Theoretical Economics Letters, 2019, 9, 1393-1410
http://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Erratum to "Testing and Predicting Volatility Spillover—A Multivariate GJR-GARCH Approach" [Theoretical Economics Letters, 2019, 9, 83-99]

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Received: October 24, 2018
Accepted: January 26, 2019
Published: January 29, 2019

The original online version of this article (Testing and Predicting Volatility Spillover—A Multivariate GJR-GARCH Approach" [Theoretical Economics Letters, 2019, 9, 83-99]. https://doi.org/10.4236/tel.2019.91008) unfortunately contains some mistakes. The author wishes to correct the errors.

Testing and Predicting Returns and Volatility Spillover—A Multivariate DBEKK Approach

Abstract

The aim of this paper is to assess the dynamic interdependence among Stock, Bond and Money market of Australia. The proposed diagonal BEKK (DBEKK) model allows for market interaction which provides useful information for pricing securities, measuring value-at-risk (VaR), asset allocation and diversification and, assisting financial regulators for policy implementation. The purpose of this paper is to examine the return and volatility spillovers for Stock, Bond and Money markets. Historical data on Stock, Bond, and T-bill of Australia's domestic financial markets from 4 April 2006 to 20 June 2016, for a total 883 observations are analyzed. The DBEKK model is estimated by QMLE. The DBEKK model is used as it is the only multivariate conditional volatility model with well-established regularity conditions and known asymptotic properties [1]. Application of the model to Australia's domestic Stock, Bond, and Money markets reveals that the domestic financial markets are interdependent and volatility is predictable. Empirical findings suggest return and volatility spillovers from Bond and Money markets to Stock market, Tbill to Bond, and Bond to Tbill. Further the negative spillovers were detected utilizing partial co-volatility in the DBEKK model. The empirical findings of this paper quantifies the association among the security markets which can be utilized for improving agents' decision-making strategies for risk management, portfolio selection and diversification. Based on these results, dynamic hedging strategies could be suggested to analyze market fluctuations in the financial markets.

Keywords

Diagonal BEKK, QMLE, Diversification, Spillovers, Partial Co-Volatility, Bond Market, Stock Market, Money Market

1. Introduction

Security traders in the financial markets make their "buy" and "sell" decisions based on the information available in the financial markets. The amount of risk associated with a series of returns, however, depends on the arrival of the so-called "good" and "bad" news that continuously spread throughout the financial markets in every moment of time. Since "news" is not directly observable thus returns are stochastic and volatile. An interesting feature of asset price is that "bad" news seems to have a more pronounced effect on volatility than does the "good" news. This asymmetric "news" is associated with the innovation distribution of losses and gains in the financial markets, which plays a vital role in determining the leverage effect on asset volatility. Black [2] finds that the leverage effect is caused by the fact that negative returns have greater influence on future volatility than the positive returns. To understand the dynamics of simultaneous presence of "news" and "leverage" effect on volatility, one is required to develop functional forms of the expected returns and volatility of return processes of a financial time series, while the return can be model as an autoregressive integrated moving average (ARIMA) process. There are three main ways of modelling financial volatility, such as implied volatility, realized volatility, and conditional volatility, McAleer *et al.* [1]. In this paper we use the conditional volatility approach to specify volatility function.

In developing dynamic volatility models, there are two strands of modelling conditional volatility *i.e.* the univariate and multivariate volatility modelling respectively. Engle [3] first introduced univariate autoregressive conditional heteroskedasticity (ARCH) model for measuring and predicting asset return volatility. This model is useful because it captures some stylized facts such as volatility clustering and thick-tail distribution of return series. Bollerslev [4] extended the ARCH model which allows for the effect of past volatility in the expanded ARCH model. This extension is widely known as the generalized ARCH (or GARCH) model. Tsay [5] derived Engle's [3] ARCH model from random coefficient autoregressive process, see McAleer [6]. Although useful, the basic ARCH/GARCH models are incapable of capturing leverage effects on volatility. Leverage effect is the tendency for volatility to decline when returns rise and rise when returns fall, Enders [7]. Black [2] first discovered the leverage effect that exists in the financial data and confirmed by French *et al.* [8]. Various types of volatility models, within the univariate framework, have been developed in the literature to address both the theory and empirical issues of the model, namely the news asymmetry, volatility clustering, thick-tail, non-normality, and risk premium in the financial returns. For example, Nelson [9] develops an Exponential GARCH (EGARCH); Engle and Ng [10] provide nonparametric tests for asymmetry between news and volatility, and Glosten *et al.* [11] propose asymmetric GARCH model. The asymmetric GARCH of Glosten *et al.* [11] is generally known as threshold GARCH (TGARCH, GJR-GARCH or AGARCH) model. McAleer [12] showed that GJR captures asymmetry. In the risk-return framework there was another development of the univariate ARCH/GARCH model, in which the first moment of a series is allowed to include the information generated by the second moment of the returns series. This specification is capable to deal with investor/agent's demand for compensation for holding risky assets. This extension is widely known as ARCH-in-Mean (or ARCH-M) model developed by Engle *et al.* [13]. Further extension such as GARCH-M, asymmetric GARCH (AGARCH) can be found elsewhere.

The first two moments respectively called mean and variance of return series have been investigated extensively in the univariate finance literature to understand the trading dynamics of risk and returns in the financial asset markets, for example Bollerslev [14] and Bera [15], among others. These articles use various modeling issues e.g. functional form and dependence. Joint estimation of the univariate mean-variance model reported elsewhere uses t-distribution or generalized error distributions (GED) as one might not want to perform a maximum likelihood estimation using normal distribution, because the normality assumption of unconditional volatility of innovation might not hold [7].

The Second strand of volatility modelling has been emerged from modelling volatilities of returns within the multivariate framework. Within this framework the shocks to volatility from one market are allowed to affect both the risk and return of the other markets. The dynamic dependence of multivariate financial assets provides a rich sources of volatility transmission that helps the investors to play active role in financial transactions. Specifically, the multivariate extension to univariate GARCH allows volatility spillovers and leverage effects across markets jointly. Directional causality between assets returns can be established among the securities by statistical testing. The multivariate extension to univariate model was first introduced by Engle, Granger, and Kroft (1984) in the ARCH context, and Bollerslev *et al.* [16] in the GARCH context. This multivariate GARCH is known as VEC model because of its structure. Further development of the multivariate volatility model is the Baba-Engle-Kraft-Kroner [17] BEKK model proposed by Engle and Kroner [17]. This model allows for dynamic dependence between the volatility series and guarantee that the covariance matrix of volatility is positive definite. This property is a requirement of a statistical model. But the interpretation of the model parameters is not straight forward.

In practice, the BEKK model is frequently used for conditional volatility modelling of financial series. Recently, Chikashi Tsuji [18] used BEKK to examine linkages between French and German Stock index returns. Gounopoulos *et al.* [19] used a BEKK model, to examine the linkages between insurance companies, currency exposures of US, UK, and Japanese banks and Stock returns. Similarly, Long *et al.* [20] analysed the conditional time-varying currency betas for five developed and six emerging financial markets by applying a BEKK model. Employing a BEKK model, Caporale *et al.* [21] tested the impact of exchange rate uncertainty on net equity and net Bond flows and on their dynamic linkages. The objective of Olson *et al.* [22] is to evaluate whether commodities have an effective function as a hedging tool for equity investors. Employing a BEKK model, they computed time-varying hedge ratios for the US equity index. Cardona *et al.* [23] examined the volatility transmission between US and and Latin American financial markets.

McAleer *et al.* [24], Ling and McAleer [25] showed that the quasi maximum likelihood estimators (QMLE) of the Diagonal BEKK (DBEKK) model are consistent and asymptotically normally distributed. Thus QMLE based inference and tests are valid in the DBEKK volatility models. However, asymptotic normality of the QMLE of the BEKK can't be proved except some conditions imposed on the parameters. McAleer *et al.* [1] developed a CCC VARM-AGARCH model as an extension of univariate asymmetric volatility model of Glosten *et al.* [11]. Ling and McAleer [25] have proved consistency and asymptotic normality of the QMLE based estimation of VARMA-GARCH model parameters. McAleer *et al.* [1] established strictly stationarity & ergodicity of the VARMA-asymmetric GARCH model. They established consistency and asymptotic normality of the QMLE estimators under certain conditions. The CCC model of Bollerslev [14] and DCC model of Engle are the reparametrization that connects Covariance & correlation matrices.

Very recently Chang *et al.* [26] in modelling volatility spillover between energy and agricultural markets draws attention on the use of commonly applied Full BEKK specification for estimating conditional volatility. They have argued that QMLE based parameter estimates of Full BEKK model has no asymptotic properties and hence there has no valid statistical tests for testing volatility spillover effects in BEKK. Similar is the case for DCC-Volatility models. They have argued using DBEKK instead of Full BEKK, because DBEKK models have stochastic validity for likelihood function and QMLE has the desirable statistical properties for developing statistical inference and tests. McAleer *et al.* [24] showed that the QMLE of the parameters are consistent and asymptotically normally distributed. Chang *et al.* [27], Chang *et al.* [26] [28], McAleer [12], Allen and McAleer [29], McAleer [30], are only a few important articles discussed about the statistical distributional issues for estimating and testing Full BEKK, DBEKK and triangular BEKK (TBEKK) volatilities spillovers. Chang *et al.* [26] suggested that the existence of multivariate eighth moments cannot be verified for the existence of distributional properties of the Full BEKK specification. Hence no valid test exists for testing volatility spillover effects. It is stated in the above papers that the application of the Full BEKK and TBEKK has no verifiable asymptotic properties. In this paper we explore the multivariate conditional mean and conditional volatility models using DBEKK specification to find out returns and volatility spillovers of Australian domestic Stock, Bond, and Money markets.

This paper is organized as follows. In Section 2, model and methodology is discussed. Section 3 describes the sources of data and statistical properties of the data. Real application of the models is reported in Section 4. Finally Section 5 concludes the paper with future research directions.

2. Methodology

To apprehend the dynamic interdependence of asset returns and volatility spillovers, we utilize multivariate autoregressive conditional mean and diagonal BEKK (DBEKK) conditional volatility models. The DBEKK model can be estimated by the quasi maximum-likelihood (QML) method. The quasi maximum-likelihood estimates are consistent and asymptotically normally distributed, McAleer *et al.* [1]. But the Full BEKK model has no statistical distributional properties. Hence QMLE is not appropriate for testing volatility spillovers in BEKK model, McAleer *et al.* [1], McAleer [30], Chang *et al.* [26]. On the other hand, the QMLE is appropriate for testing volatility spillovers in Diagonal BEKK, McAleer *et al.* [1]. We describe the DBEKK model below.

2.1. The Multivariate DBEKK-Volatility Model

Let $r_r = \left(r_{1t}, r_{2t}, \cdots, r_{Nt}\right)'$ be a vector of returns of N number of assets at time index t ($t = 1, 2, 3, \cdots, T$). The set of information available at time t is denoted by $\Im_{t-1}$. We assume that the dynamic multivariate security returns r_t can be adequately represented by a vector autoregression of order p conditional on the information set $\Im_{t-i}$ as

$$r_t \mid \Im_{t-1} = \Phi_0 + \sum_{l=1}^{p} \Phi(l) r_{t-l} + \varepsilon_t \tag{1}$$

where, $E\left(r_t \mid \Im_{t-1}\right) = \Phi_0 + \sum_{l=1}^{p} \Phi(l) r_{t-l} = \mu_t, \text{say}$, and $\Phi(l) = \left(\Phi_{ij}(l)\right)$ is the $N \times N$ coefficient matrix of the lagged dependent variable of the mean model. The $N \times 1$ intercept vector is denoted by Φ_0 and $\varepsilon_t \mid \Im_{t-1} = H_t^{0.5} e_t$, where $e_t = \left(e_{1t}, e_{2t}, \cdots, e_{Nt}\right)'$ is the independent and identically distributed (*iid*) random vectors of order $N \times 1$ with $Ee_t = 0$ and $Ee_t e_t' = I_N$, where I_N is an Identity matrix of order $N \times N$. The symmetric conditional variance-covariance matrix H_t of order $N \times N$ is defines as follows.

$$H_t = E\left(\varepsilon_t \varepsilon_t' \mid \Im_{t-1}\right) = E\left[\left(r_t - E\left(r_t\right)\right)\left(r_t - E\left(r_t\right)\right)' \mid \Im_{t-1}\right] \tag{2}$$

Model (1) with (2) can be written more compactly as $r_t \mid \Im_{t-1} \sim D\left(\mu_t, H_t\right)$, where $D(.,.)$ is some specified probability distribution. Or, equivalently as $\varepsilon_t \mid \Im_{t-1} \sim D(0, H_t)$. Various parameterizations for H_t have been proposed in the literature, for example, Bollerslev *et al.* [16], Engle [31], Tse and Tusi [32] among others.

Thus our model of return and volatility of returns takes the following form.

Return:

$$r_t \mid \Im_{t-1} = \Phi_0 + \sum_{l=1}^{p} \Phi(l) r_{t-l} + \varepsilon_t, \varepsilon_t \mid \Im_{t-1} \sim D(0, H_t) \tag{1'}$$

Volatility:

$$H_t \mid \Im_{t-1} = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B' \tag{2'}$$

The parameters Φ_0 is the intercept vector and $\Phi(l)$ is the coefficient matrix of the autoregression of lag order l for the mean model. The matrix C is a $N \times N$ lower triangular matrix such that CC' is symmetric and positive definite matrix containing the intercepts parameters of the conditional volatility model (2'). The matrices $A = (\alpha_{ij})$, $B = (\beta_{ij})$, $i, j = 1, 2, 3, \cdots, N$, are each $N \times N$ matrices of short-run and long-run weight parameters, respectively. The model (2') is generally known as Full BEKK model, Engle-Kroner [17].

In model (2') if the matrices A, and B, are diagonal we get a model what is called diagonal BEKK (DBEKK). We will treat model (2') as the DBEKK model with diagonal A, and B matrices. So model (2') is our DBEKK model of conditional volatility.

2.2. Estimation of the VAR-DBEKK Model

The mean-variance model (1') & (2') can be estimated jointly under non-normality by utilizing the quasi-maximum-likelihood (QML) method. The estimates thus obtained are called quasi maximum likelihood estimate (QMLE). Note that the asymptotic validity of the QMLE holds only for the DBEKK models, McAleer *et al.* [1]. The multivariate quasi maximum-likelihood estimates can be obtained by maximizing the following log-likelihood function

$$l(\theta) = -0.5NT\ln(2\pi) - 0.5\sum\left(\ln|H_t| + \varepsilon_t H_t^{-1}\varepsilon'_t\right) \tag{3}$$

where $l(\theta)$ is the log-likelihood function, N is the number of assets, T is the number of observations, θ is the full parameter set of the models (1') and (2'). ε_t and H_t are as defined above, and $|H_t|$ is the determinant of H_t.

Numerical optimization routine in RATS Estima can be used to maximize $l(\theta)$. In the case of non-normality, the resulting estimates are known as Quasi Maximum-Likelihood Estimate (QMLE) of θ.

Following the recent research by Chang *et al.* [26] and Chang *et al.* [27], we report the partial co-volatility spillovers for the DBEKK model only. Chang *et al.* [26] define the partial co-volatility spillovers as follows.

Partial co-volatility spillovers: $\dfrac{\partial H_{ij,t}}{\partial \varepsilon_{k,t-1}}, i \neq j, k = \text{either } i \text{ or } j$ and

See also Chang *et al.* [27] for notational consistency of the definition. The matrix H is as defined in (2') with diagonal A, and B matrices. The empirical calculations provided in Section 4.

2.3. Tests for Return Spillovers and Causality

Refer to the multivariate volatility model of Section 2.1, the following hypotheses are of interest to test for return spillover effects across assets by the Granger (1969, 1981) causality test. Considering three assets portfolio, the following hypotheses can be tested.

Return Spillovers from Asset *j* and *k* to Asset *i* ($i \neq j \neq k = 1,2,3$)

1) Return spillovers from Bond and T-bill to Stock

$H_0 : \phi_{12} = \phi_{13} = 0$ against $H_1 : \phi_{12} \neq \phi_{13} \neq 0$.

2) Return spillovers from Stock and T-bill to Bond

$H_0 : \phi_{21} = \phi_{23} = 0$ against $H_0 : \phi_{21} \neq \phi_{23} \neq 0$.

3) Return spillovers from Stock and Bond to T-bill

$H_0 : \phi_{31} = \phi_{32} = 0$ against $H_1 : \phi_{31} \neq \phi_{32} \neq 0$.

The above spillover tests are the causality tests in Granger's sense. The tests are performed using restricted unrestricted version of F-test. The F-test results are reported in the empiric al Section 4.

3. Data and Preliminary Results

Historical data on Stock, Bond, and T-bill of Australia's domestic market from 4 April 2006 to 20 June 2016, for a total 883 observations are used for analysis. The data was retrieved from Bloomberg database. The daily returns, in percentages, for Stock (all ordinaries), Bond (5-year maturity rate), and T-bill (90 day bank accepted bill) are constructed by the following growth rate form.

$$r_{it} = 100 \times \ln\left(\frac{p_{it}}{p_{it-1}}\right), \quad i = 1,2,\cdots,N\ ; \quad t = 1,2,\cdots,T \tag{6}$$

The variable p_{it} denote the nominal price of the i-th asset at time t and the variable r_{it} is the percentage log returns (or the growth rate) of the i-th asset at time t, p_{it-1} is the one-period lag of p_{it}, and $\ln(.)$ is the natural logarithm of the argument. N is the number of asset and T is the time index.

Data Property and Preliminary Results

In this section we provide graphical means to explore the data properties. First we plot the return series and the squared return series. Then we provide summary statistics in Table 1. We use RATS package for empirical computation of this paper.

Figure 1, shows the time plots of daily log returns, in percentage, of (a) Stock, (b) Bond, and (c) T-bill. The volatility seems to be larger during June 2008-December 2008 and August 2011-February 2012 for Stock returns; October

Table 1. Basic statistics of the return series from 4 April 2006 to 20 June 2016.

Statistics	Stock	Bond	T-bill
Mean (%)	−0.034 (0.337)	−0.095 (0.045)	0.003 (0.216)
Yearly mean (%)	−8.806	−24.61	0.78
Stdev (%)	1.062	1.413	0.065
Yearly stdev (%)	17.09	22.74	1.05
Min	−4.249	−6.278	−0.389
Max	5.529	4.667	0.740
Skewness	−0.203 (0.0140)	−0.273 (0.0009)	1.414 (0.0000)
Excess kurtosis	2.241 (0.0000)	1.816 (0.0000)	20.675 (0.000)
LB(20)	22.591 (0.309)	58.436 (0.000)	21.441 (0.372)
$LB^2(20)$	504.046 (0.0000)	469.484 (0.0000)	40.316 (0.0040)
JB- χ^2 (2) Test	190.841 (0.0000)	132.348 (0.0000)	16020 (0.0000)
Tsay Ori-F(10,865) Test (lags 4)	4.442 (0.0000)	3.139 (0.0006)	2.619 (0.0028)
McLeod and Li Test (lags 4)	331.257 (0.0000)	254.087 (0.0000)	21.067 (0.0206)
ARCH (LM) Test (lags 4)	36.360 (0.0000)	16.974 (0.0000)	4.404 (0.0000)

Note: p-value is in parentheses.

2008-April 2009, August 2011-December 2011, March 2012-November 2013, and March 2015-December 2015 for Bond; and occasionally around December 2009 and July 2011-August 2011 for T-bill. Time plot of daily log returns highlighted that Bond market is affected the most by the global financial crisis (GFC) while T-bill is least affected as T-bill is for short term and 5 year Bond market is for long term. Therefore, the three Australian financial markets are affected simultaneously with some variation.

Figure 2 shows some dependence in the individual asset returns with high peaks. This is further confirmed by the Ljung-Box [33] test reported in summary Table 1 below. The jumps are particularly associated with global financial crisis (GFC) periods for all of the series as the jumps are around 2008-2009 and 2011-2012 and 2015 for Stock; 2008-2009, 2011-2012, 2014-2015 for Bond; and occasionally around 2009 and 2011-2012 for T-bill. The spikes and the LB-Q statistics on the squared series suggests that the percentage changes of the series have some ARCH effects.

Table 1 provides various statistics to judge the data properties. In particular, all of the return series are significantly skewed and are heavy-tailed distributions. The later property reveals that the series exhibits volatility clustering. This shows that the rare tail-events have longer effects. The mean of the Stock and T-bill are insignificant while the average Bond return is significant at the 5% level. Serial correlation up to 20 lags for Stock and T-bill are insignificant but Bond returns are serially dependent. The squares series, however exhibits serial dependence in the second moment for all of the series. Both the Tsay [34] and McLeod and Li

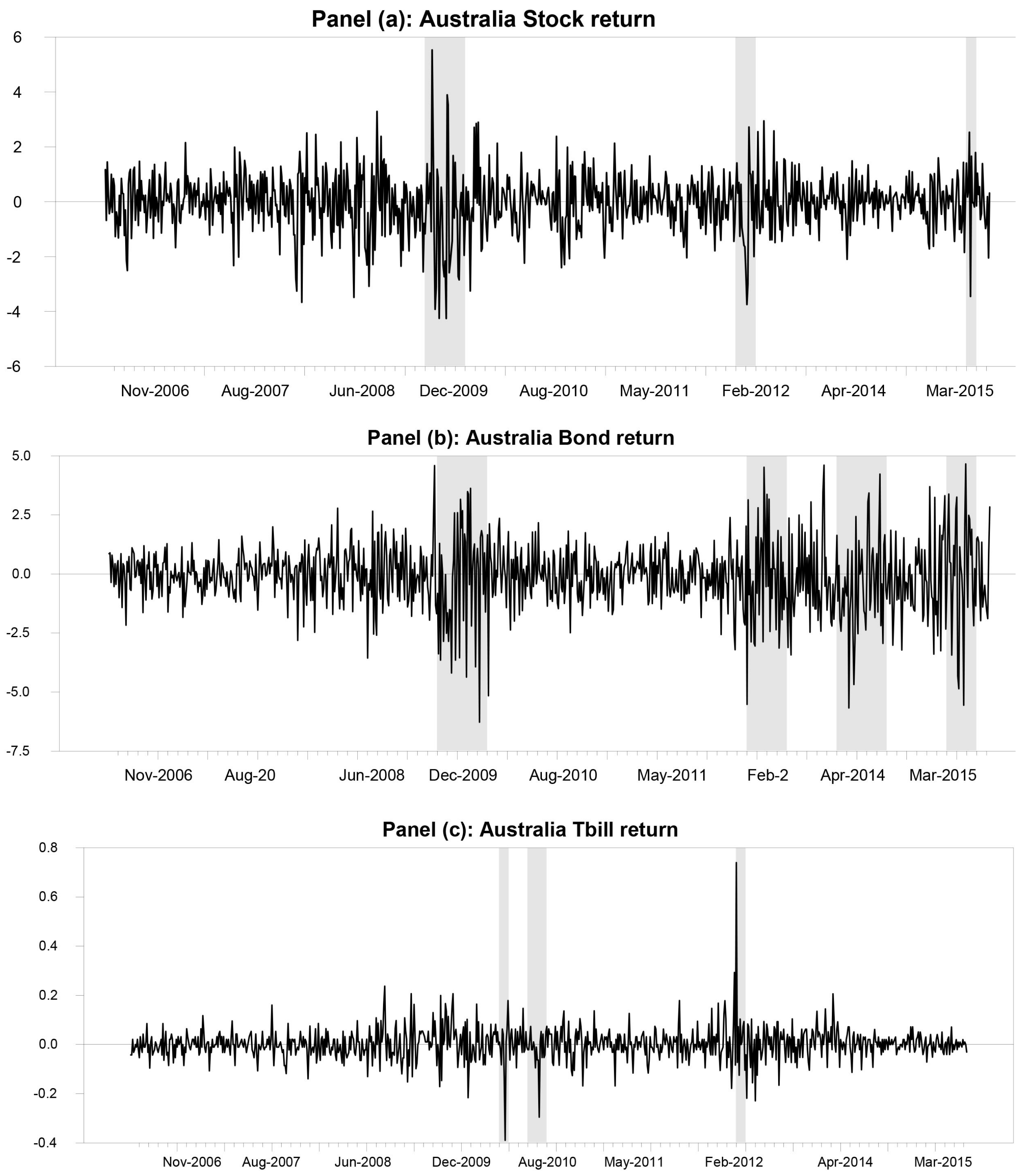

Figure 1. Time plot of daily log returns in percentage from 4 April 2006 to 20 December 2016 (x-axis representing the time dimension and y-axis representing the percentage log returns).

[35] tests supports for nonlinearity in all of the series. Existence of conditional volatility in all series is supported by the Engle [3] ARCH test. Further, the normality of all of the series is rejected by the Jarque-Bera [36] test. We have also applied Tiao-Box [33] test for cross-correlation to all of the series the series (not

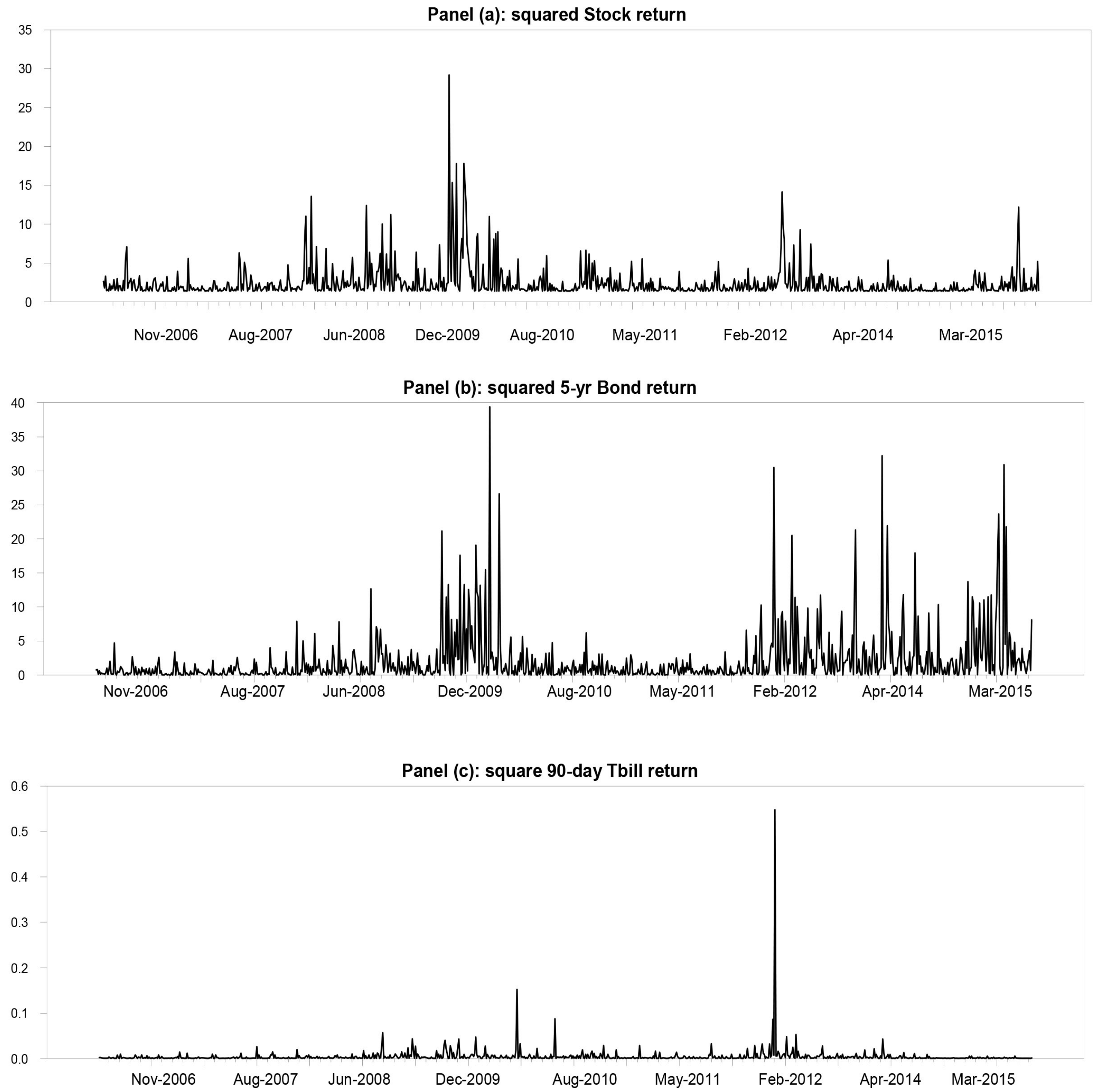

Figure 2. Time plot of the squared return series.

reported, can be obtained from the author), some significant negative and positive cross-correlation exists among the variables at different lags. The series are further tested for unit root nonstationary by augmented Dickey-Fuller, Phillip-Perron, and KPSS tests. The test results are provided below.

All of the tests results indicate that the series are not unit root processes. The test results of Table 1 and Table 2 reveal that we jointly model the observed facts of the first and second moments of the data generating process to investigate dependence structure of the variables within the multivariate framework, which is discussed below.

Table 2. Stationarity/non stationarity tests of the return series from 4 April 2006 to 20 June 2016.

Return Series	ADF test with lag = 5	PP test with lag = 5	KPSS test with lag = 5
Stock	−11.783***	−28.572***	0.138
Bond	−10.953***	−31.442***	0.456
T-bill	−12.277***	−30.040***	0.257

***Significant at 1% level. Note: The Null hypothesis for KPSS is stationary while ADF and PP tests the null hypothesis of non-stationarity.

4. Estimation of the Model

In this section we report the QMLE results of the VAR(1)-DBEKK(1,1) model of Australia's Stock, Bond, and Money markets. We apply the AIC, BIC, and HQ criteria to select the order of the vector autoregression (VAR) of the mean model. We select order 1 for VAR because among the three criteria both BIC and HQ select VAR of order 1. In the univariate case, there was overwhelming support to GARCH (1,1) order volatility model, Bollerslev [4]. Considering these empirical facts, we thus proceed to fit jointly a VAR (1)-DBEKK(1,1) model. The estimated model is reported in Table 3.

In Table 3, the conditional mean of the Stock equation shows significant lagged effects of the Bond and T-bill variables. While the Bond equation shows significant intercept and the lagged T-bill variable. For the T-bill equation only lagged Bond is significant. The results show that there a bi-directional causality exist between Bond and T-bill. The Stock does not explain its own lag.

The estimated variance-covariance (VCV) matrix of the DBEKK are reported in Table 4. The estimates of the diagonal elements of the matrix A are quite similar for a_{22} and a_{33}, however the element a_{11} is very low in compared with a_{22} and a_{33} the main diagonal elements b_{11}, b_{22}, and b_{33} of the B matrix are fairly similar and higher than the corresponding $a_{ii}, i = 1,2,3$, which is usually as usually found in univariate GARCH conditional volatility models. These are the short and long run volatility weights. The multivariate statistics diagnostics and the hypothesis tests, are provided below.

The Ljung-Box (LB) test results reported in Table 5, fails to suggest any model inadequacy of serial dependence of the errors of theVAR-DBEKK model. Nyblom stability test [37] indicates no overall parameter instability at the 5% level of significance. The AIC and BIC has no clear evidence of model selection of DBEKK. In this paper, we tried to find out a way to provide an adequate model for multivariate conditional volatility.

4.1. Granger Causality and Return Spillover Effects of Stock, Bond, and T-bill of the Conditional Mean Model

In this section we conduct the return spillover test utilizing The Granger Causality tests based on the F-statistics as provided below.

The test results in Table 6 suggests that there are significant return spillovers running from Bond and T-bill to Stock returns in Australia's domestic assets

Table 3. The VAR(1) mean model for Stock, Bond and Money markets.

	Φ_0	Φ		
	Constant	Stock_{t-1}	Bond_{t-1}	T-bill_{t-1}
DBEKK Mean model				
Stock_t	0.0329 (0.0246)	−0.0418 (0.0317)	0.0918*** (0.0232)	−2.0359*** (0.5325)
Bond_t	−0.0525** (0.0250)	−0.0113 (0.0223)	−0.0208 (0.0245)	0.8143* (0.4747)
T-bill_t	0.0003 (0.0011)	−0.0002 (0.0009)	0.0019** (0.0009)	0.0256 (0.0258)

Note 1: standard error is in parenthesis. Note 2: "*" denote significant at 10% level, "**" denote significant at 5% level and "***" denote significant at 1% level. Note 3: $\Phi_0 = \begin{pmatrix} \phi_{10} \\ \phi_{20} \\ \phi_{30} \end{pmatrix}$ $\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix}$.

Table 4. DBEKK conditional volatility model for Stock, Bond and Money markets.

DBEKK

$$C = \begin{pmatrix} 0.1873^{***} & 0 & 0 \\ (0.0313) & & \\ 0.0354^{***} & 0.1307^{***} & 0 \\ (0.0122) & (0.0129) & \\ -0.0009^{*} & -0.0070^{***} & -0.0039^{***} \\ (0.0005) & (0.0005) & (0.0005) \end{pmatrix}$$

$$A = \begin{pmatrix} -0.0511 & 0 & 0 \\ (0.0459) & & \\ & 0.2778^{***} & 0 \\ & (0.0095) & \\ & & 0.2377^{***} \\ & & (0.0069) \end{pmatrix}$$

$$B = \begin{pmatrix} 0.9449^{***} & 0 & 0 \\ (0.0128) & & \\ & 0.9599^{***} & 0 \\ & (0.0020) & \\ & & 0.9589^{***} \\ & & (0.0014) \end{pmatrix}$$

Note: standard error is in parenthesis. "*" denote significant at 10% level, "**" denote significant at 5% level and "***" denote significant at 1% level.

Table 5. Multivariate statistics diagnostics.

VAR-DBEKK
$\text{LB-Q}(10) = 94.5567(0.3506)$
$\text{LB-Q}^2(10) = 109.0536(0.0838)$
$\text{loglikelihood} = -952.6214$
$\text{Nyblom} = 5.8497(0.08)$
$\text{shape} = 5.7486$
$\text{AIC} = 2.2236$
$\text{BIC} = 2.3754$

Table 6. Return Spillovers and Granger causality test in the mean-model.

Mean Model	Granger causality		F-statistic
	From	To	
VAR-DBEKK	Bond and T-bill	Stock	34.750*** (0.0000)
	Stock and T-bill	Bond	0.2.306 (0.100)
	Stock and Bond	T-bill	1.428 (0.2397)

Note 1: p-value is in parentheses. Note 2: "***" indicate 1% significance, "**" indicate 5% significance level, and "*" indicate 10% significance level.

markets However, there is no significant causality running from Stock and T-bill to Bond, and, from Stock and Bond to T-bill in the DBEKK model. This observation suggests that Australia's domestic asset markets are interlinked and partially transmitting return shocks across domestic asset markets with some reservations. This information is useful in planning for future investment decisions both by individuals and financial institutions to minimize risk.

4.2. Conditional Volatility Models

In **Table 4**, we have reported the DBEKK volatility modeling. This model is useful for volatility predictions and decision making purpose by the investor and institutions in the multiple financial asset markets. Because this is the only multivariate volatility model which has the sound statistical properties for feasible statistical inference based on the QMLE of the model parameters, see for example McAleer *et al.* [2], McAleer [30], Chang *et al.* [26], Chung *et al.* [27]. We will investigate the volatility spillover effects in the DBEKK model. From our empirical results we observe that the GARCH weights (see the diagonal of the B matrix) of the DBEKK models are larger than the ARCH weights (see the A matrix). The similar observation usually found in the univariate GARCH (1,1) models, Bollerslev [4]. The DBEKK structure of volatility enjoy the asymptotic properties of QMLE and is applicable for developing spillover tests. In the context of DBEKK the co-volatility spill overs can be tested by the approach stated in Chang *et al.* [27]. Following Chang *et al.* [26] and Chang *et al.* [27] we apply the definitions of the volatility spillovers and compute partial co-volatility and Full co-volatility spillovers bellow.

4.3. Partial Co-Volatility Spillovers

Following Chang *et al.* [26] and Chang *et al.* [27], we report the co-volatility spillovers with DBEKK model as follows.

Partial co-volatility spillovers with DBEKK

$\frac{\partial H_{\text{stock,bond},t}}{\partial \varepsilon_{\text{stock},t-1}} = a_{11}a_{22}\varepsilon_{\text{stock},t-1} = 0.00078$ Stock shock negatively spillovers to average co-volatility of Stock and Bond.

$\frac{\partial H_{\text{stock,bond},t}}{\partial \varepsilon_{\text{bond},t-1}} = a_{11}a_{22}\varepsilon_{\text{bond},t-1} = 0.00074$ Bond shock negatively spillovers to average co-volatility of Stock and Bond.

$\frac{\partial H_{\text{stock,Tbill},t}}{\partial \varepsilon_{\text{stock},t-1}} = a_{11}a_{33}\varepsilon_{\text{stock},t-1} = 0.00067$ Stock shock negatively spillovers to average co-volatility of Stock and T-bill.

$\frac{\partial H_{\text{stock,Tbill},t}}{\partial \varepsilon_{\text{Tbill},t-1}} = a_{11}a_{33}\varepsilon_{\text{Tbill},t-1} = -0.00003$ T-bill shock positively spillovers to average co-volatility of Stock and T-bill.

$\frac{\partial H_{\text{bond,Tbill},t}}{\partial \varepsilon_{\text{bond},t-1}} = a_{11}a_{33}\varepsilon_{\text{bond},t-1} = 0.00063$ Bond shock negatively spillovers to average co-volatility of T-bill and Bond.

$\frac{\partial H_{\text{bond,Tbill},t}}{\partial \varepsilon_{\text{Tbill},t-1}} = a_{11}a_{33}\varepsilon_{\text{tbill},t-1} = 0.00016$ T-bill shock positively spillovers to average co-volatility of Bond and T-bill.

The negative sign effect of shock is an important indicator for tradeoff and asymmetry. The above partial co-volatility spillover is calculated at the average shock.

4.4. Pattern of Change in Predicted Volatility and Correlations

The estimated model satisfies most of the desirable properties, namely model adequacy, parameter consistency, volatility clustering and asymmetric effects. As mentioned before, a good forecast model must capture all stylized facts of the data. In this regard, VAR-DBEKK model can be used for modelling and predicting volatility and correlation of return volatilities. Note that the only multivariate model qualifies for volatility spillover test is the DBEKK model. The graph displays time plot of the predicted time varying volatility and correlations.

Figure 3 shows in-sample and out-of-sample predicted volatilities and correlations of volatilities between assets. The main diagonal of Figure 3 display the predicted volatility and the off-diagonal graphs display the predicted correlations of volatilities. The predicted volatilities of Bond, Stock and T-bill exhibit changes of the pattern of movement over time. The out-of-sample volatility prediction of each of the security is tranquil. This could be the recovery of the GFC. The prediction of correlation of volatility between Stock and Bond, and T-bill and Stock are both positive in the 100-step-ahead prediction. But a mix of both negative and positive during in-sample prediction. However, both the in-sample and out of sample prediction of correlation of volatility between T-bill and Bond are negative. This carries useful information about the asset markets interaction and trade-off, which is consistent with our previous findings. The volatility prediction is monotonically decreasing in all cases after 2011. The out-of-sample prediction is tranquil for T-bill but the Stock and Bond price volatility continues to fall. The overall predicted Bond return is more volatile than the predicted Stock returns during 2011. There were some tranquil periods both in Bond and T-bill volatility predictions during the mid-2007 and a severe peak in all of the securities' volatility during 2011-2012. All those are the European financial crisis periods.

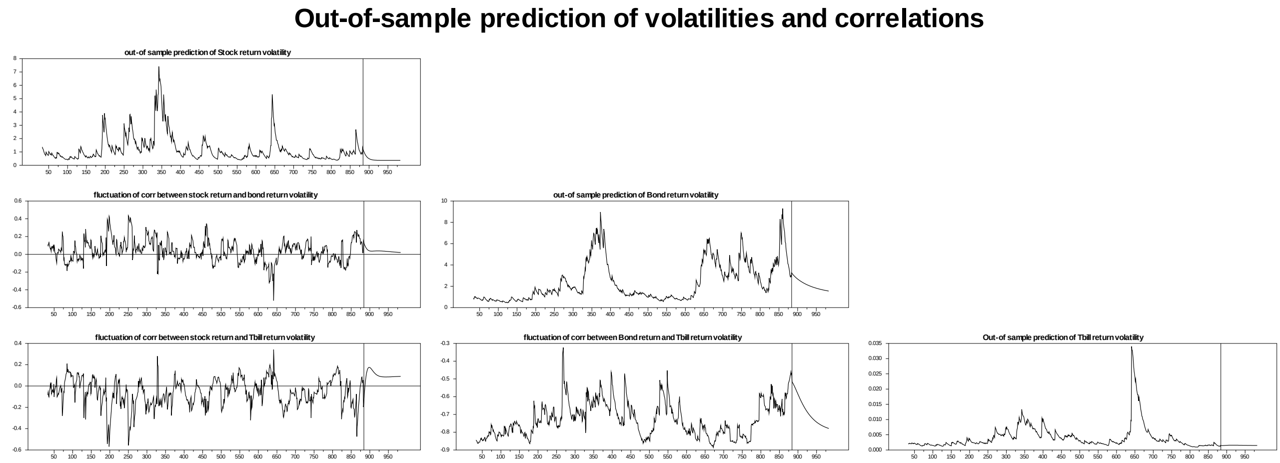

Figure 3. Predicted volatility and correlations of Stock, Bond, and T-bill in DBEKK model.

5. Conclusions

In this paper we investigate the impact of return and volatility of return spillovers in the multiple asset markets using VAR-DBEKK model. We also investigated the causality of Stock, Bond and T-bill in the conditional mean model. To our knowledge, application of the DBEKK in Australia's domestic Stock, Bond, and T-bill markets jointly is the first. The results of this paper show that Australia's domestic asset markets are interdependent in general, although there are some variations. The Granger causality test suggests that there are significant return spillovers running from Bond and T-bill to Stock in Australia's domestic assets markets. However, there is no significant causality running from Stock and T-bill to Bond, and, from Stock and Bond to T-bill.

Significant return spillovers from Bond and T-bill markets to Stock markets found in the DBEKK, model by the Granger causality tests. It is found that the causality running from T-bill to Bond and vice versa, implying bi-directional causality exists between Bond and T-bill in the DBEKK model. Time plot of the daily log returns highlighted that the domestic Bond market is affected by the global financial crises (GFC), while T-bill is least affected as T-bill is more liquid than the Bond market. We also found negative partial co-volatility spillovers of Stock and Bond with the DBEKK. The negative correlation between T-bill and Bond returns volatility indicates that there is a tradeoff between Bond and T-bill markets. This information is useful and vital for asset management and portfolio diversification strategies. Stock and Bond volatility correlations is a mix of both positive and negative but with some noticeable negative correlation is reported between these two assets during 2011 and 2012. Volatility correlations between asset returns are important for policy makers' asset allocation through diversification during trading under uncertainty. In general the DBEKK model adequately fits the data by the LB and the Nyblom tests. The short and the long run weight parameters are found to be significant with some reservation. The dynamic interactions among assets simultaneously affect investor's expectation of trading securities in Australia's domestic financial markets. The approach of this paper can be extended to investigate spatial dependence of co-volatility & correlation spillovers across countries and, the asymmetric effects of news for modelling and predicting returns and volatilities simultaneously in the international financial markets for global investment policy decision purposes.

Acknowledgements

The authors are most grateful to the Editor-in-Chief and Editorial Adviser of the Theoretical Economics Letters for valuable comments and suggestion provided to me in finalizing the article.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] McAleer, M., Hoti, S. and Chan, F. (2009) Structure and Asymptotic Theory for Multivariate Asymmetric Conditional Volatility. *Econometric Reviews*, **28**, 422-440. https://doi.org/10.1080/07474930802467217

[2] Black, F. (1976) Studies of Stock Price Volatility Changes. In: *Proceedings of the* 1976 *Meeting of the Business and Economic Statistics*, American Statistical Association, Washington DC, 177-181.

[3] Engle, R.F. (1982) Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, **50**, 987-1007. https://doi.org/10.2307/1912773

[4] Bollerslev, T. (1986) Generalized Autoregressive Conditional Hetroskedasticity. *Journal of Econometrics*, **31**, 307-327. https://doi.org/10.1016/0304-4076(86)90063-1

[5] Tsay, R.S. (1987) Conditional Heteroscedastic Time Series Models. *Journal of the American Statistical Association*, **82**, 590-604. https://doi.org/10.1080/01621459.1987.10478471

[6] McAleer, M. (2019) What They Did Not Tell You about Algebraic (Non-)Existence, Mathematical (IR-)Regularity and (Non-)Asymptotic Properties of the Dynamic Conditional Correlation (DCC) Model. *Journal of Risk and Financial Management*, **12**, 61. https://doi.org/10.3390/jrfm12020061

[7] Enders, W. (2014) Applied Time Series. John Wiley and Sons, Hoboken.

[8] French, K.R., Schwert, G.W. and Stambaugh, R.F. (1987) Expected Stock Returns and Volatility. *Journal of Financial Economics*, **19**, 3-29. https://doi.org/10.1016/0304-405X(87)90026-2

[9] Nelson, D.B. (1991) Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, **59**, 347-370. https://doi.org/10.2307/2938260

[10] Engle, R.F. and Ng, V.K. (1993) Measuring and Testing the Impact of News on Volatility. *The Journal of Finance*, **48**, 1749-1778. https://doi.org/10.1111/j.1540-6261.1993.tb05127.x

[11] Glosten, L.R., Jagannathan, R. and Rukle, D.E. (1993) On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*, **48**, 1779-1801.

https://doi.org/10.1111/j.1540-6261.1993.tb05128.x

[12] McAleer, M. (2014) Asymmetry and Leverage in Conditional Volatility Models. *Econometrics*, **2**, 145-150. https://doi.org/10.3390/econometrics2030145

[13] Engle, R.F. and Granger, C.W. (1987) Co-Integration and Error Correction: Representation, Estimation, and Testing. *Econometrica*, **55**, 251-276. https://doi.org/10.2307/1913236

[14] Bollerslev, T. (1990) Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model. *The Review of Economics and Statistics*, **72**, 498-505. https://doi.org/10.2307/2109358

[15] Bera, A.K. and Jarque, C.M. (1980) Efficient Tests for Normality, Homoscedasticity and Serial Independence of Regression Residuals. *Economics Letters*, **6**, 255-259. https://doi.org/10.1016/0165-1765(80)90024-5

[16] Bollerslev, T., Engle, R.F. and Wooldridge, J.M. (1988) A Capital Asset Pricing Model with Time-Varying Covariances. *Journal of Political Economy*, **96**, 116-131. https://doi.org/10.1086/261527

[17] Engle, R.F. and Kroner, K.F. (1995) Multivariate Simultaneous Generalized ARCH. *Econometric Theory*, **11**, 122-150. https://doi.org/10.1017/S0266466600009063

[18] Tsuji, C. (2017) How Can We Interpret the Estimates of the Full BEKK Model with Asymmetry? The Case of French and German Stock Returns. *Business and Economic Research*, **7**, 342-351. https://doi.org/10.5296/ber.v7i2.12071

[19] Gounopoulos, D., Molyneux, P., Staikouras, K.S., Wilson, O.S.J. and Zhao, G. (2013) Exchange Rate Risk and the Equity Performance of Financial Intermediaries. *International Review of Financial Analysis*, **29**, 271-282. https://doi.org/10.1016/j.irfa.2012.04.001

[20] Long, L., Tsui, A.K. and Zhang, Z. (2014) Estimating Time-Varying Currency Betas with Contagion: New Evidence from Developed and Emerging Financial Markets. *Japan and the World Economy*, **30**, 10-24. https://doi.org/10.1016/j.japwor.2014.02.001

[21] Caporale, G.M., Ali, F.M. and Spagnolo, N. (2015) Exchange Rate Uncertainty and International Portfolio Flows: A Multivariate GARCH-in-Mean Approach. *Journal of International Money and Finance*, **54**, 70-92. https://doi.org/10.1016/j.jimonfin.2015.02.020

[22] Olson, E., Vivian, A. and Wohar, E.M. (2017) Do Commodities Make Effective Hedges for Equity Investors? *Research in International Business and Finance*, **42**, 1274-1288. https://doi.org/10.1016/j.ribaf.2017.07.064

[23] Cardon, L., Gutiérrez, M. and Agudelo, A.D. (2017) Volatility Transmission between US and Latin American Stock Markets: Testing the Decoupling Hypothesis. *Research in International Business and Finance*, **39**, 115-127. https://doi.org/10.1016/j.ribaf.2016.07.008

[24] McAleer, M., Chan, F., Hoti, S. and Lieberman, O. (2008) Generalized Autoregressive Conditional Correlation. *Econometric Theory*, **24**, 1554-1583. https://doi.org/10.1017/S0266466608080614

[25] Ling, S. and McAleer, M. (2003) On Adaptive Estimation in Nonstationary ARMA Models with GARCH Errors. *The Annals of Statistics*, **31**, 642-674. https://doi.org/10.1214/aos/1051027884

[26] Chang, C.-L., McAleer, M. and Wang, Y.-A. (2018) Modelling Volatility Spillovers for Bio-Ethanol, Sugarcane and Corn Spot and Futures Prices. *Renewable and Sustainable Energy Reviews*, **81**, 1002-1018. https://doi.org/10.1016/j.rser.2017.07.024

[27] Chang, C.-L., McAleer, M. and Zuo, G. (2017) Volatility Spillovers and Causality of Carbon Emissions, Oil and Coal Spot and Futures for the EU and USA. *Sustainability*, **9**, 1789. https://doi.org/10.3390/su9101789

[28] Chang, C.-L., Li, Y. and McAleer, M. (2018) Volatility Spillovers between Energy and Agricultural Markets: A Critical Appraisal of Theory and Practice. *Energies*, **11**, 1595. https://doi.org/10.3390/en11061595

[29] Allen, D.E. and McAleer, M. (2018) Theoretical and Empirical Differences between Diagonal and Full BEKK for Risk Management. *Energies*, **11**, 1627. https://doi.org/10.3390/en11071627

[30] McAleer, M. (2019) What They Did Not Tell You about Algebraic (Non-)Existence, Mathematical (IR-)Regularity and (Non-)Asymptotic Properties of the Full BEKK Dynamic Conditional Covariance Model. *Journal of Risk and Financial Management*, **12**, 66. https://doi.org/10.3390/jrfm12020066

[31] Engle, R. (2002) Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. *Journal of Business & Economic statistics*, **20**, 339-350. https://doi.org/10.1198/073500102288618487

[32] Tsui, Y.K. and C.A.K. (2012) A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-Varying Correlations. *Journal of Business & Economic Statistics*, **20**, 351-362. https://doi.org/10.1198/073500102288618496

[33] Ljung, G.M. and Box, G.E. (1978) On a Measure of Lack of Fit in Time Series Models. *Biometrika*, **65**, 297-303. https://doi.org/10.1093/biomet/65.2.297

[34] Tsay, R.S. (1986) Time Series Model Specification in the Presence of Outliers. *Journal of the American Statistical Association*, **81**, 132-141. https://doi.org/10.1080/01621459.1986.10478250

[35] Mcleod, I. and Li, W.K. (1983) Diagnostic Checking ARMA Time Series Models Using Squared Residual Autocorrelations. *Journal of Time Series Analysis*, **4**, 269-273. https://doi.org/10.1111/j.1467-9892.1983.tb00373.x

[36] Jarque, C.M. and Bera, A.K. (1987) A Test for Normality of Observations and Regression Residuals. *International Statistical Review*, **55**, 163-172. https://doi.org/10.2307/1403192

[37] Nyblom, J. (1989) Testing for the Constancy of Parameters over Time. *Journal of the American Statistical Association*, **84**, 223-230. https://doi.org/10.1080/01621459.1989.10478759