

Applications of Quantum Physics on Resistivity, Dielectricity, Giant Magneto Resistance, Hall Effect and Conductance

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Abstract

Quantum theory with conjecture of fractional charge quantization, eigenfunctions for fractional charge quantization, fractional Fourier transform, Hermite function for fractional charge quantization, and eigenfunction for a twisted and twiggled electron quanta is developed and applied to resistivity, dielectricity, giant magneto resistance, Hall effect and conductance. Our theoretical relationship for quantum measurements is in good conformity and in agreement with most of the experimental results. These relationships will pave a new approach to quantum physics for deciphering measurements on single quantum particles without destroying them. Our results are in agreement with 2012 Physics Nobel Prize winning Scientists, Serge Haroche and David J. Wineland.

Keywords

Quantum Resistivity, Quantum Dielectricity, Giant Magneto Resistance, Quantum Hall Effect and Conductance

1. Introduction

Experimental results in quantum physics since last three decades brought significant changes in our understanding. The discovery of quantum Hall effect in heterostructure semiconductors results in the Nobel Prize winning award for the year 1987 to Von Klitzing [1]. With this discovery, the experimental results of quantum

conductance are reported by Van Wees *et al.* [2] in the two dimensional electron gas of a GaAs-AlGaAs heterostructure. The visible range photons used to illuminate water molecules are studied with absorption and Fourier transform infrared spectroscopies [3]. The biological specimens are also considered for chromotherapy [4]-[7]. A new conjecture of fractional charge quantization with newly developed theory is coined to look into the shape of eigenfunctions, determine the energy eigenvalues and validate the quantum scattering [8]. Meanwhile, new experimental results on giant magneto resistance (GMR) to enhance storage capacity with charges are reported. This discovery of GMR led Albert Peter and Paul Gruebber to win the Nobel Prize for the year 2007 [9]. During the last decade (2000-2010), surprising results are noticed on dielectrics and dielectricity. A new quantum theory, with our conjecture of charge quantization, on dielectricity is presented in which we modify the Clausius Mossotti and Debye equations [10]. The same quantum theory of dielectricity is applied on Faujasite-type molecular sieves and on dolomite [11] [12], respectively. The quantum theory of dielectrics and dielectricity is further extended and modified by using Hermite function for fractional quantum states and fractional Fourier transform .

Now, we witnessed again new exciting experimental results on individual quantum systems which led the Nobel Prize winning award in physics by Serge Haroche and David J. Wineland in the year 2012. We studied American Institute of Physics (AIP) reports of 2012 prize winning award and all relevant research papers [13]-[21]. Most of the experimental results of physics Nobel Prize winners like Von Klitzing, Albert Peter and P. Gruebber, Haroche and Wineland fit to our “conjecture of fractional charge quantization” and indeed “theory”. A new theory is described “how charge being a constant entity, on anelectron in the momentum space is fractionally quantized while interacting with a photon, with twisting and twigging effects of an electron quanta” [22] [23]. The eigenfunction for an electron quantum wire or string with sub-quanta (twigs) on its lateral surface at different locations namely above its surface, at the surface and within the sub-quanta and the electron string with beaded fractional quantized states for the fractional charges are determined [23].

2. Results and Discussions

The fractional Fourier transform (FRFT) of order α of $x(t)$ is defined by Almeida [24]

$$\begin{aligned} \mathfrak{F}_\alpha [x(t)] &= X_\alpha(u) = \int_{-\infty}^{\infty} x(t) K_\alpha(t, u) dt \\ &= \begin{cases} \sqrt{\frac{1-j \cot \alpha}{2\pi}} e^{j\frac{u^2}{2} \cot \alpha} \int_{-\infty}^{\infty} x(t) e^{j\frac{t^2}{2} \cot \alpha - jut \csc \alpha} dt & \text{if } \alpha \text{ is not a multiple of } \pi \\ x(t) & \text{if } \alpha \text{ is a multiple of } 2\pi \\ x(-t) & \text{if } \alpha + \pi \text{ is a multiple of } 2\pi \end{cases} \end{aligned} \quad (1)$$

where α is a rotational angle in the time-frequency plane, and \mathfrak{F}_α is the FRFT operator. For $\alpha = \pi/2$ the kernel coincides with the kernel of Fourier transform (FT). Saleem Iqbal *et al.* developed fractional Fourier integral theorem and fractional Fourier Cosines and Sines transforms [25]. [23] developed Hermite function for the fractional quantum states, *i.e.*,

$$H_{n_f}(\xi) = 2^{n_f}; 0.1 \leq n_f \leq 0.9. \quad (2)$$

Equation (2) is consistent with other definitions of Hermite polynomials. Saleem Iqbal [21] obtained the eigenfunction for a twisted and twigged electron quanta by using Equations (1) and (2), *i.e.*,

$$\psi_{n_f} = e^{in_f \alpha} = \left(\frac{\alpha}{\sqrt{\pi} 2^{n_f}} \right)^{\frac{1}{2}}. \quad (3)$$

Equation (3) represents plane wave for a rotation vector alpha (discussed in [21]) for all corresponding fractional quantum numbers, *i.e.*,

$$0.1 \leq n_f \leq 0.9 \text{ and } 0.17 \leq \alpha \leq 1.53. \quad (4)$$

We shall use Equations (1)-(3) and relation (4) to obtain interesting results for different cases of physics

problems.

2.1. Case I—Quantum Resistivity

We know that the electrical resistivity is the inverse of conductivity. The electrical conductivity according to Drude Model (classical) is defined as:

$$\sigma = \frac{ne^2\tau}{m_e} \quad (5)$$

where n is the number of charge carriers, e the charge of an electron, τ the relaxation time and m_e the effective mass of an electron. With the advent of single electron transistors (Spintronics), one could expect quantum conductivity across the interface states. The single electron tunneling will follow a helicon profile with each turn of the helix corresponding to fractional quantum states (charges are fractionally distributed on sub-quanta, *i.e.*, twigs). Changing n with n_f (Equations (2) and (4)), e with $Q_{n_f} = (q_1, q_2, \dots, \text{onsub-quanta})$ and m_e with $\frac{E}{c^2}$, $c = \lambda\nu$, $p = \hbar\kappa$, $\hbar = \frac{h}{2\pi}$ and $\tau = \frac{1}{\nu_{nf}}$ in Equation (5), we have

$$\sigma_q = \sum_{n_f=0.1}^{0.9} \frac{2\pi n_f}{\hbar} \left(\frac{Q_{n_f}}{\kappa_{n_f}} \right)^2 \quad (6)$$

where κ_{n_f} is fractional wave number. $\frac{Q_{n_f}}{\kappa_{n_f}}$ can be determined from Raman spectroscopy [26]. Q_{n_f} can be

determined from relative heights of Raman peaks. The helicon profile of an electron is due to spinning or gyroscopic motion. To our conjecture, the dual nature of a quantum particle is a metaphoric states, *i.e.*, it simultaneously behaves as particle and quanta. The fractional quantized state of charge in the momentum space are the manifestations of gyroscopic constant, $g^2/\hbar c (0.02-0.08)$. Equation (6) shows that the quantum conductivity follows periodicity of fractional quantum numbers, *i.e.*, $0.1 \leq n_f \leq 0.9$ and is inversely proportional to quantum action (energy becomes oscillatory). The quantum resistivity is the inverse of quantum conductivity, *i.e.*,

$$\rho_q = \frac{1}{\sigma_q} = \sum_{n_f=0.1}^{0.9} \frac{\hbar}{2\pi n_f} \left(\frac{\kappa_{n_f}}{Q_{n_f}} \right)^2 \quad (7)$$

Remember that the conductivity is different from conductance.

2.2. Case II—Quantum Dielectricity and Giant Magneto Resistance (GMR)

The mesoscopic fields in a cavity are the manifestations of quantum mechanical dipole moments (fractional charge quantization to a single electron or many electrons systems) due to either molecules, atoms, ions or even the charge, being a constant physical entity, of an electron in the momentum space while interacting with photons. To our conjecture the quantum mechanical dipole moment is a fractional charge quantization, *i.e.*,

$$\text{quantum mechanical dipole moment} = \sum_{n_f=0.1}^{0.9} \hbar Q_{n_f}, D \equiv \lambda \quad (8)$$

where D is the displacement of charge either on an electron or in many electrons system, λ the wavelength of the interacting photons and “ \equiv ” congruent operator. Using fractional Fourier transform (FRFT), Hermite function for the fractional quantum states, *i.e.*, Equations (1) and (2), quantum mechanical dipole moment [Equation (8)] and the quantum theory of dielectric susceptibility is obtained with a constant [27]. The constant is ascribed to giant magneto resistance is discussed and the calculation of quantum electric susceptibility of dielectric material with particular reference to mesoscopic fields in a cavity is established in [27] [28], *i.e.*,

$$\chi_p = \left(\frac{4\pi p_o^3}{n_f \alpha_q E_m} \frac{g^2}{\hbar c} \right)^{\frac{1}{3}} \epsilon^{1/3} \epsilon^{2/3} \quad (9)$$

where p_o is the polarization in a cavity at zero kelvin, $0.1 \leq n_f \leq 0.9$, α_q the quantum electron polarizability (orientation of sub-quanta (twigs) of an electron string or wire either due to single electron or many electrons system), E_m the molecular field inside the cavity, gyroscopic constant (0.02 - 0.08), $g^2/\hbar c$; ϵ' the real permittivity and ϵ'' the imaginary permittivity. They ascribe the constant the GMR, *i.e.*,

$$\text{GMR} = \left(\frac{4\pi p_o^3}{n_f \alpha_q E_m} \right)^{\frac{1}{3}} \frac{g^2}{\hbar c}. \quad (10)$$

The most attractive quantum electrodynamic potential of an electron or electron quanta (the interior of which is envisaged as a potential well and is defined by the strength of the quantum well)

$$\gamma_{n_f} = \frac{2}{\hbar} (\pi \mu r_e n_f \hbar)^{1/2} = \left(\frac{4\pi \mu r_e n_f}{\hbar} \right)^{1/2}; 0.1 \leq n_f \leq 0.9 \quad (11)$$

where μ is the reduced mass of electron (equivalent to quanta of electron), r_e is the radius of electron varying with the depth of the quantum well and n_f is the fractional quantum numbers corresponding to varying strips of the depth of quantum well. GMR is associated with quantum electrodynamic (QED) potential in a cavity with mesoscopic fields preferably due to fractional charge quantization. The concept of quantum capacitance is also floated [11] which follows the shape/profile of Gaussian tail. The fractional charge quantization if oriented in a preferential direction will results in to GMR.

2.3. Case III—Quantum Hall Effect (QHE)

The megnetoresistance in quantum Hall effect should depend on magnetic field when an electron (charge as a constant physical entity) is fractionally quantized with twisting and twiggging of an electron quanta. This is why we are interested in quantum Hall effect on heterostructure semiconductors by Von Klitzing [1]. The electric field is fractionally quantized with a gap of quantum Hall resistance, *i.e.*, $R_q \sim \frac{h}{e^2} \sim 25813 \Omega$. To our understanding, this resistance is a manifestation of twisting and twiggging effects of an electron quanta. This is visible in our Equations (9) and (10), with a gyroscopic constant, $\frac{g^2}{\hbar c}$, *i.e.*, $0.02 \leq \frac{g^2}{\hbar c} \leq 0.08$. Magnetoresistance is of two types, one is longitudinal and the other is transverse. The longitudinal magnetoresistance is associated with magnetic field parallel to the current. The excitonic quantized Hall state at total Landau level filling factor is unity with longitudinal component vanishing and Hall component developing. The Lorentz force, in QHE, for a single electron, is

$$F = \frac{dp}{dt} = -e(\bar{E} + \bar{v} \times \bar{B}) - \frac{p}{\tau}. \quad (12)$$

Changing \bar{E} with E_Q , *i.e.*, electric field due to fractional distribution of charges in sub quanta or twiggs on an electron wire or string, \bar{v} with $\frac{\hbar k}{m_e}$, m_e with $\frac{E_{n_f}}{c^2}$ where E_{n_f} is the energy due to sub-quanta of an electron and τ with τ_Q where τ_Q is the relaxation time for twiggs on an electron wire. After simplification of Equation (12) with substitutions, the quantized Lorentz force due to single electron is

$$F_q = \frac{dp}{dt} = -e \left[\bar{E}_{Q_{n_f}} + 2\pi \left(\frac{v_{n_f}}{k_{n_f}} \right)^2 \times \bar{B} \right] - \frac{\hbar k}{\tau_{Q_{n_f}}} \quad (13)$$

where $Q_{n_f} = \sum_{n_f=0.1}^{0.9} Q_{n_f}$, v_{n_f} is an integrated vibrational frequency of each of the twiggs at different fractional quantum numbers, *i.e.*, $0.1 \leq n_f \leq 0.9$. In quantum Hall effect, the current is not independent of time because the fractional charge in their corresponding sub-quanta (twiggs) of an electron is dependent on twisting time or energy operator. Thus, we change the following relationships of classical Hall effect, *i.e.*,

$$-e\bar{E}_x - \omega_c (\hbar k)_y - \frac{(\hbar k)_x}{\tau} \neq 0, \quad -e\bar{E}_y - \omega_c (\hbar k)_x - \frac{(\hbar k)_y}{\tau} \neq 0 \quad (14)$$

$$\begin{aligned} -e\bar{E}_{Q_x} - \omega_c (\hbar k)_y - \frac{(\hbar k)_x}{\tau} &= -\frac{\hbar}{i} \frac{\partial}{\partial t} = i\hbar \frac{\partial}{\partial t} \\ -e\bar{E}_{Q_y} - \omega_c (\hbar k)_x - \frac{(\hbar k)_y}{\tau} &= -\frac{\hbar}{i} \frac{\partial}{\partial t} = i\hbar \frac{\partial}{\partial t} \end{aligned} \quad (15)$$

Using eigenfunction ψ_{n_f} for an electron (Equation (3)) in Equation (15), we get two sets of energy eigenvalue equations

$$\begin{aligned} &\left[e\bar{E}_{Q_x} - \omega_c (\hbar k)_y - \frac{(\hbar k)_x}{\tau} \right] \psi_{n_f} \\ &= i\hbar \frac{\partial}{\partial t} \psi_{n_f} = i\hbar \frac{\partial}{\partial t} e^{in_f \alpha} = i\hbar (in_f) e^{in_f \alpha} \frac{\partial \alpha}{\partial t} = -\hbar (n_f) \psi_{n_f} \frac{\partial \alpha}{\partial t} \end{aligned} \quad (16)$$

$$\begin{aligned} &\left[-e\bar{E}_{Q_y} + \omega_c (\hbar k)_x - \frac{(\hbar k)_y}{\tau} \right] \psi_{n_f} = i\hbar \frac{\partial}{\partial t} \psi_{n_f} = -\hbar \frac{\partial}{\partial t} \psi_{n_f} \frac{\partial \alpha}{\partial t} \\ E_{op} \psi_{n_f} &= -\frac{i\hbar}{2\psi_{n_f}} \cdot \frac{\partial \alpha}{\partial t} \end{aligned} \quad (17)$$

where α is rotational angle of FRFT ((defined in Equation (1)) in (time, frequency) plane. Equation (16) can be solved by considering $-e\bar{E}_{Q_x} + \omega_c (\hbar k)_y$ and $-e\bar{E}_{Q_y} + \omega_c (\hbar k)_x$ as Hermitian Hamiltonian operators. The cyclotron frequency for each of sub-quanta, *i.e.*, twigs) on the lateral surface of an electron string will be different from each other despite the fact that they are integrated on a lateral surface of an electron wire, as a consequence of which, we shall encounter GMR. The twisting time of an electron quanta for each sub-quanta will vary. This shows that $\frac{\partial \alpha}{\partial t}$ will also vary with different rotation angles and with the frequency of each

sub-quanta. The classical Hall coefficient usually depends on the number of charge carriers and also on moderate to high magnitude fields. For QHE, the Hall coefficient becomes insignificant due to single electron and due to fractional charge quantization either on a single electron or many electron system. The GMR is enhanced, especially due to the transverse component of the magnetic field. Therefore, it is suggested that the Hall coefficient in QHE should be replaced by drag coefficient or resistance known as quantum Hall resistance, *i.e.*, the drag resistance is quantized in terms of $\frac{h}{e^2}$. The classical cyclotron frequency is defined as:

$$\omega_c = \frac{eB}{m_c} \quad (18)$$

e is changed with $\sum_{n_f=0.1}^{0.9} Q_{n_f}$, m_e with $\frac{E_{n_f}}{c^2}$ and c with λv_{n_f} . λ is also changed with $\frac{2\pi}{k_{n_f}}$ in Equation

(17). After substitutions and simplifications, Equation (17) is changed in to quantum cyclotron frequency for each of sub-quanta on the lateral surface of electron wire, *i.e.*,

$$\omega_{cq} = \frac{2\pi}{\hbar} \sum_{n_f=0.1}^{0.9} B \left(\frac{Q_{n_f}}{k_{n_f}} \right) \left(\frac{v_{n_f}}{k_{n_f}} \right). \quad (19)$$

With resonance Raman Scattering in the fractional regime, $\left(\frac{Q_{n_f}}{k_{n_f}} \right)$ and $\left(\frac{v_{n_f}}{k_{n_f}} \right)$ can be easily determined.

2.4. Case IV—Quantum Conductance

Quantum conductance was first experimentally observed by Wees *et al.* [2]. They observed that the conductance did not increase continuously but rather in quantized steps of $\frac{2e^2}{h}$, where h is Planck's constant.

When the electronic mean free path of a wire exceeds the wire length, the wire behaves like an electron wave guide. Each wave guide mode or channel (ballistic conductors) contributes an amount G_0 to the total conductance of the wire, *i.e.*,

$$G_o = \frac{2e^2}{h} = \frac{2}{h/e^2} = \frac{2}{R_{qH}} \quad (20)$$

where R_{qH} is the quantum Hall resistance, *i.e.*, $R_{qH} = \frac{h}{e^2} \sim 25813 \Omega$. Usually, we know that the combined effect of Ohmic resistance and dynamic resistances (capacitive reactance and inductive reactance) is called impedance. The inverse of the impedance is termed as admittance. To our opinion, the inverse of the quantum Hall resistance is quantum conductance thus Equation (19) is modified as

$$G_o = 2G. \quad (21)$$

We consider the current density j equal to current I , *i.e.*, $j = -ev(\mu_1 - \mu_2)$, where v is the velocity of electron, μ_1 and μ_2 are chemical potentials connecting the two reservoirs adiabatically for a one dimensional wire and e the charge of an electron. Since $(\mu_1 - \mu_2)eV$ is the electromotive force to drain the current in between the two reservoirs and V is equal to voltage. The resulting conductance G will be determined as follows:

$$\begin{aligned} I &= \frac{V}{R_{qH}} \Rightarrow V = I \cdot R_{qH} = -ev(\mu_1 - \mu_2)R_{qH} \\ \Rightarrow \frac{1}{R_{qH}} &= \frac{I}{V} = \frac{-ev(\mu_1 - \mu_2)}{V} = G \end{aligned} \quad (22)$$

$$\Rightarrow G_o = 2G = \frac{-2ev(\mu_1 - \mu_2)}{V} = G. \quad (23)$$

With our conjecture of fractional charge quantization, change e with Q_{n_f} , v with $\frac{\hbar k_{n_f}}{m_e}$, m_e with $\frac{E_{n_f}}{c^2}$, c with λv_{n_f} , we get the modified definition of

$$G_o = -\frac{2}{V} \sum_{n_f=0.1}^{0.9} Q_{n_f} v_{n_f} (\mu_1 - \mu_2) \quad (24)$$

where v_{n_f} is the velocity of sub-quanta or twigs on the lateral surface of an electron string or wire. Looking carefully Equation (23) and comparing with Equation (21), G_o can be regarded as conductance for fractional quantized charges on sub-quanta. The current density or current due to twigs (sub-quanta) on the lateral surface of an electron wire, according to our calculations is now defined by the following relationship

$$j \text{ or } I = -ev(\mu_1 - \mu_2) = \sum_{n_f=0.1}^{0.9} Q_{n_f} (\mu_1 - \mu_2) 2\pi \left(\frac{v_{n_f}}{k_{n_f}} \right)^2 \quad (25)$$

where v_{n_f} is the fractionally quantized frequency of twigs. Equation (24) can be calculated for data from resonant Raman scattering in the fractional Hall regime. The velocity v_{n_f} in Equation (23) for each of the twigs on the lateral surface electron wire can be determined from cyclotron frequencies of the corresponding twigs and, of course, with resonant Raman Scattering.

3. Conclusion

Formulas for quantum resistivity (Quantum conductivity) and quantum conductance are developed by using

fractional Fourier transform. Formulas for quantum behaviour of dielectricity and giant magneto resistance are suggested by using fractional Fourier transform. Formulas for quantum Hall effect following the fractional electric field are suggested. Raman and resonance Raman spectroscopy are suggested for measuring diverse parameters pertaining to quantum behaviour of resistivity, dielectricity, GMR, Hall effect and conductance.

References

- [1] Klitzing, V. (1987) Quantum Hall Effect in Heterostructure Semiconductors. *American Physical Society News Letter*.
- [2] Van Wees, B.J., Van Houten, H., et al. (1988) Quantized Conductance Point Contacts in a Two Dimensional Electron Gas. *Physical Review Letters*, **60**, 848-850. <http://dx.doi.org/10.1103/PhysRevLett.60.848>
- [3] Yousaf, S., Raza, S.M., et al. (2008) Absorption of Radiant Energy in Water: A New Conjecture and Theory of Charge Quantization in Chromotized Water Samples. *Science International (Lahore)*, **20**, 189-195.
- [4] Yousaf, S., Raza, S.M. and Masoom, Y. (2008) Colours as Catalysts in Enzymatic Reactions. *Journal of Acupuncture and Meridian Studies*, **1**, 139-142. [http://dx.doi.org/10.1016/S2005-2901\(09\)60034-0](http://dx.doi.org/10.1016/S2005-2901(09)60034-0)
- [5] Yousaf, S., Raza, S.M. and Masoom, Y. (2011) A Case History of Treatment of Cutaneous Leishmaniasis by Chromotherapy. *Chinese Medicine*, **2**, 43-46. <http://dx.doi.org/10.4236/cm.2011.22008>
- [6] Yousaf, S., Raza, S., Masoom, Y. and Samad, A. (2009) Effect of Different Wavelengths on Superoxide Dismutase. *Journal of Acupuncture and Meridian Studies*, **2**, 236-238. [http://dx.doi.org/10.1016/S2005-2901\(09\)60060-1](http://dx.doi.org/10.1016/S2005-2901(09)60060-1)
- [7] Yousaf, S., Raza, S.M., Masoom, Y., et al. (2011) Effect of Different Colours in the Visible Region on Leishmania Tropic. *Advances in Biosciences and Biotechnology*, **2**, 380-384. <http://dx.doi.org/10.4236/abb.2011.25055>
- [8] Yousaf, S., Raza, S.M. and Ahmed, M.A. (2008) Newly Developed Recursive Relationship for Fractional Quantum States and Associated Energy Eigen Values. *Science International (Lahore)*, **20**, 255-260.
- [9] Peter, A. and Grubber, P. (2007) Giant Magnetoresistance. *APS News Letter*, **16**, No. 10.
- [10] Rehman, F., Raza, S.M. and Ahmed, M.A. (2009) Quantum Theory of Dielectricity and Its Application to Dolomite. *Science International (Lahore)*, **21**, 29-32.
- [11] Jabeen, S., Raza, S.M., et al. (2012) Quantum Mechanical Analysis on Faujasite-Type Molecular Sieves by Using Fermi Dirac Statistics and Quantum Theory of Dielectricity. *Journal of the Chemical Society of Pakistan*, **34**, 251.
- [12] Gormani, M., Rehman, F., et al. (2006) Quantum Behaviour of Dielectric in Dolomite of Balochistan, Pakistan. *Journal of the Chemical Society of Pakistan*, **28**, 414-416.
- [13] Haroche, S., Gilles, N., et al. (2006) Step by Step Engineered Entanglement with Atoms and Photons in a Cavity. *AIP Conference Proceedings*, **551**, 143.
- [14] Haroche, S., Gilles, N., et al. (1995) Manipulating Quantum Fields with a Single Atom in a Cavity. *AIP Conference Proceedings*, **329**, 30. <http://dx.doi.org/10.1063/1.47571>
- [15] Britton, J., Liebfried, D., et al. (2009) Scalable Arrays of Paul Traps in Degenerate Silicon. *Applied Physics Letters*, **95**, Article ID: 173102. <http://dx.doi.org/10.1063/1.3254188>
- [16] Maitre, X., Hagley, X., et al. (1997) Quantum Memory with a Single Photon in a Cavity. *Physical Review Letters*, **79**, 769. <http://dx.doi.org/10.1103/PhysRevLett.79.769>
- [17] Derrick, E.B., Haroche, S. and Raimond, M. (1996) Characterising Whispering-Gallery Modes in Microspheres by Direct Observation of the Optical Standing Wave Pattern in the Near Field. *Optics Letters*, **21**, 698. <http://dx.doi.org/10.1364/OL.21.000698>
- [18] Knight, J.C., Dubreuil, N., et al. (2000) Single Photons Nondestruction Clarified. *Physics Today*, **53**, 92. <http://dx.doi.org/10.1063/1.1325252>
- [19] Haroche, S. and Daniel, K. (1989) Cavity Quantum Electrodynamics. *Physics Today*, **42**, 24-30. <http://dx.doi.org/10.1063/1.881201>
- [20] James, C.B., Steven, J.W. and David, R.J. (2001) Time Measurement at the Millanum. *Physics Today*, **54**, 37. <http://dx.doi.org/10.1063/1.1366066>
- [21] Iqbal, S., Sarwar, F., Raza, S.M. and Rehman, A. (2015) How Fractional Charge on an Electron in the Momentum Space Is Quantized? *ASRJETS*, **14**, 265-272.
- [22] Iqbal, S. (2012) Analysis and Applications of Fractional Fourier Transform. Unpublished PhD Thesis, University of Balochistan, Quetta, Pakistan.
- [23] Iqbal, S., Sarwar, F. and Raza, S.M. (2016) Eigen Functions for a Quantum Wire on a Single Electron at Its Surface and in the Quantum Well with Beaded Fractional Quantized States for the Fractional Charges. *JAMP*, **4**, 320-327. <http://dx.doi.org/10.4236/jamp.2016.42039>

- [24] Almeida, L.B. (1994) The Fractional Fourier Transform and Time-Frequency Representations. *IEEE Transactions on Signal Processing*, **42**, 3084-3091. <http://dx.doi.org/10.1109/78.330368>
- [25] Iqbal, S., *et al.* (2012) Fractional Fourier Integral Theorem and Fractional Fourier Cosine and Sine Transforms. *Science International (Lahore)*, **24**, 233-238.
- [26] Lefrant, S., Mulazzi, E. and Mathis, C. (1994) Raman Spectra of n-Doped Transpolyacetylene Systems: Experiments and Theory. *Physical Review B-I*, **49**, 13400-13407.
- [27] Iqbal, S., Jabeen, S., Sarwar, F. and Raza, S.M. (2016) Quantum Theory of Mesoscopic Fractional Electric Fields in a Cavity of Viscous Mediums. *WJCMP*, **6**, 39-44. <http://dx.doi.org/10.4236/wjcmp.2016.61006>
- [28] Jabeen, S. and Jabeen, S. (2013) Dielectric Behaviour of Indigenous Rock Materials? PhD Thesis (Supervised by Dr. Syed Mohsin Raza), University of Balochistan, Quetta, Pakistan.