_5 and C_6 are selected to rank the grindability of granite. Decision matrix is normalized via Equation (10) and weighted normalized decision matrix is formed by using Equation (11). The values of decision matrix, normalized decision matrix and weighted normalized matrix are given in **Table 6**.

Positive and negative ideal solutions are determined by taking the maximum and minimum values for each criterion via Equations (12) and (13):

$$A^+ = \{0.0192, 0.1735, 0.1376, 0.0894\}, \quad A^- = \{0.0158, 0.0810, 0.0533, 0.0357\}$$

Then, the distance of each method from PIS (positive ideal solution) and NIS (negative ideal solution) with respect to each criterion are calculated, with the help of Equation (14). Then, closeness coefficient of each granite is calculated by using Equation (15) and the ranking of the granites are determined according to these values. The grindability ranking of granites are also shown in **Table 7** and **Figure 1** in the descending order of priority.

4. Laboratory Tests

4.1. Equipment and Parameters

The test machine is CNC Machining Center SPEED Y2000 imported from CMS Company in Italy. Cutting tool is diamond profiling wheel (ASS10105, produced by the You-oriented Company in Italy). The process parameters are the cutting speed (50 m/s), the feed speed (1000 mm/min) and cutting depth (14.5 mm). The grinding forces are measured to evaluate the grindability of granites.

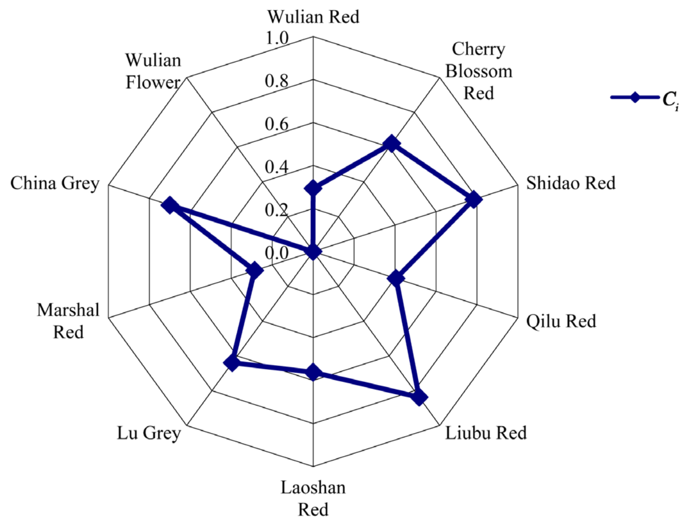

Figure 1. The grindability ranking of granites based on C_i.

Table 6. Decision matrix, normalized decision matrix and weighted normalized matrix.

Granite type	Decision matrix				Normalized decision matrix				Weighted normalized matrix			
	C_1	C_2	C_5	C_6	C_1	C_2	C_5	C_6	C_1	C_2	C_5	C_6
Wulian Red	67.55	34.30	92.89	8.542	0.308	0.294	0.171	0.188	0.017	0.126	0.055	0.037
Cherry Blossom Red	75.25	45.06	149.90	14.10	0.343	0.386	0.276	0.310	0.019	0.166	0.088	0.060
Shidao Red	64.29	44.19	234.37	13.63	0.293	0.379	0.431	0.299	0.016	0.163	0.138	0.058
Qilu Red	69.01	29.70	162.00	13.97	0.314	0.255	0.298	0.307	0.018	0.109	0.095	0.060
Liubu Red	75.64	43.36	203.97	20.89	0.344	0.372	0.375	0.459	0.019	0.160	0.120	0.089
Laoshan Red	71.88	29.76	208.29	18.06	0.327	0.255	0.383	0.397	0.018	0.110	0.122	0.077
Lu Grey	67.25	47.11	149.90	14.10	0.306	0.404	0.276	0.310	0.017	0.174	0.088	0.060
Marshal Red	70.22	25.2	147.88	12.87	0.320	0.216	0.272	0.283	0.018	0.093	0.087	0.055
China Grey	70.19	38.41	214.40	15.0	0.320	0.329	0.394	0.329	0.018	0.142	0.126	0.064
Wulian Flower	62.28	22	90.77	8.35	0.284	0.189	0.167	0.183	0.016	0.081	0.053	0.036

Table 7. Rankings of the grindability of granites according to C_i values.

Granite type	D^+	D^-	C_i	Rank
Wulian Red	0.1092	0.0454	0.2936	8
Cherry Blossom Red	0.0580	0.0951	0.6213	5
Shidao Red	0.0330	0.1196	0.7838	2
Qilu Red	0.0824	0.0560	0.4046	7
Liubu Red	0.0226	0.1162	0.8375	1
Laoshan Red	0.0668	0.0855	0.5614	6
Lu Grey	0.0575	0.1019	0.6391	4
Marshal Red	0.1013	0.0405	0.2857	9
China Grey	0.0424	0.0987	0.6993	3
Wulian Flower	0.1362	0.0001	0.0005	10

4.2. Experimental Results and Data

The grinding force is regarded as the criterion of grindaility, and the evaluating criterion can be constituted. By laboratory tests the grinding forces were measured and the experimental results are shown in **Table 8**.

4.3. Data Analysis Results and Discussion

According to **Table 8**, the fifth granite in ranking is Liubu Red, it has a maximum value of grinding force. On the opposite side, Wulian Flower has a minimum value of grinding force. Contrast with **Table 7**, the relationship between grinding force and closeness coefficient of the studied granites (C_i) has a highly significant correlation. As grinding force increases, C_i value increases. These results confirm the results of new ranking. It is concluded that the new ranking method of granite is reasonable and acceptable for evaluating the grinding force of granites.

For evaluating the grindability and properly selecting the tool and grinding parameters for a new granite type, only the petrographic analysis and mechanical property testing are needed. Based on the data, the information about the grindability prediction can be obtained by data processing, statistical analysis and fuzzy operation instead of a number of grinding tests. This new ranking method of granite grindability by means of fuzzy mathematics is reasonable and acceptable.

5. Conclusions

1) The grindability is affected by the SiO_2 content, quartz content, compressive strength and flexural strength of the granite.

2) The criteria of grindability, *i.e.* the grinding force is affected by the above mentioned factors in a different trend. So, the rights of the factors are distributed in different ways for evaluating the grindability using different criteria.

Table 8. Experimental results and evaluating criterion of grindability by grinding force.

Test no	Granite type	Grinding force [N]	
		Prediction and class of grindability	Test result
1	Wulian Red	V (400 - 500)	437.47
2	Cherry Blossom Red	IV (500 - 600)	562.76
3	Shidao Red	I (800 over)	826.15
4	Qilu Red	V (400 - 500)	450.19
5	Liubu Red	I (800 over)	897.24
6	Laoshan Red	V (400 - 500)	486.23
7	Lu Grey	III (600 - 700)	676.28
8	Marshal Red	VI (400 below)	384.29
9	China Grey	II (700 - 800)	792.84
10	Wulian Flower	VI (400 below)	318.57

3) The grinding force can be chosen as the criterion according to the need of production for ranking the grindability of a granite in order to select a suitable tool and the determine the optimum grinding parameters.

4) This new ranking method of granite grindability by means of fuzzy mathematics is reasonable and acceptable. For evaluating the grindability of a new granite type, only the petrographic analysis and mechanical property testing instead of a number of grinding tests are needed to obtain the information about the grindability prediction.

Acknowledgements

The research was financially supported by the Science and Technology Foundation of Shandong Labor Vocational and Technology College (Grant No. 2015KJZ003).

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