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Two Dimensional Heisenberg Exchange Interaction in the Magnetization Studies of Multiferroic

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ABSTRACT

Multiferroics are novel classes of materials that exhibit cross-coupling of mutually excluding phenomena, *i.e.* magnetism and ferroelectricity. In recent years, the coexistence of ferroelectricity and magnetic orderings has become a hot issue and drawn considerable attentions due to the promising applications to these days technology and the fundamental science involved in these classes of materials. The microscopic origins of magnetism and ferroelectricity differ fundamentally, while the real mechanism of ferroelectricity is still under debate. In the present work, we have started from a simple method Heisengerg hamiltonian and an interaction term resulting from electric field coupling with the magnetic spins with anisotropic limit, demonstrated that magnetization can be manipulated by electric field and anisotropic field in agreement with results experimentally observed. In the multiferroic thin film system the magnetic field tends to play a role in stabilizing the spins in preferred orientations and induces a coupling of magnetism and ferroelectricity that opens a route to switch magnetization with electric polarazation and vice-versa.

Keywords: Multiferroic; Multiple Ferroic Orders

1. Introduction

Multiferroics are unique classes of materials whose fundamental state is both magnetic and ferroelectric [1]. In these materials magnetism and ferroelectric ordering coexist no matter how their course of existence varies [2]. The microscopic description of magnetism is based on the localized magnetic moments, mostly due to the partially filled *d* or *f* shell transition-metal or rare-earth elements electrons in which the exchange interactions between these localized moments exhibit magnetic orderings. Whereas the origin of ferroelectrics is wide range (can be due to: Charge ordering, lone pairs, etc.) and forbids the d shell electrons in contrast to magnetism [3,4]. Thus, multiferroics couples the two insurmountable phenomena. They remarkably show strong coupling between charges, spins, lattice and orbital degrees of freedom and the interplay between various competing orders gives rise to interesting features and varieties to these classes of materials. The overlap of strongly correlated magnetic and ferroelectric orders enables the spinnability of one domain over the other [5-7]. For instance, in ferromagnetic materials the magnetization M is easily switchable parameter on an application of external stimulating magnetic field, *H* and in ferroelectrics the polarization *P* is also a controllable parameter by an ex-

ternal electric field, *E*. However, on the other hand, in multiferroics these switchable order parameters, *P* and *M* are cross-linked, *i.e.* magnetization can be switched by external electric field and polarization can also be tunned by an applied magnetic field. The magnetoelectric spectra in $GaFeO_3$ is found to be proportional to the photon energy which depends on the direction of magnetization [8-10].

2. Theoretical Model

We consider the spin Hamiltonian of the multiferroic thin film system that exhibits frustrated spin interactions [11,12]. The Hamiltonian of this anisotropic many body interaction system on the influence of external fields can be expressed by [13-15],

$$H = -\sum_{ij} J_{ij} S_i \cdot S_j - g\mu_B H \sum_i S_{iz} - \mu E \sum_i S_{iz} + D\sum_i S_{iz}^2 \quad (1)$$

where $S_i(S_j)$ is spin operator at sites $i(j)$. J_{ij} is the exchange coupling that depends on the relative positions of neigbhoring spins and is defined by $J_{ij} = J(R_i - R_j)$. The constant g is the gyromagnetic ratio, and the second term of Equation (1) is the effective Zeeman field with an external deriving field while in this model chosen to be parallel to the $+z$ axis to line up all the spins in the same direction. Whereas, the third term includes an electric field

E that interacts with the spins in the system. The constant μ describes the coupling strength between the spins and the radiation field that is either intrinsic or extrinsic [16,17]. The terms μ_B and *D* are the Bohr magneton and magnetic anisotropic (or spin wave stiffness) constants, respectively [17,18]. The total spin operators in the Hamiltonian are:

$$S^2 = \sum_i (S_i)^2$$

$$S_z = \sum_i S_{iz}$$

The ground state of N identical atoms of spin *S* is defined as

$$S^2 |0\rangle = NS(NS+1)|0\rangle$$

$$S_z |0\rangle = NS|0\rangle$$

The total spin operator at any site *i* has components S_{iz}, S_{ix} and S_{iz} that should be treated independently with an identity S_i. $S_i = S(S + 1)$. Transferring the spin operator problems into many body interaction systems upon replacing with bosonic creation and annihilation operators expressed as

$$S_{i^+} = S_{ix} + S_{iy} = (2S)^{12}\left(1 - \frac{a_i^+ a_i}{2S}\right)^{\frac{1}{2}} a_i \qquad (2)$$

$$S_{i^-} = S_{ix} - S_{iy} = (2S)^{12} a_i^+ \left(1 - \frac{a_i^+ a_i}{2S}\right)^{\frac{1}{2}} \qquad (3)$$

where $a_i^\dagger$ and a_i are the creation and annihilation operators satisfying the commutation relation $[a_i, a_i^\dagger] = \delta_{il}$ It is also possible to transform this bosonic operators to magnon variable operators b_k and $b_k^\dagger$ so that these operators can be related to the bosonic operators [19,20]

$$b_k = N^{\frac{-1}{2}} \sum_i e^{ik.x_i} a_i$$

and

$$b_{k^+} = N^{\frac{-1}{2}} \sum_i e^{-ik.x_i} a_i^+$$

where the inverse transformation is

$$a_i = N^{\frac{-1}{2}} \sum_k e^{-ik.x_i} b_k$$

and

$$a_{i^+} = N^{\frac{-1}{2}} \sum_i e^{ik.x_i} b_k^+$$

At low excitation *i.e.* where the interaction is predominated by bilinear spin variables, neglecting the interaction corresponding to higher order terms, the reduced Hamiltonian is of the form

$$H = -JNZS^2 + DNS^2 - g\mu_B HNS - \mu ENS$$
$$+ \sum_k \left(-2JZS(\gamma_k - 1) - 2DS + g\mu_B H + \mu E\right)b_k^+ b_k . \qquad (4)$$

Here, *z* is defined to be the number of nearest neighbor spins. In this Hamiltonian we consider only the term bilinear to the magnon variables

$$H_m = \sum_k \left(-2JZS(\gamma_k - 1) - 2DS + g\mu_B H + \mu E\right)b_k^+ b_k \qquad (5)$$

where, $\gamma_k = \sum_\delta e^{-ik.\delta}$.

This simplified interaction term is equivalently expressed as $H_m = \sum_k n_k \omega_k$, if we write

$$\omega_k = -2JZS(\gamma_k - 1) - 2DS + g\mu_B H + \mu E \qquad (6)$$

The dispersion equation on the approximation that $|k \cdot \delta| \ll 1$ can be reduced to

$$\omega_k = JSa^2 k^2 - 2DS + g\mu_B H + \mu E , \qquad (7)$$

so that

$$\omega_k = \beta k^2 + \gamma \qquad (8)$$

The above expression is a dispersion relation with some additional term $\gamma = -2DS + g\mu_B H + \mu E$ that tends to show how the dispersion term depends on the field components. Moreover, the constant $\beta = JSa^2$.

3. Results and Discussion

The spin-wave dispersion function of the multiferroic thin film system is obtained as a function of the wave vector and the field components.

A significant change in the dispersion relation is observed as depicted by **Figure 1** above at different γ values [21]. This can be used to modify the behavior of the multiferroics up on appropriate selection of the magnetic
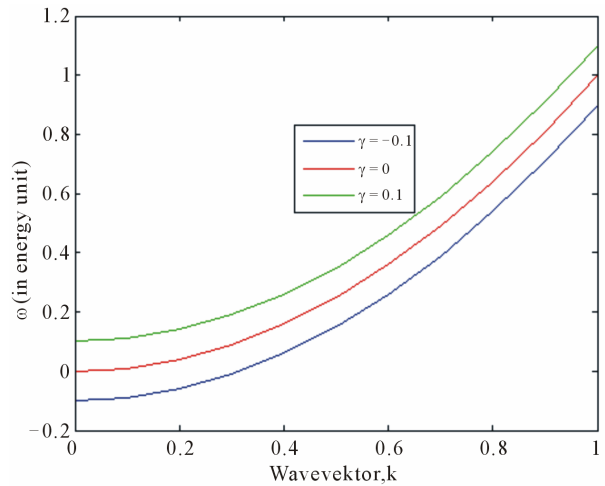

Figure 1. (color online) The figure indicated shows the variation of dispersion function for different gamma values.

and electric field values in some orientations. The inter-dependence of the magnetic and electric fields (their couplings) in multiferroic materials is also analyzed based on the spin-wave occupation number, $\sum_k b_k^+ b_k$.

The change in magnetization is obtained to be

$$\Delta M = \frac{\mu_o k_{B^{\frac{3}{2}}}}{4(\pi\beta)^{\frac{3}{2}}} e^{-\frac{\gamma}{k_B T}} T^{\frac{3}{2}}$$

where $\gamma = \varsigma + \mu E$ and $\varsigma = -2DS + g\mu_B H$ and the change in magnetization versus temperature at some specific γ is shown in **Figure 2** below for low temperature limits.

Consequently, the magnetization is a function of the γ values at low temperature as indicated by **Figure 3** shown below [22,23]. Thus, the electric field dependence of
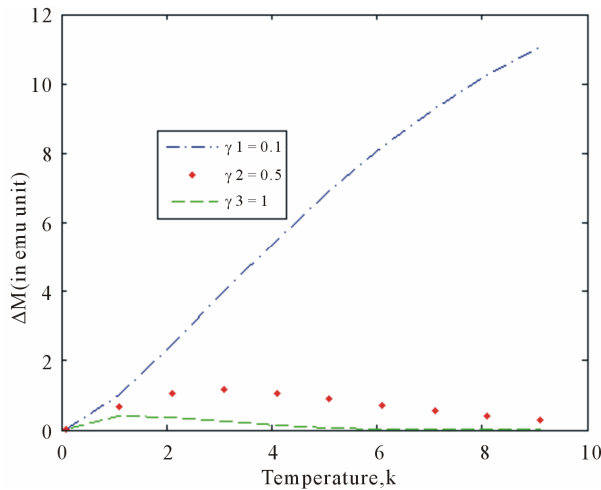

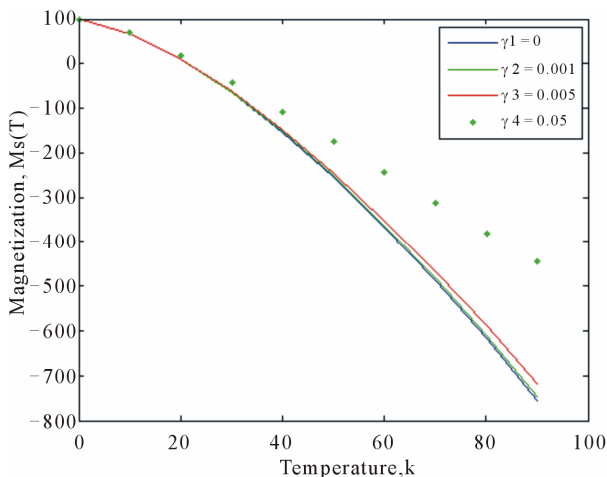**Figure 2. Change of magnetization versus temperature at different gamma values.**

Figure 3. Dependence of magnetization on the parameter γ, which can be tuned by explicitly on the applied fields and the nature of magnetic anisotropy.

magnetization can be easily deduced from the expression $\gamma = \zeta + \mu E$ where $\zeta = -2DS + g\mu B\ H$ for some specific values of the parameters, D, S, and H at a particular temperature.

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